COLLABORATIVE APPROACHES FOR TEACHING AND LEARNING GEOMETRY

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A Dissertation

Submitted to

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DECLARATION

To the best of my knowledge and belief this thesis contains no material previously published by any other authors/scholars except where due acknowledgement has been made. This thesis contains no materials which have been accepted for the award of any other degree or diploma in any university.

_____ 24 May, 2022

Dilip Kumar Shrestha

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DEDICATION

This work is dedicated to my late mother Nani Maya Shrestha, my father Nandlal Shrestha, my life partner Parbati Dangol and daughter Rhea Shrestha to whom I am always indebted. Their inspiration, expectation and trust in me turned on me to be magnetic energy to achieve this academic height.

Master of Philosophy in Mathematics Education of *Dilip Kumar Shrestha* entitled: *Collaborative Approaches For Teaching And Learning Geometry* was presented on 24 May, 2022.

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I understand and agree that my dissertation will become a part of the permanent collection of the Kathmandu University Library. My signature below authorized the release of my dissertation to any reader upon request for scholarly purposes.

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ABSTRACT

An abstract of the dissertation of *Dilip Kumar Shrestha* for the degree of *Master of Philosophy in Mathematics Education* presented at Kathmandu University, School of Education on 24 May, 2022.

Title: *Collaborative Approaches For Teaching And Learning Geometry*

Abstract Approved

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Prof. Bal Chandra Luitel, PhD — Dissertation Supervisor

Asst. Prof. Binod Prasad Pant — Dissertation Supervisor

As a mathematics teacher at the secondary level, I have been facing various problems in my teaching profession. In my teaching experience, many students showed disinterest and low satisfaction with mathematics learning. They were studying mathematics because it was a compulsory subject at school. In this sense, most of the students tried to memorize various mathematics problem solving and concepts. I tried such practices in my teaching as well. But, very few students were successful in the learning process. The result of such a scenario would lead the majority of the students as poor achievers in mathematics subjects. The major causes of such problems occurred from our daily teaching and learning strategies that would focus on less engagement of students, hesitation in participation in classroom tasks and group discussion.

In that scenario, I have planned to solve the problems of my students who were facing such problems in their mathematics learning. It was the result of observation of changes in the perception of students in mathematics learning through collaborative teaching and learning approach which was guided by two perceptions;

constructivism theory and collaborative learning approach under the paradigm of interpretivism. In the classroom implementation, I made a complete action plan for 18 days. I developed the plans including mathematical contents such as a similar and congruent triangle, relation between the area of parallelograms and triangles, geometry circle, etc. Thereafter, I applied such action plans in Grade-X of a private school in Kathmandu valley through an action research method. The research embraced collaborative teaching and learning approach as an appropriate pedagogy for engaged learning. In the meantime of implementing an eighteen days action plan, I collected students' perceptions in mathematics learning for a couple of weeks, taking interviews with the four key participants, maintaining daily field notes and writing my own reflection.

The collaborative learning approach is focused on students' engagement in the learning process to provide equal opportunities for sharing their knowledge and understanding. Therefore, this study highlighted how the collaborative approach helped my students to understand geometrical concepts in a meaningful way. I conclude that collaborative approaches help the students to challenge their underlying beliefs about mathematics as a difficult subject and make them more responsible and accountable for what they discussed during their learning process. Moreover, the results showed that the collaborative approaches are more helpful to make the active engagement of students through pair/group discussion, questioning, concept mapping and shared knowledge.

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ABBREVIATIONS

B.Ed	:	Bachelor of Education
CDC	:	Curriculum Development Center
CSC	:	Computer Supporting Collaboration
ERO	:	Education Review Office
ICT	:	Information and Communication Technology
KUSOED	:	Kathmandu University School of Education
M.Ed	:	Master of Education
MPhil	:	Master of Philosophy
SEE	:	Secondary Education Examination
SLC	:	School Leaving Certificate

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CHAPTER I

INTRODUCTION

In this chapter, I present the research agenda affirming the conception of my research. I began with a contextual incident that occurred in my teaching and learning career. I have focused on a particular challenge that I faced in my teaching geometry at the secondary level. While doing so, I have applied emerging learning theories and my past experiences of teaching and learning. I have discussed why this research is useful and how the problem arises in the real scenario. Along with these, I discuss why collaborative teaching and learning are helpful and necessary in the teaching field. This chapter is the foundation of the research agenda through which I got motivated to work.

Background of Study

There were different values in my mind as a teacher and educational reader. It was my relatively bitter experiences that I faced several problems in geometry teaching as a novice teacher. I am extremely influenced by the ideas of Lerman (2000) who argued that mathematical meaning, thinking, and reasoning are the consequences of social activities and interactions among social beings. It is the knowledge that can be acquired through collaborative discussion and sharing knowledge based on prior experiences and works. Upon my teaching practice and experience of student's life, I realized that the teacher- centered method of teaching and learning is not sufficient for mathematics teaching. Indeed, our mathematics curriculum and pedagogy largely emphasized that the content introduction through teacher-centric rather than student-centric activities. In this regard, students consider mathematics a difficult subject and show little interest in learning. When I chose

collaborative teaching and learning approaches, it underlined the collaborative work, group discussion and shared knowledge of students to learn mathematics concepts in meaningful ways. I have briefly elaborated my teaching and learning experiences *as a student* and *as a teacher* below;

My Learning Experiences: As a Student

As I talk about my school life, I had completed my school life (Grade I to X) from a community school situated in the Western part of the Dhading district. As I recall my past schooling, I had numerous uncertainties, bitter experiences and various queries in mathematics learning. It could be any day in September 2001; I was a student in Grade VIII. When my mathematics teacher entered the classroom, he started to say, '*Today we are going to start a new topic that is 'Geometric Triangle*'. He continued, " *Here we are going to learn about two concepts: first is verification experimentally proof and second is theoretically proof."* He wanted everyone to understand the topic well, but the students were not able to move forward as per his wish. He added, *"The verification experimentally proof can be proven by using geometrical instruments like scale, protector, and compass and theoretical proof can be proven by using axioms and postulates. I will give you some related problems and practice questions for you to understand it more clearly".* When I reflect on those moments, it seems that he had a fixed plan for his teaching. He was to complete the chapter and control the classroom. He had been teaching mathematics for a decade. Further, he used to fill the knowledge into our *empty minds*. I had too many misperceptions concerning the process of his teaching and learning strategies. Eventually, my friend dared to ask a question related to the problem-solving of this chapter. After finishing his question, the aggressive teacher said, "*You don't know even these simple things?" This procedure you already learned in the previous Grade.*

Where was your mind when I was teaching such concepts in previous classes? The teacher shouted to us, *pay attention, otherwise*……. ". Being a passive listener, I copied solutions from the board. He added some more examples from the exercise. We just soundlessly observed him. After his explanation, he started to do the solution on the board by speaking aloud the lines and explaining the process. As we finished, sir said, "*Now you all go to the exercise part and start to do question number XXX. These are similar type as an example.*" The whole class was silent. Most of my friends, including me, could not answer those questions.

This was one of the situations in my schooling where my teachers taught in class. My teacher used a teacher-centered teaching approach that was too beneficial for rote memorization and routine problem solver. But, I would look for conceptual understanding of mathematics concepts rather than content memorization. Such types of incidents were commonly repeated in my school life. As I remember that situation, the teacher used to make a very minor space for group discussion and less encouragement to idea- sharing. There was almost no space for students to share hidden abilities and understanding. Numerous times, I used to share my difficulties with my colleague in the learning process. I would ask for their help in problem-solving. I used to get common answers from those friends as – "*Do rigorous practice until and unless the problem-solving is not fully memorized, mathematics is such a type of subject which is a collection of formulae. So, if you are capable of memorizing them comprehensively, then that will be enough!*" So, I had changed my mindset that mathematics was a subject that would demand rote memorization and precise practice. As the level was increasing, my bitterness toward mathematics learning was also growing up. From Grade-IX, we had another mathematics teacher. I had heard from my senior brothers and sisters that his ways of mathematics teaching were different.

His teaching was based on more practical ways. It would motivate every student to actively participate in the learning process. So, I was very much curious as well as a little bit afraid of attaining his classes due to my deep-seated belief in mathematics learning as rote memorization and algorithmic problem solving that was influenced by my previous teacher. But I wanted to get new experience in his teaching during the very first class.

I still remember the period when my teacher taught us "*properties of triangles*" in Grade- IX. It was a revised course, the same as Grade-VIII. He entered the classroom carrying a bundle of paper and we were curious about that. He stated, "*While learning the properties of triangles, you must know about verification experiments of each triangle*". After discussion, he distributed each of us a piece of blank sheet. He said, "*Everyone please listen to me, today we are going to demonstrate 'that the sum of the interior angle of a triangle equals to* 180^0 *by using a paper cutting method. So, all of you make a big triangle on the blank paper with the help of a scale and pencil*". Even though this triangle's property was already learned in Grade- VIII, we were more curious about his teaching styles because we hadn't got such a practical learning environment in previous Grades. Now, my teacher was going to demonstrate such a property of triangles with the involvement of students. That was my first experience engaging in the learning process without hesitation. We were very excited that it was our first mathematical activity in the mathematics class besides writing and doing class work. One of my friends asked "*Sir, how big is the triangle? Is there any measurement of sides"* He replied, "*First, have a discussion with your friends and make a triangle as big as you want*?" The teacher gave us authority in our decision. We made a big triangle with different measures on three sides. While we were involved in that activity, he was also joining us by helping to construct a

triangle. He continued,*" Now, cut the angles continuously and mark the different arcs*. He added, *keep in mind, the marking arc length of each angle might be the same and cut above the marking angle of each angle.* We did as we were taught. There was a free choice about taking the arc length of each angle. We were able to do that procedure and cut three different angles. Then, we attached these three different cutting pieces. We were shocked because it made a *semi-circle.* He gave us a few minutes to discuss it. All my friends were busy to attached three arcs together. Meanwhile one of the friends said, *"Sir it is a semicircle, while we measure the angles it is* 180^0*."* Some others added *"After attaching all three angles, it has a common straight line whose angular measurement is* 180^0". Our teacher said, "*Yes, you are correct.* And he added *"the sum of all interior angles is* 180^0*"*. That was the first day someone appreciated our work.

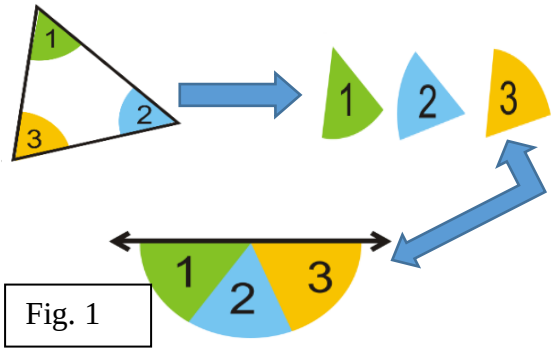

Fig. 1

Similarly, sir taught us other properties of triangles, similar triangles, congruent triangles and properties of quadrilaterals in the same way. I still remember those happy moments. This is one of the events which hit in my mind, how effective the class would be conducted if the mathematics content were learned by various activities with discussion. It could be more interesting if a teacher played a vital role as a facilitator instead of the instructor in the classroom. We did such types of activities in the other few chapters as well. When we reached Grade X, the scenario became worse because it was the impulse of "*Iron gate*" SLC (School Leaving Certificate, now Grade-X is considered as SEE). So, the major target of teachers' was to complete the chapters in time and go for rigorous practice. During my SLC time, I had one mindset that mathematics was just a combination of numbers and formulae,

and it deals with right and wrong answers with no gray area in between (Lewis, 2011). I still remember that moment when I had practiced those mathematical problems so many times and I knew their final answers before writing their answers. Then I attended the SLC examination and got a good result in mathematics. After my SLC, I got enrolled in the *Humanities stream* with major subjects such as mathematics and social science. At that level, my thinking became more deep-rooted to conventional learning because the condition was even worse regarding much precise memorization without understanding. As with my schooling, I still believed in rote memorization and rigorous practice to get more marks in mathematics. Doing so, I was upgraded to another level. I implemented such practice in my learning. I got good marks in mathematics as well as other subjects. It upgraded me to undergraduate level. After completing my intermediate level, I got enrolled in an institutional college for my undergraduate education in the education stream with a major subject as mathematics. But once again the scenario was similar to my intermediate level. Teachers were focused on content-based teaching strategies and contents were relatively more complicated than intermediate level. The target of teachers was completing the course. Intentionally, students should be involved in rote memorization and that was mostly concerned with exam orientated. There was also the enthusiastic struggle to get the concept of projective geometry, real analysis, higher algebra and some others.

In my learning experiences, I got a few opportunities to learn mathematics through different approaches such as the activity-based teaching approach, group discussion and some others. Mostly I found the *chalk and talk* method, rather than the modern teaching and learning approach. I have experienced that the teaching was largely concerned with course completion and thorough practice. Conventional

practices were directly proportional to my learning experience in different levels of mathematics learning. Hence, such practices directly or indirectly shaped my mindset about teaching and learning mathematics in the present situation. I started to consider somehow more strongly that the mathematics teaching and learning process was the same at almost all levels.

My Teaching Experiences: As a Teacher

It could be any day in October 2006. I had finished my board examination of Bachelor's second year. My friend offered me to join a private school to teach mathematics in middle school. After doing certain job recruitment procedures, I got an opportunity to teach mathematics as a middle school mathematics teacher. I was a novice teacher but I had some ideas of teaching and learning. I had gained some ideas of teaching from my teachers who had taught me at various levels. So, I tried to recap all of those teaching styles of my teachers. I chose their teaching strategies, blended them and prepared planning myself. I had a mindset that students could not get the mathematical concept unless teachers don't explain them adequately and they do not practice enough. So, I believed that precise description with a detailed solving process was obligatory to make students good at mathematics (Rashidov, 2020). In mathematics teaching, we introduced mathematics content through integrating data in the form of images, figures, and texts as problem-solving from numeric to schematic for improvement of students' performance. I was not an in-service teacher even though I was a pre-service teacher. I had some ideas and skills on how I could prepare lesson plans and how to deliver content among the students. That was my first teaching experience as a teacher in the classroom. When I entered Grade VIII, I was very afraid to introduce myself, but I tried. There were huge numbers of students. I was slightly nervous. I asked the students, *"Which chapter is going on?"* I heard that

some chapters had been completed by their previous mathematics teacher. That was the middle year of the academic session. In a loud voice students said," *HCF and LCM of algebraic expression*". I remembered my mathematics teacher when I was in Grade VIII. I began to imitate him. He was a very authoritarian and algorithmic problem solver (Nurkaeti, 2018). As Pant (2017) stated, it is more difficult to avoid deep-seated belief in algorithmic problem-solving in mathematics. It seems that students had their belief system on the process of some algorithmic methods of solving problems. I tried to explain the Highest Common Factor (HCF) and Lowest Common Multiple (LCM) together with examples on white board. Then I asked them to copy down the solution to the problem in their note copy from the board. Although they were copying, I added them to copy what I had written on board even including *dots, equals to* sign as well. As the class bell was going to ring, I turned the exercise book and asked them, "*Everybody, turn page number XXX. Do all the questions exactly in the same process as I taught*". Some were very happy because I gave them lots of examples rather than their previous teacher but the majority of them with sad faces said, "*We will try to do our best*". The bell rang and my class time was over. I left the class and headed towards another class according to the class schedule.

In that way, my teaching profession was going on. Whether my students liked that or not, I was very much confident about my teaching practice and even sometimes parents were appreciating me. Low-achieving students also tried to complete their tasks due to my presence. I believed in algorithmic processes in mathematical problem solving (Pant, 2017). I maintained a silent classroom for effective learning. In that sense, I looked for *pin-drop* silent types of classroom environments in my class and it was considered to be the best technique to control the students. This is the fact that, generally, teachers are active participants in teaching

and learning activities. In my professional experiences, school administrators and academic heads are less encouraged to interact, discuss, and engage in the learning process. They want a silent classroom. Students are expected to be passive listeners and a receiver. The teacher pours the knowledge into students' empty minds. In a mathematics classroom, teachers are usually found to be solving mathematics problems in an algorithmic process. In the Nepali context, students are less participating in the learning process because there is a less significant amount of group discussion and interactive environment in the classroom. The classroom environment and the real-life situation seem to be isolated because the mathematics classrooms do not deal with the basic concept (Maharjan, 2009). This shows that most of the problems done in the classroom are not based on real-life situations and students are submissively engaged in the classroom. In our context, teaching mathematics is based on the problems (Pant, 2017) that are prescribed by our textbook. Most of the teachers in the classroom start with a problem of the exercise and demonstrate the solution to the problem. Mathematics textbooks are designed for a dogmatic approach to repeat the same style of problems.

My teaching career was moving in the same way. I accomplished my bachelor's and master's degree too. So, soon after completing my master's degree, I was engaged in different private colleges as a mathematics lecturer for about five years. Within that teaching career, I met a lot of colleagues from different universities. As I came to know about Kathmandu University from my friends, it drew my attention more. I heard that the teaching and learning process of this university was more practical than other universities. I decided to get enrolled in Kathmandu University School of Education (KUSOED) for my MPhil degree in mathematics education. This became a turning point for my teaching and learning practice. I got

acquainted with different kinds of teaching and learning approaches including ICT-based teaching approach, collaborative teaching approach, project-based teaching approach, problem-solving teaching approach, group work and so many. I got various ideas to shift my deep-rooted lecture-based teaching approach to various new approaches in my teaching field. I had a strong belief that if the mathematical content were explained in a *pin-drop* silence, it could be appropriate for conceptual understanding. I might have that kind of thinking because of my past experiences in my school time where there used to be around 70 students in a classroom and teachers used to tell us to open a particular page number from our book and directly start writing the solution of the problems. So, we were supposed to copy the process of problem-solving from the board and go for precise practice whether we understood those steps or not. I had a few questions in my mind," *Is there any alternative way of learning mathematics? Is there any role for students while learning mathematics rather than just precise practice and rote memorization?"* If we just concentrate on mathematics teaching and learning, there are many issues regarding teaching methods. These are level of understanding, teacher's perspective, school and classroom environment and deep-seated perspective of parents regarding students' achievement. Due to this reason, rather than understanding the conceptual meaning and creating the mathematical knowledge, they seemed to be only focused on rigorous practice and rote memorization. The excessive portion of the Nepali classroom environment is under control by the teacher as an instructor. There is a decontextualized situation of real-world experience and mathematics problem solving for the students (Luitel, Pant & Shrestha, 2020). They are unable to find the connection between the things they are taught in the classroom setting and the real world of mathematical applications. As

Boaler (2000) stated, they could transfer the mathematical knowledge they had learned in school to their real-world mathematics situation.

These types of disconnected scenarios may doubt both teachers and students in the teaching and learning process. This situation could be handled by group work, shared knowledge, and collaborative work. Collaborative work helps the students for construction / reconstruction of their prior knowledge through their involvement. Collaborative learning approaches construct the knowledge through social negotiation, a less competitive manner among learners for recognition (Brown & Duguin, 2000). Mathematics teaching and learning in collaborative approaches emphasize working together in groups and encourage all group members to share their understanding through group discussion (Johnson, et. al., 2007). In that sense, the collaborative teaching and learning approach is a condition in which two or more people learn or attempt to learn something together. However, it invites students to share their understanding creatively and critically. The learning environment will be open and meaningful due to the direct involvement of learners in the learning process. It makes the classroom alive and engaging. This approach is two-way communicative learning. It helps to share the knowledge, authority and opportunities among students and teachers (Birgin & Acar, 2020).

Mathematics teachers normally understand collaboration as a group task for learners. But, collaborative learning is defined in various ways. It seems to me that it is a constructivist model of learning. While we work together and share ideas to accomplish the same goals, our role is socially represented. While we define our role we should have our responsibility and duty. Collaborative learning is student-centered rather than teacher centered. Student-centric learning is more effective rather than passive one where the role of teachers become instructors only (Horn, 2012). While

we focus on a collaborative approach in the teaching and learning process, teachers become a facilitator of students. Teacher wants to change ideas, make plans and solutions to achieve goals. Therefore, it is helpful for students for social and personal development. Collaborative teaching-learning is beneficial for teachers to reduce teacher talking time. It increases students' engagement in problem-solving. It seeks a better extent of coherent input, an easy classroom atmosphere and better enthusiasm for learning. Collaborative teaching and learning techniques provide the focal keys for teaching students to process skills that are needed to work successfully within a group. We need to construct knowledge that is constructed through social activities. Social interaction helps to facilitate the students to interact with peers at more advanced levels.

We can use different approaches in teaching mathematics if we challenge to embrace those new teaching approaches at school in the present society. It makes students' competitive in a global market. To fulfill this desire, we could switch the traditional methods of teaching mathematics with pioneering teaching strategies, which include collaborative discovery, inquiry learning, project-based learning and problem-solving methods. Whenever we facilitate this type of opportunity for students, teachers and students should interact with each other. Students should be providing ample chances to be involved in the teaching-learning process of mathematics. Teachers should think about creative manners such as an environment where students become more enthusiastic to interact or collaborate with the teacher as well as colleagues. This research was conducted, seeking the essence of collaborative teaching approaches in mathematics classes.

Problem Statement

Being a mathematics teacher, I have focused on a teacher-centered teaching and learning approach in my teaching. When the teachers are at the center of teaching, that situation mightn't enhance students' creativity in the teaching and learning process (Felder & Brent, 2010). In my experience, many Nepali teachers only tried to solve mathematics problems and allowed the students to remember the problem-solving process in rigid techniques. Indeed, I always raised one question "*why couldn't we use further methods of teaching for meaningful learning*?" There are several modern techniques for teaching and learning mathematics. But, many mathematics teachers believe that mathematics could be learned from precise practice and rote memorization from the dogmatic process. They believe *one size fit all* (Stoehr, 2017). I was also desperately influenced by such an approach in my educational career as being *a teacher* or as *a student*. So, I highlighted a teacher-centered teaching approach in my teaching.

As per ERO (2018), the performance level of students is at the underperforming level in mathematics achievement. Accordingly, ERO's study indicates that 32 out of 100 students fall below basic level in mathematics achievement. It seems that huge mass of students are underperforming in mathematics subjects. The report suggested that around 50% of students are struggling with the simple mathematical calculation. To improve mathematical learning achievement, an emphasis should be laid on the underperforming students instantly through the application of modern pedagogy. Teachers should provide the students with many opportunities to learn in many ways through various methods of teaching and learning. So, I decided to implement a collaborative teaching and learning approach in my classroom to solve the problems/issues that I faced in my teaching profession.

This approach could be more helpful to my teaching and learning process by switching from teacher-centered to students' centered. It emphasizes the student-centric teaching approach to avoid the negative concept of the students in mathematics and to promote meaningful learning. This approach encourages the students to encounter the questions through group discussion. Students get the opportunities to share their own knowledge among friend-circle. The role of a teacher is a facilitator who scaffolds the students in a difficult situation in the learning process.

In my experience, the mathematics curriculum is based on different values. According to Belbase, (2011), the mathematics curriculum of Nepal has been guided by a view of mathematics as a culture-free and pure body of knowledge. Nepali connotation of the curriculum is mainly as subject matter to study (Belbase, 2011). Mathematics subject experts have been extremely influenced by the contents of mathematics. Mathematics curricula have been extensively influenced by the context of perspectives and beliefs of mathematics subject experts. Mathematics curricula explore geometry as the concrete body of mathematics. It describes the objectivity, rigid evaluation system and teaching methods. In that scenario, the teaching and learning process should be transformed from teacher-centered to student-centric. The aim of mathematics teaching and learning should be making an individual acquire mathematical knowledge, where teaching is mostly influenced by how to solve the mathematical problems and acquire reasoning methods (Özerem, 2012). In addition, to acquire mathematical concepts one should be able to visualize the diagrams in real objects. So, teachers should facilitate the students to connect geometrical problems with their prior knowledge through group discussion, sharing knowledge and working together.

My students were good at algorithmic-problem solving with a competitive mindset but they were not good enough to work in groups, share ideas, and develop a common understanding. In that situation, students were more attentive to achieving more marks in mathematics subjects. This scenario could have created a trench between low achiever students and high achiever students. Implementing a collaborative learning approach could be helpful to the students where students get the opportunity to engage in the learning process and share knowledge amid their friend circle. Collaborative learning emphasizes working together to accomplish shared learning goals and working less competitively to achieve a goal (Laal & Ghodsi, 2011). When students are more engaged in the learning process through a collaborative manner, it might change the perception of students working in a group rather than in a competitive sense.

Purpose of the Study

The purpose of this study is to explore the collaborative teaching and learning approach in teaching mathematics. This research sees from the lens of pedagogical perspective to understand the concepts of Euclidean geometry. It enhances students' engagement in constructing knowledge with their lived experiences and practices to make meaningful learning. In addition, the study improved my practices of teaching geometry using collaborative approaches at the secondary level.

Research Question

How can I engage my learners in the meaning-making process while teaching and learning geometry through a collaborative approach?

Rationale of Study

The real understanding comes out when the learners practically get involved in activities and feel their own actions. Group work and pair work bring collaboration

between students. These approaches focus on interaction. This research may provide the cornerstone of the process of teaching and learning mathematics in the context of Nepal. This research study might be used to change self and other teachers who are concerned in the educational sector. However, reflecting on my past and present, knowingly or unknowingly, I have continuously experienced that collaborative teaching among students could be used for meaningful learning. As a qualitative research approach, this study figures out the students' perception, and meaningful learning through collaborative techniques in teaching mathematics. It also signifies the influences of the student's engagement in the learning process to promote their learning outcomes.

The insights from the study contribute to the teachers and students, particularly to the mathematics teachers and future researchers who are interested in researching the field of collaborative practices in the mathematics classroom. However, in general, it can help the mathematics teachers to adopt the modern techniques in teaching and learning. It will help them to make effective teaching and learning in a diversified classroom. Lai (2001) says that children appear to enter school with high levels of interest and expectation. However, collaborative practices tend to increase students' interest in learning mathematics with full passion as they progress through school. The findings of the research will help curriculum designers and policy-makers to understand how to implement a collaborative approach in the mathematics classroom.

This study motivates mathematics teachers to make their students engage and involve themselves in their various mathematical contents. It motivates students to learn concepts in an applicable manner rather than rote memorization and rigorous practice (Pant, 2017) with the actual understanding and meaning behind them. This

study will play a role to improve my educational practices as well as other mathematics teachers and teacher-researchers for setting research areas. It helps to develop the strategies and apply them in the classroom environment, which might significantly reduce the impassiveness of mathematical learning from the real social and communal practices and also from the detachment of learners in the learning process.

Delimitation of the Study

This study intended to apply collaborative techniques to improve the teaching practice concerning the mathematics concepts of geometry. Among the various aspects of the collaborative teaching approach, my major focus was how this approach could help me to enhance my mathematics teaching and meaningful learning through students' engagement. Mathematics concepts can be acquired through shared knowledge and collaboration with the active engagement of students in the learning process. My study commonly focused on how the collaboration encouraged the students to active participation in teaching and learning mathematics. As stated by Dillenbourg (1999), there are three types of collaborative learning namely peer collaboration, group collaboration and computer-supported collaborative learning (CCSL). In my study, I chose group collaboration as my learning type where I selected my research agenda, and my students designed their products and created the environment to facilitate mathematics queries.

A single pedagogy might not be applicable to all types of learners (Lee, Capraro, & Carpraro 2018). In my experience, in mathematics classroom, there are diverse backgrounds of the students. These diverse backgrounds of the learners might be dislocating intelligence, family, society, and ethnical group. This study is also limited to learners of diverse intelligence. While implementing collaborative

techniques in the classroom, it can be incorporated with different approaches such as the cooperative learning approach, activity-based teaching approach, inquiry-based teaching approach and others.

Chapter Summary

In this chapter, I raised the major issues and agendas of my research study. I shortly narrated my experiences in mathematics teaching and learning as a teacher and as a student respectively. To make it more precise, I shared some of the incidents in my classes. I collected the voices of mine as well as other learners in which we could find the various problems and obstacles that they were/are facing in their learning phases such as the issues of interest and motivation for free space to think and respond. I delimited my study by stating why I preferred a collaborative approach. While talking about the problem, I included some of my personal experiences and how and why our students were not performing well in mathematics learning. To support my research work, I made one research question for the study. In the following chapter, to support my study, I have discussed the available literature on the themes and theories along with the current practices.

CHAPTER II

LITERATURE REVIEW

A literature review plays a remarkable role in the study. It gives us the roadmap for moving around the research process. It supports the development of concrete concepts and valid conclusions. Since the present research concerns teaching and learning mathematics through collaborative techniques, theories are drawn from engaged learning, mathematical thinking, and a constructivist approach to mathematics for meaningful mathematics learning. This chapter explores the existing theories, themes and practices related to study, envisioning mathematical thinking for meaningful and engaging learning. The coherence of this chapter reviews the thematic domain, and then theoretical, and finally empirical reviews of the previous research conducted in a similar area.

Thematic Review

This section articulates the main supporting themes for the study. I present the standpoint of collaboration in 21st-century learning skills, collaborative teaching and learning approach, collaborative techniques in teaching and learning mathematics, culturally relevant and meaningful mathematics learning. I present the thematic review for the collaborative learning approach based on its definition and meaning. In addition, I elaborate on how a collaborative teaching and learning approach is effective for mathematics class in meaningful learning.

Collaborative Learning Approach

In my bitter experiences, teaching mathematics is generally limited to the four walls of the school building. It is rarely acknowledged that mathematics can be learned outside the classroom. As stated by Margaret (2017, p. 12), "*thinking beyond*

the box" is hardly practiced. In that sense, I started a pedagogical shift in my teaching practice to more progressive rather than usual teaching. As my observation, nowadays Nepali mathematics teachers are also adopting progressive teaching approaches to teaching mathematics. The pedagogical desire in mathematical classrooms that offer to engage students in a meaningful learning often viewed in opposition to conventional education (Matusov, 2021). In this view, conventional teaching refers to long-established culture of practice that society traditionally used in schools. Contrary to the conventional teaching approach, collaborative teaching and learning shifts the pedagogical desire by changing teachers' perception and by enhancing students' collaborative work for meaningful learning. In this regard, progressive education provides meaningful learning to the students to achieve pedagogical desire. Collaborative learning approach is an exceptional learning approach to achieving progressive learning. In my observation, I found that some schools in Nepal have started to think beyond the classroom setting for learning mathematics concepts. Schools are adopting project-based, activity-based, and peer discussion types of teaching methods in their regular classroom activities. These are experimented with forms of pedagogies to engage learning activities to construct meaning by *learning by doing* (Anzai & Simon, 1979). Among many pedagogical perspectives in the teaching-learning process, the collaborative learning approach is one of the emerging pedagogies which address the social-cultural and contextually relevant mathematics concepts in meaningful ways.

Collaborative learning is an umbrella term for a variety of educational approaches involving a joint intellectual effort by students, or students and teachers together (Rades, 2016). In this approach, students usually work in groups of two or more than two, share knowledge, engage in group tasks and have less competitive

sense in the learning process. In the competitive situation of today's world students need to have core knowledge and understanding in addition to other skills such as critical thinking and innovation, communication and collaboration (Alshwaier & Areshey, 2012). In that sense, collaborative learning allows students to solve the problem in an authentic problem-solving method which leads the students to be more reflective and responsible in the learning process. It enables them to make deep and meaningful connections to what they are learning with their everyday experiences. It encourages group work and promotes students to learn new skills for their everyday life. Collaborative learning characterizes the significant change away from the typical lecturer method in the classroom. Smith and MacGregor (1992) says, "In collaborative learning for collaborative classes, the lecturing/ listening/note-taking process may not disappear entirely, but lives alongside other processes that are based in students discussion, activities and active participant work with the help of different materials" (p. 233).

Collaborative techniques encourage the students to involve in group discussion and allow them to share knowledge through active participation in learning activity and acquire knowledge through logical discussion and analysis. In this regard, collaborative techniques refer to shared knowledge, shared authority, communication-driven, group work and teachers' as a facilitator. Since students are at the center of this type of learning approach, students' interest is more valued and, in the case of problems, the teacher acts as an effective adherent or facilitator to provide materials and resources for the learning (Westwood, 2008). They are considered to be effective tools for teaching and learning mathematics scaffolding activities through group tasks, frequent opportunities to share innovative ideas, knowledge construction on problem-solving, and others. According to Dillenbourg (1999), collaborative learning is a

pedagogical method or psychological process that emerges in Collaboration. Here, "one observes the two or more people who have learned and collaboration is viewed as the mechanism which caused learning" (P.4). This is insight for the two or more people working on the same goals in constructive meaning-making.

In this way, a collaborative technique is an effective teaching and learning technique for addressing the key skills of 21st-century skills. It is learning from working collaboratively with an individual representing diverse cultures, religions and lifestyles in a spirit of mutual respect and open dialogue in personal, work, and community contexts. It gives us more insight that helps the learners to be supported in meaningful learning through interactive and critical discussion on the problem. Students work in a group; the result may embed to knowledge sharing, critical thinking, and knowledge construction. The main theme of collaboration is to generate the questions and find the solution based on group tasks. It is for individual idea sharing within the group where the teacher facilitates the problem-solving.

21st- Century Learning Skills

In recent years, there has been cumulative focus on conceptualizing and implementing 21st-century skills in teaching and learning process (Teo, 2019) which includes 3'R like "rigor", "relevance" and "real world skills" (Mccoog, 2008 cited in Smith & Hu, 2013, p. 89) . These real-world skills include "communication", "collaboration" and "critical thinking" (Teo, 2019, p.2) that demanded progressive teaching and learning. The 21st-century skills include several frameworks for categorizing the skill and knowledge required for participation in the workplace and in society (Lai &Viering, 2012). Education policy makers around the world are recognizing that students need a broad range of skills such as communication, critical thinking, collaboration and problem- solving, and creativity in order to thrive in the

future (Kim & Care, 2018). In this regard, communication and collaboration are key learning skills to achieve 21st-century learning skills. According to P21 (2009), 21st-century skills are core knowledge instruction such as critical thinking, problem-solving, communication and collaboration. Collaborative skills demonstrate aptitude too effectively and respectfully with various teams and willingness to help make necessary compromises to accomplish a common goal. As Dillenbourg (1999) and Hunter (2006) state, collaboration has been described as a skill that encourages learning mechanisms to be enacted. Collaborative learning encourages the students to work in the group and share prior knowledge through group discussion. In this regard, teachers act as a facilitator and have resources at hand to help them in learning then encouragement to the students in shared knowledge.

The National Research Council (2011) outlined several justifications for collaboration's status as a key 21st-century skill. Different frameworks of 21st-century skills place collaboration as a learning skill (P21, 2015), and interpersonal skill (NRC, 2011) or a way of working. These frameworks have different conceptualizations of collaboration as a construct, and in terms of its interaction with other skills (Lai &Viering, 2012). In that sense, collaborative learning skills enhance the students' ability and performance in the conceptual formation to achieve 21st - century skills. A 21st-century skill provides value and motivation to students, and can help to structure pedagogical approaches (Swan, Shen & Hiltz, 2006). Support of 21st-century skills favors student-centered methods. Students were allowed to collaborate, work in a group to accomplish the same goal, and engage with the community (Rotherham & Willingham, 2009). Hence, collaborative learning is a situation where two or more people learn or attempt to learn something together to achieve the same goals. Collaboration is defined as working in a group, achieving compromises, and

getting the possible results from sharing knowledge in a group. Collaboration may be the most difficult concept among the four C's of 21st-century skills (Swan, Shen & Hiltz, 2006). The key element of collaboration is willingness i.e., all participants have the will to sacrifice parts of their own ideas and adopt others to get results.

The sorts of collaboration and communication give the logical cognitive capacity to the students that appear to lie at the center of the teaching and learning process. Indeed, in the digital age of the 21st-century, schools feed the learner with facts and knowledge (Teo, 2018) with the formulation and application of a collaborative approach in the classroom. In that sense, 21st- century education doesn't merely exist as inanimate outlines and procedure leaflets, but as real-life in and through the classroom practices within teachers and students.

Collaborative Techniques in Teaching and Learning Mathematics

There are many appropriate examples of teaching and learning mathematics using collaborative approaches. After reviewing the concept of collaborative learning, I came to develop my mindset in a way that collaborative approaches emphasize on group discussion, creating a learning environment for active participation, motivating the students and bringing positive change in the learning of mathematics. But, in the traditional view, the mathematics subject was considered an abstract body of knowledge, an independent and pure thought system of symbolic expressions (Taylor & Luitel, 2007), and a subject demanding precise practice and rote memorization. In this way, by separating one of the most inseparable disciplines from the perspective of group work or group tasks, mathematics was being learned and taught for a long duration of time. Eventually, with changes in the perspectives of society, countries, and the world, there are emerging numerous modern approaches to teaching and learning (Maass et al., 2019). Among those different approaches, the collaborative

learning approach has become an essential teaching and learning pedagogy (Emam et al., 2019; Roth, 2020). In this approach, students need to interact, communicate and debate in designed studies with their friends and facilitator. Moreover, "Collaborative learning is a teaching strategy that is applied with small teams of students of different levels of ability, and where all members participate to deliver the assigned task" (Emam et al., 2019, p. 164). In the traditional teaching and learning approach, teachers extensively focused on rote memorization of solving techniques of mathematics problems. If they are allowed to group discussion on mathematical concepts formulation and explore the phenomena not only from the point of examination, they will also be able to construct the actual meaning in the context of their lived experiences (Atweh et al., 2013). Mathematical knowledge and concepts can be learned from their real-life experiences and real-life problems.

As Emam et al., (2019) stated, collaborative teaching and learning will be effective for meaningful learning if it is integrated into a course or an assignment that has been designed to be student-centered. Collaborative classrooms articulate the pedagogical shift from teacher-centered to students centered. Students should be active problem-solvers, contributors, and discussants in the classroom. It might transfer the power of students working together and engage them in concept gaining perspective. Collaborative learning is a learning approach to involve students in solving mathematical problems, completing tasks, and creating a productive learning environment.

Engaged and Meaningful Learning

I always remember the proverb of Howard Gardner "you learn your best when you have something you care about and can get pleasure in being engaged in". It is clear that learners engage in the learning process that is more meaningful and

interesting. Engaged learning is a learning opportunity to involve students in self-learning, practice real application of knowledge across contexts and interact with other perspectives. Gardner (2007) stated, students' involvement in the learning activities provides adequate opportunities to think creatively. In that sense, I realize that a collaborative learning approach might create the students' involvement in learning and get them opportunities to explore their hidden aptitude through group work, group interaction and shared knowledge. As a result, students can demonstrate the concept of mathematics by providing evidence that they can recognize, label and diagram the formation of mathematics problems or facts. Conceptual understanding imitates the student's aptitude to reason in a setting involving the careful application of concept definitions, relations, or representation (Al-Mutawah et al., 2019). It is the comprehension of not only what to do, but also what is done. In that sense, engaged learning creates the environment for students to discuss in the group though the way they think about it differs. So, collaboration gives the platform to the students in the sense of constructing new knowledge with active engagement in the teaching and learning process. Learning by understanding is essential to empower students to solve mathematical problems that they will certainly face in the future.

Engaged learning encourages students to examine "how concepts translate into practice, how they expect and value greater personal involvement from students, and how they oblige students to link action and understanding" (Gardner, 2007, p. 14). The link between action and understanding of learners is of equal value in a collaborative approach. The main aim of this approach is to demonstrate students' ideas and knowledge through group work, knowledge sharing and knowledge construction. Accordingly, the research study intended to improve the learning experiences through students' collaboration. As Moore (2013) stated, effectively

engaged learning pedagogies induce the learning to look carefully at their experiences, question their own assumptions, and imagine how things might be different. In this regard, a collaboration of students' activities supports engages learning to enhance the minimum learning outcomes of learners. Students' involvement is a major concern in teaching and learning. Minimum learning outcomes of the learners could be perceived when they are involved in different activities through collaborative work.

Culturally Relevant and Meaningful Mathematics Learning

I have practiced my own Newari culture from my childhood. There are different mathematical concepts attached to our culture. If the school mathematics is connected through the cultural context of Newari culture, it could be relevant to teaching mathematics. But, Luitel (2013) stated that the mathematics classroom of Nepal had been dominated by the Eurocentric view of mathematics. It looks at mathematics as a culturally free subject and it doesn't have any cultural relevance. So, mathematics has been a foreign subject (Luitel, 2009). In my experience, learning is more fruitful if the learner could relate his/her understanding to real-life experiences. I have experience of teaching mathematics through cultural relevance. When I was teaching fractions in Grade-V, I got very confused about the concept of fraction. At that time almost all my students were Newar (*One of Ethnic groups of Nepal*) and I related the fraction concept with making *Bara* (One of Newari Food). In the Newari community, after preparing *Bara*, it is divided into different parts and distributed among god, goddess and family members. These different parts of *Bara* give a more authentic concept of the fraction to the students. In this sense, mathematics was developed from long cultural practices. If learning mathematics is directly linked with culture and real-life practices, the learning will be more meaningful.

Ethnomathematics is one of the approaches in mathematics which relates mathematics with mathematical modeling and anthropology (D'Ambrosio, 2006). Ethnomathematics helps to humanize the lessons and topics of mathematics and includes all students. It enhances the self-confidence level of students with the feeling of ownership as they work with their own cultural views and practices in mathematics (Belbase et al., 2021). So, ethnomathematics may support me while conducting collaborative teaching and learning techniques in my class and students will benefit from the cultural integration in mathematics learning. Orey and Rosa (2021) believed that mathematical knowledge was developed by members of dissimilar cultural groups. In that sense, the members of distinct cultural groups described their culture in their own terms and values. Mathematical knowledge would be a deeper understanding of traditional multicultural views of mathematics practices. In this regard, ethnomathematics allows one to find and focus on creating a deeper understanding of how members of different cultures actually solve the problems within their cultural practice through local communities.

Criticizing the view of mathematical knowledge isolated from real-life that is just a set of symbols, numbers, and figures, the modern view about mathematics knowledge is directly or indirectly related to or is a part of human life (Heaton, 2017). These concepts have been accepted by most modern educators. This thoughtful concept has been forwarded by mathematicians and philosophers like Aguirre and Zavala (2013), Gay (2000) and Kitchen (2005) while talking about culturally relevant mathematics teaching (CRMT). CRMT focuses on learning mathematics integrating mathematics thinking, language, culture and social justice (Aguirre & Zavala, 2013). Apart from this, CRMT shares the power among the learners to perceive the authentic learning environment through cooperative and collaborative learning for social

justice. Incorporating mathematical concepts with culturally relevant practice gives more insight and joyful learning to the students. The practice can be interconnected through interaction in a group. It draws related cultural activities in the classroom. It could give authentic mathematical concepts and enable learners to construct their own meaning. So, collaborative learning techniques might promote the learning of meaningful mathematics in a relevant context.

Theoretical Referents

When I went through Taylor (1997), I found that there are several approaches to collaborative learning and all of them are applied to students' lived experiences from which they continue to learn. In this stage, I have tried to select the actual real-life situation of the student's problem where they don't present through the traditional approach but through artificial context in the classroom to be solved by the class. My role is always being in the artificial context of the classroom and also providing authentic key experiences. I facilitate the students in their difficulties and create such a classroom environment to build new knowledge through collaborative work.

Collaborative learning is mostly defined as "any situation learning process where two or more people learn or attempt to learn something together" (Dillenbourg, 1999, p. 1). In collaborative learning, the problem will arise that will be trouble-free to solve through students' collaboration. So, according to Dellenbourg, to learn something we need to do more, especially a joint problem. Moreover, collaborative learning is the shared engagement of the students in a synchronized effort to solve the problem together. This approach could help the students to construct knowledge by working together to accomplish the same goals. As Johnson and Johnson (1998) advocate, collaborative learning is student-centered rather than teachers centered. Active learning is more effective than passive one. In this regard, we focus on a

collaborative approach to teaching and learning that gives more insight to the students for planning and achieving a collaborative goal. Therefore, it is helpful for students to social and personal development.

The 21st-century skills emphasized the collaboration. So, it is also considered to be an effective strategy for teaching and learning approach. Collaborative learning is broadly defined as "any instructional method in which students work together in small groups toward a common goal" (Prince, 2004, p. 223). Collaborative teaching and learning are beneficial to the students because teachers give more priority to activities for the student and teachers talk less about the subject matter. Here students find more passion related to the problem, a better amount of understandable input, a more comfortable classroom atmosphere and better motivation for learning. Johnson, Johnson, and Holibec (1988) advocated that collaborative learning provides the main key to teaching students to process skills that are needed to work successfully within a group.

However, implementing collaborative learning invites students to explore abilities through sharing knowledge among their colleagues in a critical and creative manner. In that situation, the learning environment is more open and meaningful. So, collaborative learning is two ways communication learning approaches that gives a platform to the students for sharing their knowledge, authority, and opportunity among the friend circle and facilitator. As Jacobs (2015) states, the collaborative approach is student-centric pedagogy where teachers work as a facilitator, role model and coach whereas students work by setting the goals and aims with the subject matter. Collaborative learning opens up the floor for productive discussion and interaction so that the students can connect the prior knowledge to gaining

knowledge. In this sense, students get an adequate chance to learn by setting goals, sharing knowledge and assessing their own performance.

Lev Vygotsky (1896-1934), a Russian psychologist, formulated a theory of cognitive development that is based on a student's ability to learn socially relevant tools and cultural signs. Vygotsky's major concern was with socially relevant tools like hands, hammers and computers, and cultural-based signs like language, writing, and number systems. Commonly students are engaged in interaction with their colleagues and adults who socialize them with their culture. In that situation, students work in a group and their collaboration plays a vital role in mathematics learning. Central to Vygotsky's theory of cognitive development is his theoretical construct of the zone of proximal development. He suggested that students can accomplish their work with the help of more knowledgeable others, such as peers or facilitators. The thought of a zone of proximal development was developed during the 1920s and was elaborated progressively until he died in 1934.

Vygotsky defined the zone of proximal development as " the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers" (p.86). The impression is that the individuals learn best when working together with others combined with collaboration and sharing their understanding through interaction and discussion. In that sense, the emphasis on teaching is a privilege of the ZDP which the students can't do by themselves but has the potential to accomplish with the guidance of others or peer collaboration with more skilled persons (Shabani et al., 2010).

Vygotsky, in terms of designing the tasks for learners, suggested that teaching located at or below a student's current level of understanding would not be

challenging enough to promote further development. Instruction that is beyond what a student can do is effective for stimulating learning. Therefore, teachers while designing group tasks and activities should understand the actual level of the students and design the tasks accordingly. In that sense, a collaborative teaching and learning approach could be effective for constructing students' own knowledge. Students' scaffolding is essential which refers to instructional supportive activities and social interactions that occur between the child and other individuals as they guide for effective learning and development of ZDP (Wood. et. al. 1976, cited in Lui, 2012). Students should engage in interactive activities for developing their knowledge. Thus, from the Vygotsky framework, it can be said that students construct their knowledge with the support of their teacher's scaffolding. So, teachers facilitate classroom teaching for better learning of students.

The collaborative learning approach enhances the observed behaviors of students. During collaborative activities or peer discussions they try to understand social behaviors and responses to their colleagues. In that sense, they observe social belief by connecting their subject matters and learning take place without reinforcement (Bandura, 1977). As Bandura emphasized that for meaningful and purposeful learning to take place, the observed behaviors of those being imitated should be appropriate.

Constructivism

Constructivism is a learning theory that was founded by Jean Piaget (1896-1980). It claims that learning takes place from real-life experience and is co-constructed. People construct their own understanding and knowledge of the world through things and reflection on themselves (Giesen, 2012). Thus constructivism is mainly related to how people construct knowledge from their own experiences and

practices. Different people have different mindset and perceptions on some things. They construct their knowledge differently accordingly. Beker et al. (2007) forward that constructivist teaching uses synonyms for child-centered, activity-based, inquiry-based, and inclusive teaching and learning approaches. The knowledge is constructed by the learners with their active engagement in their learning process. Similarly, learners come to the classrooms with their prior knowledge of the classroom contents. Therefore, it is important to provide an interactive environment to the learners about the related problems with their individual ideas for constructing knowledge. So knowledge is constructed with their active engagement and participation through interaction.

Human nature is self-oriented and so is the process of constructing knowledge (Bada & Olusegun, 2015). In the life cycle of human- beings, they perform according to their interests. They participate in different kinds of activities where they are benefited. According to Habarmas (1972), human interest is classified into three different categories namely *technical, practical and emancipatory* interest. Accordingly, knowledge constructed through the technical interest is objective knowledge that takes mathematics learning isolated from the practical implementation in human life. Practical interest addresses the human experiences and their cultural practices of mathematics learning connected with their real-life experiences. This research aimed to empower learners to learn mathematics through collaborative learning approaches with real-life practices.

As the collaborative teaching and learning approach emphasized practical knowledge, student-centric teaching and learning approach where students get prolonged engagement in teaching and learning activities beyond the conventional classroom structure and textbook oriented, they get the opportunity for learning in the

group both working and discussing inside the collaborative classroom structure. In another word, the collaborative teaching and learning approach focuses on a group task, discussion, and sharing understanding among their friend circle. It creates an in-depth knowledge gaining environment in the classroom and prolonged investigation through their direct observation and engagement.

Constructivism is a paradigm or world viewpoint that learning is an active, constructive process (Millar, 1989). As Bada and Olusegun (2015) argue, constructivism represents one big idea in education which is reforming education for all students' success and student-centric learning environment. In this sense, constructivism dictates that students learn new mathematics concepts constructed through prior knowledge. Students can learn those things which they can link or construct in their cognition, relating to their pre-knowledge and experiences. In my research, cognitive constructivism (Islami & Gustana, 2021) plays a vital role. Accordingly, they believed Jean Piaget is a figure of constructivism and children's ways of thinking could be figured out through individually obtained knowledge. The constructivism approach in education deals with a contextual approach. Context could not be built as limited and narrowed. Cognitive constructivism is a theory of the mental state of acquiring knowledge (Xu, 2019). This is a non-Piagetian thought. It emphasizes the utility of prescribed computational representations in understanding and developmental change. It inspires me that knowledge can be constructed through the cognitive domain. Constructivism is the theoretical approach to teaching and learning which adopts the active involvement of students in building knowledge. The major focus of this approach is to motivate students to understand the meaning of subject matters and their implementation in their daily life. The mathematician and philosopher Jean Piaget stated the 'theory of mental state' as the base of cognitive

constructivism. This paradigm of mathematics learning shifts from the behaviorist theory of classical stimulation response conditioning to modern cognitive constructivist learning.

In constructivism, the learner is an information constructor (Vaishali & Misra, 2020). The role of teachers has changed to "*guide from the side*". It takes teachers as facilitators not an instructor and encourages the students to actively participate in the learning process, fostering critical thinking, creativity, and problem-solving ability. As Naylor and Keogh (1999) state, the students can make a sense of the new situation in terms of their existing understanding and involvement in the meaning-making process which links new ideas with their existing knowledge. In that sense, constructivist learners actively create their own meaning and understanding with their prior knowledge and create their own subjective presentation of objective reality (Vos, 2018). In this regard, new information is linked to prior knowledge which is mental representation or subjectivity. The constructivist classroom provides the learner to build a new knowledge with the help of prior knowledge and gain experience. While students are allowed to participate in a collaborative environment, they find their meaning because of their real-life context. Furthermore, constructivist students are encouraged to explore possibilities, invent alternative solutions, and collaborate with other students (Roselli, 2016). This insight taught me that constructivism is a theory of learning which is based on the idea of knowledge construction with mental activity. When a learner wants to construct knowledge, there should be an active organism to seek new meaning.

Geometry serves to connect mathematics in different mathematical branches. A good understanding of geometry and its practical emphasis on spatial characteristics and relationships will help students gain increased access to abstract

concepts of mathematics (NCTM, 2000). It is essential for creating a geometrical climate that will motivate students to learn. Sun and Williams (2003) referred to the importance of constructivist learning in acquiring knowledge that required student-centered, goal-directed and real-life problems in order to find meaningful solutions. In order for students to participate in mathematics, learning is high. For this, we need to attack the classroom environment and it makes us more motivationally oriented. Constructivist pedagogical approaches advocate in a way that the mathematical course was designed to problem- solving and cooperative learning environment. In this regard, students actively participate in problem-solving mathematical exploration through hands-on activities with collaboration.

Empirical Review and Research Gap

This part of the review is based on the previous research studies done under the same research design. The number of major studies have given compelling evidence [for] collaborative teaching and other similar approach, "*incorporating interactive instruction"* (Smith et al., 2001), "*Activity based instruction (ABI) for motivating the children in mathematics learning"* (Luitel, 2019), "*A comparison of cooperative learning and collaboration in high school geometry classes including geometry in construction"*(Gates, 2019), "*Teaching and learning issues in mathematics in the context of Nepal"* (Panthi & Belbase, 2017), "*The Role of the Constructivist Learning theory and Collaborative Environment on Wiki Classroom, and the Relation between them"* (Alzahrani &Woollard, 2013), "*Collaborative Pedagogy in Architectural design studio: A Case Study in applying collaborative design*" Emam et al. (2016) and "*An insight into collaborative learning with ICT: Teachers' and students' perspectives"* (Valtonen, 2011) , *The Effect of Constructivist Instruction-Based Mathematics Course on the Attitude toward Geometry of Pre-*

Service Elementary School Teachers (Tsao,2018), *Emerging ICT Tools, Techniques and Methodologies for Online collaborative Teaching and Learning Mathematics* (Dahal et al., 2021) and "*Mathematics Educators' Perspectives on Cultural Relevance of Basic Level Mathematics in Nepal"* (Belbase et al., 2021). An empirical review in research includes the finding, conclusions and main ideas of previously conducted research along with the problems that the research had been studied about collaborative teaching and learning.

Newmann et al. (2001) conducted a large study in elementary, middle and high schools where they implemented authentic pedagogy and authentic academic performance approaches in their mathematics and social studies classrooms. The researchers conducted quantitative research among 128 students in 23 schools in Chicago. They found that the inquiry-based approach in teaching led students to higher academic achievement with a deeper understanding of the subject matter. The result showed that the differences between the high- and low-performing students greatly decreased. They came up with three essential research findings that through inquiry-based learning, learners go through substantive conversion called disciplined inquiry- they construct knowledge of the subject matter and hence they will apply or connect that knowledge with the world beyond their classroom. The researchers however, could not focus on inquiry-based group work and sharing priory knowledge among the students' group for engaging in learning.

With the motive of identifying the belief system of mathematics teaching in the collaborative classroom, Valtonen, (2011) had conducted research and concluded findings in his dissertation. The research had been conducted from the perspectives of teachers' and students' in collaborative learning with information and communication technology (ICT) in 93 secondary and polytechnic level teachers, and 1370 secondary

level students. The dissertation links closely to different development projects within the area of ICT in education and also to the work done with teachers' in-service training and teacher-student training at the University of Eastern Finland. This research study was conducted by using both a cross-section survey strategy and a case study strategy consisting of five studies discussing the five different topics by using quantitative and qualitative research approaches as well as a mixed-method research approach. In conclusion, my personal interest in the research of ICT in education is to outline pedagogical models and scripts i.e. using different online environments, especially social software and mobile technologies to support learning mathematics. The findings have disclosed that there was still a gap between theory and practice in teaching and learning mathematics. Computer support collaborative techniques have been highlighted in the research but group work and group knowledge sharing might be alternative ways of teaching mathematics for meaningful learning. In this regard, collaborative learning techniques pushed up the students' learning from the ground level to the optimum level of the continuum incorporating the learners' motivation and engagement.

Chu et al. (2012) had conducted a research study on the topic of " Developing upper primary students' 21st-century skills: Inquiry learning through collaborative teaching and Web 2.0 technology". The researchers had focused on the importance of 21st-century skills in Honk Kong primary education and teaching strategies to foster students' 21st century skills development. Their aim was to provide specific and practical guidelines to teachers who wish to explore the use of the collaborative method in their teaching. This research study was a qualitative analysis. The fundamental concept of this approach was built on the notion of constructivist teaching which considers teachers as facilitators in guiding their learning activities

and scaffolding them. Researchers summarized the research while implementing inquiry project-based learning through a collaborative teaching approach and showed the positive impact on both students and teachers. Through this initiation, students were found learning "how to learn", being motivated to perform better and developing new and improved skills. Students –Students' collaboration for effective learning throughout group collaboration could not be seen in this research.

Emam et al. (2016) had conducted a research study on the topic of "*Collaborative Pedagogy in Architectural design studio: A Case Study in applying collaborative design*". This research mostly focused on architecture students involved in the collaborative process for architectural design. This research aimed to discover how a collaborative design studio could be improved to enhance each participant's contribution to the design process. The researcher used quantitative analysis among 92 students from Alexandria University, Egypt. The result of the research study was that the collaborative learning method was an effective learning method to increase the motivation of students. It would help the students share knowledge and increase learning capacity through using collaborative learning tools. Students' collaboration for group works could not be addressed by this research study. My research study was to fulfill that research gap.

Panthi and Belbase (2017) conducted a research study on the topic "*Teaching and Learning Issues in Mathematics in the Context of Nepal "*. In this research paper, the researchers have discussed major issues of mathematics teaching and learning in Nepal. They claim that issues come from theories such as social and radical constructivism. The issues related to social aspects are gender issues, language issues, social justice issues and issues related to the achievement gap. They highlighted different issues of teaching and learning mathematics and the idea of resolving them

practically. The researchers discussed similarities between constructivist learning theory and collaborative learning environment, the role of the teacher in the classroom, learner activities in the classroom, learning techniques, employing wiki learning environment, and the relation between two terms in wiki classroom. In the part of the conclusion, they argue that "Students learn from each other and benefit from activities that require them to articulate and test their knowledge as well as group work provides an opportunity for clarifying and refine their understanding of concepts through discussion and rehearsal with peers' '. Authors discussed issues of mathematics in the context of Nepal. To solve the issues of mathematics in the context of Nepal group work and peers discussion could be helpful in such conditions.

Tsao (2018) studied on the topic of "The Effect of Constructivist Instruction-Based Mathematics Course on the Attitude toward Geometry of Pre-Service Elementary School Teachers". The purpose of this study was to investigate the attitude toward geometry of pre-service teachers and to explore the effects of the constructivist teaching approaches. Researcher used Utley Geometry Attitude Scales (UGAS) as data collection method and implemented quantitative research methodology. They concluded that teachers' attitude toward geometry and their experience played a crucial role to facilitate positive mathematics attitudes in elementary students. This research is more helpful to draw my perception in geometry teaching through constructivist lenses.

Gates (2019) researched on the topic of "A *comparison of cooperative learning and collaboration in high school geometry classes including geometry in construction*". This study involved observation in geometry class to document the use of the instruction strategies of cooperative learning. This research study was based on quantitative research. While the researcher had observed the classroom, he didn't

reveal significant differences in the quality and quantity of small group work on geometry in construction versus other geometry classes. The result of the retrospective pretest showed a higher percentage of all students in geometry indicating teamwork and collaboration in teaching and learning. This research study basically focused on higher-level students to enhance the ability of 'geometry in construction". In my research agenda, this research could be helpful to see the students' performance in teaching geometry through collaborative techniques.

Luitel (2019) conducted a study on "*Activity-based instruction (ABI) for motivating the children in mathematics learning*". In this research, he tried to explore his practices and pedagogical strategies in classroom teaching through activities. The research project was conducted in a private school in the Kathmandu district, sampling Grade seven students'. The researcher set fifteen days plans to use activity-based instruction methods for teaching. The main intention of the research was to motivate the learners to learn mathematics through activities. The researcher believed that self-reflection helped to transform the positivist teacher into an interpretive facilitator as well as an activity-based instructor. The researcher took a base of data collected through participants' interviews, drawing self-reflection, observation, pre-test/post-test assessment and narrative meaning-making. The study gave insight into this research for the use of action plans in the context of collaborative learning approaches.

Dahal et al. (2020) conducted a research study on the "Emerging ICT tools, techniques and methodologies for online collaborative teaching and learning mathematics". Author highlighted the strength of possible integration of emerging ICT tools, techniques and methodologies to facilitate the online or offline task and assessment of students, and trainees in a collaborative manner. They implemented

qualitative research and action research as a research methodology. The educational trainers, course facilitators, and educational researchers were participants in the research study. They created instructional collaborative tools such as forum discussion, workshop, chat, comment box, google docs, slides, sheets, and Jamboard and wiki. The study indicates the establishment and usage of Moodle cloud and Google apps integrated with collaborative tools to make teaching and learning mathematics more effective. This study could be helpful to my research agenda in a way that it informs me of ways to engage students in collaborative ways and construct collaborative activities.

Belbase et al. (2021) conducted a study on "Mathematics Educators' Perspectives on Cultural Relevance of Basic Level Mathematics in Nepal". The focus of the research paper was to explore mathematics educators' perception of the cultural relevance of basic level mathematics in Nepal. The methodology of the research study was qualitative analysis by in-depth interviews of five purposively selected mathematics educators. The major theme of the research agenda was teaching in the mother tongue, contextualized ethnomathematics, and local knowledge in the curriculum as a teaching approach. The researcher argued that the provision may reduce the hindrance in mathematics learning due to compulsory learning in a second language. The findings of the research made advocacy for the implementation of local cultural knowledge in basic level mathematics classes for effective teaching and learning. This research supported me to contextualize the local cultural knowledge in the mathematics curriculum by using cultural artifacts and teacher practice. This research study couldn't address cultural knowledge through group work and a communicative manner.

These empirical referents became the base of my study to implement collaborative teaching and learning approaches in the area of teaching and learning. I believe that the collaborative teaching and learning approach is not enough to solve problems in mathematics teaching but it could be more helpful to the students to broaden their ideas about dealing with working together, group discussion and sharing knowledge through active engagement in the learning process.

Chapter Summary

This chapter discussed the supporting literature for this research study. It reviewed the themes, theories and practice related to collaborative teaching and learning approaches. The chapter began with discussion of the conventional method of learning, the transmission of knowledge, problem-solving, and creative thinking. The chapter has established collaborative teaching and learning as more than just a pedagogical method. It enhances the learning capabilities and provides exposure for the learners to get a better understanding of the topics. For this research, I have chosen action research as a tool which has been discussed in detail in the following chapter dealing with methodology.

CHAPTER III

RESEARCH METHODOLOGY

This section portrays the methodology applied in this research. Considering various research paradigms of the research study, this chapter explains the epistemology, ontology and axiology related to the study as well as the methods applied for the study and ethical considerations. Besides, it includes samples taken for the study and the detailed steps of research. Explaining some major steps integrated into the research study, this chapter explains the detailed methodology of action research that has been implemented in the research.

Ontology

The Freirean ontological view contends that people have a vocation to act upon and transform their own world individually or collectively (Shaul, 2005). So, ontology is the nature of being. Ontology is concerned with the theory of reality. So without reality, my research would not be appropriate. It means how one perceives knowledge subjectively or objectively. Moreover, the ontological assumption is concerned with the very nature or essence of the social phenomena being investigated (Andrews, 2012). From an ontological perspective, both the subjectivist and objectivist have taken for finding the appropriate strategies for using collaboration in teaching mathematics.

My ontological assumption is that all people have their subjective perceptions, involvement, and understanding which might be diverged. I believe my participants have their own perceptions, social identities, and level of understanding in learning mathematics. As an interpretative researcher, I believe in the construction of multiple knowledge and participants' perception. My research is based on subjectivism which

advocates that there is no single reality. So, at this stage, I do not believe in a single reality or truth in the world.

Epistemology

As I mentioned earlier, my ontological belief is subjective and multiple realities may exist. Malshaw (2018) believed that the knowledge is based not only on observable phenomena but also on subjective beliefs, values, reasons, and understanding. Accordingly, knowledge is constructed and co-constructed. In this sense, the harmonious rapport between researcher and participants is essential. I agree with Blaikie (2007) that epistemology considers what we can know and how what is assumed to exist can be known. So, the epistemological stance of my study is that knowledge is constructed through collaborative teaching and learning approaches. The study helps me to understand how the participant constructs the knowledge and how social and cultural values impact mathematical learning. Action research is my research methodology. I move through my reflection of class, my experiences during class, my observation of the participant, considering students' views on the collaborative techniques in teaching and learning mathematics and my major concern with comparing and contrasting the learning mathematics in past and current through collaborative techniques.

Axiology

In my experience, all people are guided by their own values. And they try to give meaning based on their value. Hence, the role of value has a crucial role in the meaning-making process in the research. Regarding collaborative learning in the classroom, there is the possibility of influencing the value of participants as well as the researcher. Hence, as Creswell and Clark (2011) mention, there is a vital role in my research study. It indicates that research is more difficult if there is value-free

research. It is concerned with value-laden. In research, I have explored my values and beliefs along with the values and beliefs of my participants which help me to explain what, why and how values are constructed.

Axiology studies the nature of value and valuation of knowledge (Cooper et al., 2016). It is the collective term for ethics and aesthetics. My research study is dependent on participants' active participation and sharing of new knowledge with their colleagues. Their beliefs and values affect this study and drive it accordingly. The researcher's value and interest also might drive the research in certain directions. In my research, there is always a chance of alternation in the claims. It is informed according to the belief system of the participants and the researcher. How they take the system of knowledge and from which background they come also affects how they create knowledge and how they react to it. In this case, the research might get driven to the area in which the participants and the researcher are interested. So, the axiology of this research study is value-laden.

Research Paradigm: Interpretivism

A research paradigm as Willis (2007) explains, is an outline that provides an ample idea about research. It provides a comprehensive picture of the nature of the research and how it is conducted. We accept that multiple realities exist in the world. There are multiple perceptions of teachers and students in the teaching and learning process. For this study, I immersed myself in their studies, feeling their pain and pleasure (Tylor & Medina, 2011). For instance, human beings cannot be studied based on mathematical truth like 3+3=6. But, sometimes it may be possible to 3+3=5 or 7 because humans have individual differences in the matter of understanding. The views of the participating students are different from one another. The interpretative paradigm supports the belief that reality cannot be created by subjective assumptions

and predictions (Alharahsheh & Pius, 2020). Hence, subjectivity ontological thinking was used in the research study.

In my research, I explored how students learn mathematics through a collaborative approach to the meaning-making process of learners as per the ontological, epistemological and axiological beliefs that I have mentioned above. It allowed me to understand my participants. Knowledge was constructed by a long process of interaction with the participants. I was aware of the notion that it is a difficult task to reference the voices of various participants when they are not present while I write the texts for my research. I realized that I need to reflect on my observation during class and try to present the subjective voice of the participants. I have tried to present my own background to make it easier for my readers to make sense of my role in research.

According to Mukhles and Ababneh (2020), the interpretive approach sees the cultural driven and historical situation in the interpretations of social life. The main objective of interpretive research is to generate reflective self or understanding of others in context or culture. Interpretive research approaches have the intention of understanding, "the world of human experiences" (Cohen & Manion, 1994, p. 36), suggesting that reality is socially constructed. These are the main keys to the interpretative research method. The interpretive researchers tend to rely upon the participants' views of the situations being studied (Creswell, 2011). They recognize the impact on the research of their own background and experience.

The interpretive paradigm is concerned typically with generating a context-based understanding of people's thoughts, beliefs, and values associated with social actions (Taylor et al., 2009). As an interpretive researcher, I attempted to seek classification, understanding, and similar situations regarding the status of reflective

writing in my research. I have focused on the meaning-making process of an individual as "I" "a researcher", "a teacher" and in the meantime "a social and cultural being" (Luitel, 2009, p. 30). My key intention was to explore the students' experiences and my pedagogical change in the meaning-making process through action research. Frerrance (2000) advocated that the term action research is used by quantitative researchers with a variety of meanings. Furthermore, when I had to use this paradigm in my research, I wrote my field notes, daily reflections, students' reflections, etc., and interpreted them with different views. I do not claim that action research is only the solution for me to underpin my research study but it is more helpful to serve the research purpose to explore the practices of my mathematical classroom.

Research Approach: An Action Research

As per my previous explanation, my research paradigm is interpretivism and its key is concerned with understanding the subjective world of the human experience. I intend to explore the participants' experience, understanding and my self-reflection on classroom teaching. Hence, action research is a key research approach of my research study.

This research has been conducted over the pathways of interpretivism paradigm as action research. During my research study, events, settings, programs, social groups, individuals or bounded systems (McMillan et al., 2004) are in-depth examined and analyzed. The word 'action research' was first coined by the US social-psychologist Kurt Lewis in the 1930's with the purpose of social mobilization and of helping against the effect of the Second World War. But later on, it has been practiced in the education field (Mills, 2000). It is directly related to practical problem solutions whatever researchers face in their educational practices. As Frerrance (2000) stated, action research is a process where participants inspect their own educational

practices systematically and carefully. So, the term action research refers to the practical solutions to the problems that are facing in our educational practices. A social setting collaborates in the diagnosis of the problem and the development of a solution based on the diagnosis (Bryman, 2012). Action research provides an opportunity for educational practitioners to reflect back on their own practices (Creswell, 2015). In that sense, it is also known as a form of self-reflective practice where a new idea is implemented to solve the current problem and reflectively analyze it and go for it if necessary (McNiff & Whitehead 2006). The present study was conducted on the real educational problems that I myself faced during my class and the improvement of my practices with their systematic solutions. The purpose of the study is to improve my teaching practices to help my students to conduct knowledge themselves by engaging in learning activities and also to help them in the meaning-making process through collaborative teaching approach. So, I chose 'action research' as the most appropriate method to apply in teaching mathematics. I got enough chances to collaborate with my research participants in the classroom setting. The problem that my students have been facing during class could be solved more effectively in this way.

Action research is only one research methodologies that we hear quite often in today's educational sectors and the teaching and learning process. Hence, action research is undertaken in a school setting that is related to a problem in teaching and learning. Moreover, it enhances collaboration between colleagues searching for new and pioneering ideas in teaching and learning. It is searching for the solution of problems which occur in the classroom and looking for ways of improving instruction and the students' achievement. Action research is a collaborative investigation process that implements an action plan for the purpose of problem-solving in educational

sectors (Reason & Bradbury, 2001). Kurt Lewin (1946) described action research as comparative research on the conditions and effects of various forms of social action and research leading to social action that uses a spiral of steps, each of which is composed of a circle of planning, action and fact-finding about the result of the action. In that sense, action research is a systematic process that allows trying to find out different ways of doing things in the classroom, until finding something that works for the researcher and students.

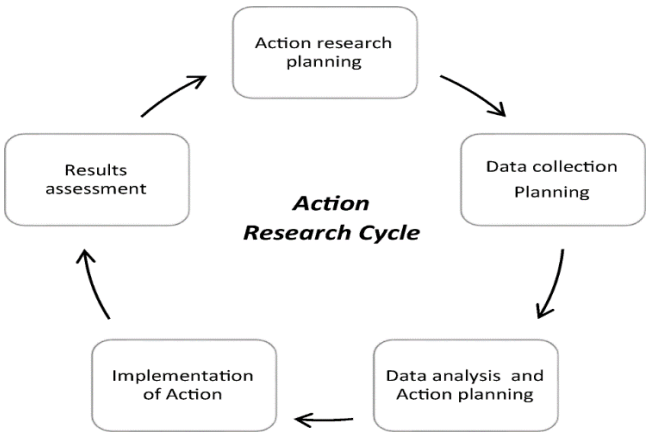

Fig: - 2: Action Research Cycle, Fagundes et al. (2017)

The action research helps to identify the specific and practical issues that the education practitioners are facing in their teaching field and empowers them to seek the solution. After that, there is the execution of the plan and reflection back about the result. This action research is an interaction coiled of mainly three steps- look, think and act as described by Stringer (2007).

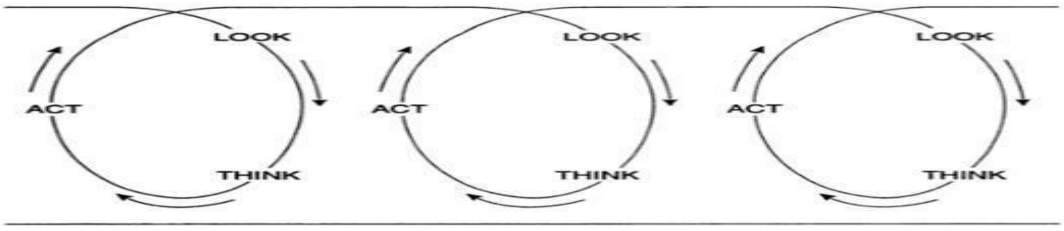

Fig. 3: Action Research Interaction Spiral

As a practitioner-researcher, while I conducted action research, I identified a problem or issue. Then, I made an appropriate plan with a practical solution and applied the action plan in the respective field. My daily reflective notes, student reflections, field notes, etc. were data/information of my research. Reflecting the whole process up to the final result, the process was repeated during field work. This shows that there are some steps for conducting action research (Creswell, 2015). They are *identification of the problem, plan of action, data collection, analysis and interpretation of information, and Reflection.*

Research Site and Participants

The site of my research study is a private school in the Lalitpur district. The research was conducted upon the students of Grade- X where I engaged professionally. Also, the participants study mathematics as a compulsory subject according to the curriculum of Nepal. On the source of the information of their recently conducted examination, most of the students had poor performance in mathematics. Investigating them further, I came to know they were the same students who had been showing lower interest in learning mathematics. So, it was more beneficial to examine the effects of collaborative approaches on the engagement of students and meaningful learning. In this process, I have taken interviews with the four selected key students. It is also a commonly used sampling strategy. Participants were selected based on their previous performance in mathematics learning. Sometimes referred to as 'judgment sampling', purposive sampling (Cohen et al., 2018) is designed to provide rich information in my research study.

Key Research Participants

The research participants were the students of Grade X which is a heterogeneous class of a private school in the Lalitpur district. The number of

participants was 16. Out of them, most of the students were less interested in learning mathematics based on my previous classes and their engagement in different mathematical activities. In this sense, the participation of students in mathematics learning was a questionable matter. In my observation, very few students had done their home assignments regularly. Fewer were regularly present in class and tangible students were very interactive in class. Among them, I have chosen four key students based on good and poor performance in mathematics based on schools' internal examination, talkative and overconfident students of class and gender, and a short description of four key participants in the interview. I have used a pseudonym instead of the participant's real name. More details of key participants are given below;

J. Shrestha

J. Shrestha is one of the active and self-motivated students of Grade X. As per my understanding, she is popular in her friends' circle as a sharp learner and bright student in learning compulsory mathematics. She participates in each and every activity and function of the school. I found after discussion with her that the family background and learning environment both were favorable for learning. She has been studying in the school for 4 years. So, she is very familiar with the teachers' circle and the teaching and learning methods of school. I thought interviewing her would give me some comparative ideas of past and present trends of teaching and learning methods of the school, and interviewed her.

S. Maharjan

According to her previous record of examination, she is regarded as an intelligent student in all subjects and mathematics as well. After discussion with her, I found that she has good support and motivation for studying at home. She has a separate study room. She has studied in this school for 5years. Talking about the

recent classes and applying a collaborative approach in class, the group members were dependent on her from the very first day of this new teaching and learning approach. In the beginning, she explained and shared her understanding with her group members to clarify their confusion.

S. Magar

So far I know, he is best known for his continuous effort in learning mathematics among his friends. But, he feels it is difficult to grab mathematical concepts, and shares that mathematics is the easiest subject for him. His performance was quite good but not satisfactory in the past examination. He has been studying in this school for six years. He had some different experiences than other friends with a new teaching and learning approach due to his shyness among friends.

B. Deula

According to other subject teachers, he was a *talkative student* in the class. He was not more concentrated on his study. When sitting with him and discussing his personal views, I found he was interested in the new things or activities in the class rather than rigorous practice and routine problems. I had an interaction with him during the class. I talked about his experiences and feelings.

Identification of the Problems

This is the first step of the action research in which the researcher identifies problems faced by students in the teaching and learning process. In this regard, action research helps to solve the problems faced by the students in the classroom (Tsolaki, et. al, 2013). The problem that I faced in my mathematics teaching and hostile experience problematized my research study. I began with my experiences of teaching and learning. I focused on problem solving of mathematics in the algorithmic process. Students couldn't get the opportunities to explore ideas and share understanding

among their friend circle. It has been effective for routine problem solvers. But, the rest of the students couldn't show their hidden ability in their performance. I was doing the same practice for many years to get good marks in examinations. When sometimes teachers asked twisted questions, good performing students (*So-called talent*) also made mistakes in problem-solving. Students learn mathematics to pass the examinations and teachers focus on making students pass and score high marks in examinations (Pokharel, 2018, p.47) In this sense, the mathematics teaching and learning process blended into a system of examination. After that, I raised one question in my mind, *"Is there any alternative approaches that could help me to teach mathematics in meaningful ways where students feel free to share their ideas and work together for constructing knowledge without competition.*" I realized that the major problem concerning my teaching experiences that I faced in teaching was that the students solve routine problems in an isolated manner. Indeed, I identified the issue that there is no collaboration and no discussion in classroom teaching. So, students didn't get the opportunities of sharing understanding. This research study is more concerned to teach geometry through a collaborative classroom for meaningful and better improvement of the students. Especially, it tries to find out ways to help those learners who have been facing problems with geometrical concepts and link between their mathematical concepts and real life.

Plan and Action

This is the second step of the action research in which the practitioner develops a plan for solving the problems. A plan is an informal statement for applying a new educational practice (Creswell, 2015). I was facing the problem of teaching mathematics by only using a direct teacher-centered approach and students' involvement in routine problem solving. To resolve such a problem, I have selected

collaborative teaching and learning approaches among many other pedagogies. To accomplish the preferred objective, I made an eighteen- day's tentative plan to carry out effective and meaningful learning through a collaborative teaching approach. According to CDC (2077), the tentative teaching hours of these chapters is sixteen hours but I planned for eighteen days. I planned group work, group discussion, and sharing of students' creativity during classes in a collaborative teaching and learning process.

Table 1

Action plan of Teaching Geometry

S.N.	Topics/Project	Estimated Days	Estimated Hours
1	Area of Plane figures	2(1st and 2nd day)	2 hours
2	Relation between Area of Triangle and Parallelogram - Properties of triangles, congruent and similar triangles - Properties of parallelograms - Area of Triangles and Parallelogram. - Relation between the area of parallelogram and triangle	10 (3rd to 12th day)	10 hours
3	Circle - Properties of Circle and related theorems	6 (13th to 18thday)	6 hours

Details of the Action plan or project refer to Annex- A attached at the end of this document.

I was curious to study how collaborative learning techniques could be useful to teach a geometric section of mathematics. Due to this reason, I choose two chapters from a geometric section of Grade X. The name of the chapters was *the relation*

between the area of parallelograms and triangles, and geometric circles. I thought that besides the algorithmic process, it has a main role in other problem solving of mathematics. These chapters seem to demand students' active engagement in learning through a collaborative learning approach. This action plan is the guideline for my research journey to conduct an effective class for meaningful learning.

Data Collection

In my research, I collected qualitative data by means of experiencing, enquiring and understanding by observation and examining the target participants. It was because the lists of high score achievements of the students couldn't provide ideas about the actual learning of students. So, I tried to gather more information through my personal observation in their natural setting. Such types of triangulation could be helpful to develop viable action plans further. It helped me to understand the actual problem in-depth (Sagor, 2005). There are various techniques of the data collection method. Out of them, Mills (2000) mentioned the three E's models for data collection techniques in action research.

Experiencing

After implementing the action plan in my regular classes, I observed the noticeable changes in students based on individual performance through participation in the different activities, working on group tasks, and sharing understanding among their friend circle. Whatever changes occurred in students' behavior, I took field notes based on my experience in natural settings. In the implementation process, some participants were not interested in such progressive teaching and learning environment even though I kept the record of those participants as well.

Enquiring

I conducted various informal as well as structured formal interviews with the participants. In that process, I observed their knowing/ understanding with the comparison of previous direct instructional teaching methods and recent collaborative learning techniques. I took regular interviews with the key participants and it could help to further improve my action plan. For that, I purposely selected key participants for detailed interviews to know how and what were their experiences about their active engagement and cooperation among their group members in the collaborative learning environment. It could be more helpful to me during my data interpretation and analysis. I interviewed some other participants for further interpretation.

Examination

I gathered the information from various sources like a reflective journal of participants, my daily field notes, and my journal on a daily basis. Also, I collected the data through audio tape, capturing photographs and videos which were taken during classroom teaching. These materials were more helpful to examine my data/ information on the field work. This material could be helpful to examine what actual learning would appear on students' sites. This was done to know their authentic understanding and difficulties of students that could be removed from the re-implementing of the action plan.

Observation as Data Collection Method

As per the nature of my ontology, epistemology, and research paradigm, I captured the personal, social and professional experiences of my students in mathematics teaching. I captured the views of students with their practices and my reflection in the classroom. Hence observation was the key method to generate data in my research. "Participant observation has been associated with qualitative methods as

the data collected by this technique tend to be predominantly qualitative" (Lacono et al., 2009, p. 39). It offers unique insights into the participants when they are reflectively exploring their experiences. In that sense, observation is independent of what is being observed, and creates individual minds which give subjective reality to the research.

The collaborative teaching and learning approach encourages students' involvement in group discussion and peer work. Students working in a group, share their ideas on the subject matter. Active engagement and shared knowledge in the meaning-making process are major concerns of a collaborative approach. In that sense, the observation method is more appropriate for my research study. Observation may be regarded as the basis of daily life for all people. We observe the performances of participants and other materials surrounding it. As a qualitative researcher, I made prolonged involvement in the research field and observed the student's behavior before research and after research. Their experiences tried to connect prior knowledge and present subject matters, responses of students in group work and sharing habits in particular subject matters. As Ciesielska et al. (2018) stated, observation is one of the important methods that can be used in research strategies like case studies and action research. In my research, while I implemented an action plan in my field, major concerns with students' activities, behavior, interaction and honesty could be dignified through observation.

In my research, the data collection and analysis of data pose particular concerns about the teaching and learning process through a collaborative approach. I continued field work and observed the participants on a regular basis. Data collection could be time-consuming and tedious, and the result could accumulate large amounts of data. In the part of subjective data collection, I considered the influence of the

researcher over participant's behaviors and the impact of the researcher's own belief on the particular subject matter (Lacano et al., 2009). The data collection and analysis process with the choice of further data collection, is dependent on the results of previous analysis. Action research adopted such a research cycle throughout field work. "Action research is systematic and oriented around the analysis of the data whose answers require the gathering and analysis of data and the generation of interpretation directly tested in the field of action" (Greenwood & Levin, 1998, p. 122). It involves the action of community and organization members who seek to improve their situation.

Analysis of the data

In this phase, I analyzed the data collected from the field and took the help of literature to conclude the finding. Data analysis refers to the process of looking back into the research experiment (Azungah, 2018) that we have conducted among a certain group of participants so that we can extract the conclusion of our study. I have drawn some useful information from field work and interacted with colleagues, instructors and research supervisors to come up with a consensual understanding through the interpretation of the result and data collected. I have interpreted the students' engagement in the learning process and motivated them for meaningful learning through collaborative techniques. I went through the collected data obtained from field work and then made my themes. For more confirmation, I triangulated the findings with my research supervisor. Then I came to conclude my themes on the proper guidelines and suggestions of my supervisors.

Reflection

The action research is an action and reflection pendulum research design. The steps of cycle could not be complete until and unless the goal is achieved. In that

sense, reflection plays a vital role in the action cycle (MacNiff & Whitehead, 2006). It is the final step of action research in which I reflected upon my research findings and conclusions. What sorts of changes are seen in implementing a collaborative approach? Are the students still working competitively? Do they connect mathematics to their real life? Can they interpret the situation? Are they working in a group for achieving goals? Does collaborative learning enhance the learning environment? Is the collaborative teaching approach effective for meaningful learning? These interpretations and conclusions were made for future improvement in teaching practice.

After the analysis of the data obtained from the field work, I reflected on my strategies and plan. At the same time, I reflected on types of attitudes and behaviors adopted by my students in the collaborative teaching and learning approach, and reflected on my findings and results with a prior understanding of students before implementing the action plan. In reflecting on my teaching, I tried to improve my practices of teaching and learning mathematics and motivated the learners to learn mathematics in a real-world situation. Also, I reflected on my teaching practices concerning how collaborative techniques helped me to improve my teaching practices and basically how my students were promoted through this teaching approach for meaningful learning. The main focus was on students' engagement in teaching and learning mathematics.

Quality Standard

The researcher has to take responsibility for portraying an accurate and truthful research process. Without making any assumptions and constructing an imaginative environment, the researcher has to attempt the research in a realistic and natural setting so that the reader and participants of the study get valid and authentic

information. I ensured quality standards during the process of my research study. In my research study, different quality standards gave more authenticity to the research study. In that sense, I used credibility, transferability, dependability (Denzin & Lincoln, 2005), pedagogical thoughtfulness and praxis to consider the quality standards in my research paradigm throughout the research conduction. I applied those criteria for maintaining quality standards during my research. Similarly, I freely adopted participants' wishes and feelings during my action plan. I allowed my participants to reflect on their understanding during class and further improve if required. In the process of data collection, I interacted with the participants and listened to their individual reflections on the class. McNiff and Whitehead, (2006) say that an action researcher works as a practitioner and his/her job is to set the standards of practice and judgment.

Trustworthiness

Trustworthiness refers to the level of confidence in data that are collected from the research field, their interpretation, and the methods that are adopted to confirm the quality of any research (Anney, 2015). So, trustworthiness generally refers to the quality of making our research work trustable from different academic perspectives. Therefore, the opinions, experiences, reactions, and reflections of my participants were taken into account for the successfully accomplished research study. The establishment of these categories of procedures and techniques helps the readers to consider our research findings trustworthy. Degama et al. (2019) stated that the quality of research is basically measured by credibility, dependability, and transferability. Nevertheless, I have taken these as my quality standards along with Pedagogical truthfulness and workability which have been discussed in the following sections.

Credibility

The study refers to the precise explanation and identification of the problem in the context of the audience and the envisioned purpose of the research (Merriam & Grenier, 2019). It produces accurate and true information and identifies the real contextual problem of the audience. Credibility relates to the prolonged engagement of the researcher in research sites and regular observation of the participants each and every minute of the activities. In my research study, I have planned Eighteen days of action plans and prolonged engagement within these days to get authentic information from the field work. Also, according to Morse and Richards (2002), credibility talks about the un-bias shape of a finding. So, I considered these standards by reviewing the pre-conducted research on action research and theory and themes supporting my research. In my research study, to be assured about my research findings, I engaged with research participants in a prolonged manner and collected dense descriptions of their responses through triangulation and examined the negative cases.

In the same way, I abstracted my research problem from the real-life problem of my classroom scenario which was challenging for me to deal with. In addition to this, its significance, and the underlying assumptions of this action research are clearly stated in the previous chapter. Keeping regular field notes and recording each and every finding built goodness and trust in my research.

Transferability

The generalization of the research findings in a different context is called transferability (Marshell & Rossman, 1999 as cited Petrov & Hospedales, 2019). In that sense, research context determines the applicability of any research finding. Hence, researchers need to demonstrate clear pictures that will describe and inform the readers in detail about what and how the research finding works. In my research

study, I provided a thorough and detailed description of my research process to balance the transferability quality standard. While doing a detailed description, I have explained my research context, research sites and procedure that I adopted during my field work. I have considered- responses of my research participants, obstacles that I faced during the process, information collection methods and the analysis of the information. My actual reason for doing all these things was to make my report systematic so that readers or any other scholars can easily understand and generalize the findings of my results and they can link them in their own context.

Dependability

The other consideration in action research is dependability. It ensures that the researcher is "consistent and dependable with the data collected" (Bloomberg & Volpe, 2008, p.86). It doesn't mean to eliminate any inconsistencies but rather acknowledging them in the process of documenting and recording data collection procedures, methods, categories and developing themes (Bloomberg & Volpe, 2008). Patton (2002) suggested the record of researcher thought, and analysis as intellectual integrity creates transparency in the research.

In the context of my research, the major sources of data were the in-depth observation of participants' behaviors, their experiences, and collaboration among friends through collaborative approach which was recorded regularly as per my action plan. The conclusion and themes drawn are directly dependent on the data collected that acknowledge their consistency. In addition to that, the procedure for the data collection was direct observation of participants through their experiences and engagement in teaching and learning with group work, presentations and reflections.

Authenticity

Authenticity refers to the relationship between researcher and participants (Ellis &Bochner, 2000). This quality disclosed the interaction between researcher and participants, context and emotional compassion that occur during the research study. In this research study, I maintained the authenticity by random selection of participants without biasness. Since my research study was based on action research, I took a whole class (*there are 16 students*) as research participants and selected four participants as key participants with their performance and gender. I maintained harmonious relations with my participants by respecting their ideas and creating a sharing environment in the classroom.

Pedagogical Thoughtfulness

In general, pedagogical thoughtfulness refers to the application of the research findings for teaching and learning activities of the readers and researcher (Van & Manen, 2016). In addition, pedagogical thoughtfulness involves the readers in critical reflection of their own practices. It helps the reader to raise the question of whether they have the same types of experiences in their context and situation. In that sense, I included lots of participants' reflections and experiences in order to invite the readers to compare and contrast with them. Pedagogical thoughtfulness increased the common sense of teachers where teacher educators became more aware of their deep-seated assumptions which guided their beliefs (Luitel, 2009). To maintain the standard of research, I used the findings of the research as a tool for teaching and learning in my regular classes. I implemented the research findings as pedagogical implications in my day-to-day teaching. Eventually, these research findings might be helpful to other mathematics teachers for shifting pedagogical insight in their mathematics classroom.

Praxis

In this study, my approach to knowledge construction is inductive. It is based on my experiences in field work. So, praxis is one of the quality standards in the field of research work. It looks at the gapping between theory and practice in the relative field. In my research study, I implemented action research for searching effectiveness of collaborative approaches in teaching and learning mathematics. Action research is conducted by adopting an interpretive approach aiming to transform the traditions of the education system (Wright, 2020). In doing this research, I negotiated with my participants to instill 21st-century skills through the implementation of the collaborative teaching and learning approach at the secondary level. The reciprocal process would be useful for me to share the ideas between me and my participants.

Ethical Consideration

Every research study is highly delicate to the moral character and ethics of the researcher. It is the ethics of the researcher to respect the beliefs, views, and ideas of participants in the research study. Negotiating and securing access, protecting my participants' information and assuring good faith are the key things involved in the ethical consideration of the action research (McNiff & Whitehead, 2006). In my research study, I followed the same methodology. To maintain the privacy of participants and respecting participants' views and ideas was my first priority. My study considered all the ethical issues concerned with the research study.

I provided all the chances to the participants to express their opinion and ideas about the study freely and openly. I tried to do my best not to use sarcasm, reprimands, accusations and personal attack during the projects. As I was aware of the effects of my style of dealing with the participant, I maintained a good and

harmonious relationship with the participants and took consent for every activity related to participants' privacy and ethics as far as possible.

Chapter Summary

Each and every individual has unique norms and rights. For those who were involved in my research study, their consent was a must. With maintaining their privacy, I included the ideas and views of participants by reconfirming my understanding in multiple seating with them. This research study adopted action research as its methodology with an interpretative paradigm. My participants were major sources of my research field and collected information.

CHAPTER IV

PERCEIVING TEACHING AND LEARNING MATHEMATICS THROUGH COLLABORATIVE TECHNIQUES

In this chapter, I have analyzed the data to synthesize the major themes that sparked during eighteen days of field work. This section includes interpretation and analysis of description based on field information such as reflection of the participants, my self-reflection and my daily records during field work. I looked upon my research journey of collaborative teaching and learning as action research and reviewed the problems. In this specific chapter, reflecting upon the days of my experiences during field work, I have discussed the findings of the field-work and collected data explained with a different heading.

Collaborative Learning Indorses Learning through Pair/Group Works

It was the very first day of my field work (1st Sep. 2021). Before starting my class activities, I divided the whole class of students of Grade-X into four different groups (with 4 members in each group) based on the alphabetic order of the first name. I mentioned the names of four different groups: Pythagoras, Archimedes, Thales and Euclid. When I initiated the discussion about the plane figure contextualized with concrete objects- I asked "*What do you understand by plane figure*?" Only a few students participated in the discussion and the rest of the students performed as if they were not interested in my concern or they were in confusion about the subject matter. Again, I added further, "*Could you please give examples of plane figures interrelating real objects*?" Those few students who were considered "*talen*t" were answering the questions without group discussion and other students

were sitting as passive listeners. As I finished my question, some students raised their hands and tried to answer.

B. Deula : *Sir ! The plane figure is those figures which can be drawn on the plane surface like a copy, book, drawing paper and many more*.

S. Maharjan: *Plane figures are considered as two dimensions. It has two dimensions x-axis and Y-axis.*

J. Shrestha: *No! No! That figure which has an area is known as a plane figure.*

Again I gave the platform to each of the students for a discussion about the above ideas. I probed the questions to each group, *"Only plane figures can be drawn on the plane surface? What do you mean by two dimensions? Does only a plane figure have an area? Not in a solid figure too?* Such questions really sparked their minds and I could see all the group members involved in the discussion. They were trying to find answers to my questions with group discussion among their group members. Still, I saw that a few students were continuously speaking but the majority of the students were just staying and looking submissively. I realized that in the class instead of discussion and group work, there was competition. I found that they had competition among good performers' so-called talented students. When the students have self-centered behavior and competitive thinking they would be more exam-oriented to achieve good marks in the exam. It leads to the students for rote memorization and thorough practice in the algorithmic process (Pant, 2017). When I observed the natural setting of a classroom, I could see that the groups that I formed were actually impractical because they do have group discussion among good performers. Obviously, low performer students could not perform their understanding in front of others due to being tagged themselves as low achiever students. My concern was that they should work in a group and get equal opportunities to share

their understanding. So, immediately I instructed them and said, "*Everyone, whatever the things that you are sharing individually with me, please once discuss in your respective groups and come up with a common understanding with suitable reasons*" I could see that it was not easy for them working on the group. But they started to work on the group that could be perceived by observing group after group.

It was the second day of field work (2nd Sep. 2021). I assigned task-1 from worksheet-2 (*See Annex- B*), which was related to the definition of plane figures and their example. The classroom was relatively silent and talented students started to write without discussion and other students were trying to copy down their copies. Immediately I stopped them to write and instructed, "*This is your group task. Everyone should involve in group work and share your ideas among your group members then come with your actual understanding*". After my instruction, I could observe that many students were involved in the discussion. Sometimes they were laughing, sometimes they criticized their friend's understanding, and sometimes they drew figures to support their claims. Still, some students were passively listing their group voices and copying down the things that they discussed. I went there and asked them a question, *why are you not engaged in group tasks*? One of them replied, *we didn't understand whatever you wanted*. I convinced them. Now they were trying to share their understanding of what they had discussed in the group. *P. Bajracharya* from the *Thales group* said, "*I understood sir, plane figures have two dimensions but figures are regular or irregular doesn't matter". "Yes sir I agree with his statement",* S. Tandukar added. "*I came to know after discussion in my group that not only plane figures have an area. Besides that, solid figures also have an area because it contains different faces of two-dimension like rectangle, square, triangle, pentagon and so on",*

J. Shrestha shared her views. I could see they had an overall discussion on their task and they started to work in a group.

It was another event of day-7(8th September, 2021). When I entered the class, the room was relatively silent but some groups were discussing in the groups. I moved around the groups and told them to share their discussion in class. I allowed them to share their views. Ms. J. Shrestha from the *Pythagoras Group* said, "*When you assigned a group task for us, then we discussed that particular task in the group with our friends. It gave us more insight because all the group members consider different understanding in the particular subject matters*". I could see that in a couple of weeks students were more accustomed to working in a group and trying to share their understanding of particular subject matters. When I moved around the group, they discussed similar triangles by drawing different shapes of the triangle but the same shape. And I asked them to construct two triangles under the following measurement, " Construct ΔABC in which AB = 6cm, BC=8cm and $\measuredangle B$=90^0, and construct ΔPQR in which PQ=6cm, QR=8cm and $\measuredangle Q$=90^0". Students were constructing triangles based on the given condition. That day they were not facing any obstacles to constructing a triangle. After constructing triangles, I let them discuss in a group- how can we show the triangles congruent? Already they had prior knowledge about it. So, some students said "*by RHS axiom*" and I added the question "*why could not use ASS axiom*?" "*ASS's condition is also held by the triangles isn't it*?" After my question Group Pythagoras added," That's *also possible*", unexpectedly Group Euclid appealed, "*Right-angled is always constant, opposite angle of right-angled is the hypotenuse and any-one other side is given then absolutely we can RHS axiom*" When I asked a question, students addressed that with their group understanding based on the discussion. Students were working on problem-solving of work sheet-7 (*See annex B*). They started to see that

the condition of the RHS axiom was given two triangles. Then they became a little bit clearer about the subject matter and had a further discussion. During this time, they had a discussion and interaction about the RHS axiom. I observed that all students were involved in interaction by drawing a rough sketch of right-angled triangles. They were trying to explain the right-angled triangle with their priory understanding. Once they finished their discussion, one group said, "*sir, we have found the condition of congruency based on the RHS axiom but we need to discuss it with you once".* I said to them, "that's *fine, please think little bit more your group understanding and make sure, let's wait for other group too*" After that, all the group started to say," *Sir we also found the condition of RHS axiom*" While they were sharing knowledge of group understand in classroom, I found that individual understanding and student's prior knowledge was diverse and varied. Out of four groups only two groups gave the correct answer but two groups were unable to find the correct condition of RHS axiom. After that, I sent them to discuss the next two groups one by one. Within this time, they had a discussion on the condition of the RHS axiom. Those groups, who couldn't get the authentic ideas about it, were trying to probe the question to the next group and the next group trying to convince them by demonstrating figures. I could see that they had very fruitful discussion and interaction. I felt very happy because students were involved in intergroup interaction. At the end of the class, they were convinced and came up with their correct answer. They understood the condition of congruency of the RHS axiom.

The above discussion and interaction of students and among groups exposes that students were active participants in the class. They were arguing very logically and creatively about the ideas in different ways. At first, they were self-arguing in comparison with their prior knowledge and interconnected to present experience that

was with their colleagues and among group members. While doing so, at first, they examined those things on their own reasoning level and later on put their views on the common floor. Having discussions among friends, and sharing their thoughts and lived experiences with pair and group members, it could be helpful to develop consequent understanding about the subject matters. This was a very good example of a learning approach that was only possible through social interaction among all participants (Kalina & Powell, 2009). This approach advocates that each of the learners is equally responsible for constructing knowledge through logical interaction and using prior knowledge based on prior knowledge (Carruthers & Worthington, 2006). In this regard, while working in pairs or in the group, all participants were equally responsible for the knowledge gaining process and incorporating group learning to seek the outcomes of subject matters.

In my observation, the students used to take less participation before I started a collaborative teaching and learning approach. When I started my class through a collaborative teaching approach in the classroom, I found students were more engaged in learning with active participation and active dialogue with their friends and teacher. In the past, I didn't allow the student to discuss in class. I was not from an education background at the beginning of my teaching career. I was not a passionate teacher (Russo &Russo, 2019). So, I was a very strict teacher and I wanted *pin-drop* types of silent class. I had a fixed plan to solve some problems in white broad asking to copy down which was very important for the exam. In that, students didn't get any opportunity to share prior knowledge of the lesson in front of the teacher and friends. Due to my strictness, students were scared to share their understanding during class. When I implemented, collaborative teaching and learning approach in class, it

motivated learners to explore their ideas and knowledge in front of their friends and to ensure the questioning habits of the teacher and friend.

In that situation, the implementation of collaborative teaching and learning techniques provided them with a new dimension to acquiring knowledge. The pair discussion helped students by decreasing the work-load as it was distributed among pairs or groups. As the same, I always remember the English proverb, "*Many hands make light work*". So, teaching and learning mathematics became fun with group discussion and pair work. Students got an opportunity to share their own thoughts and settled the knowledge sharing habit. Before implementing the collaborative learning approach, students had mathematics anxiety and disengagement in the mathematics classroom because many students had a deep-seated belief in mathematics that it was a complicated subject and only talented students could learn. The perceived fun has a positive impact on students' apparent learning, motivation, attitude, self-efficiency, and intention to do similar work (Gani, 2021). In the group discussion, they feel free to talk and interact with their colleagues' and it creates an entertaining environment for learning mathematics. After that, low achiever students may arise in a part of poor alignment between student values and the classroom (Hill et al., 2021). Low achiever students started to interact with their friends and tried to generate ideas in mathematics problem-solving. They started to think of themselves in particular subject matters and shared views on their understanding.

As students shared individual knowledge with each-other, it generated new knowledge from consensual understanding. Collaborative teaching and learning allows students to evolve their ideas in a group and discuss them with their peers in the same group (Emam, Taha & EISayad, 2018). So, I came to know that collaborative teaching and learning approach was one of the superlative learning

methods which could motivate students for effective learning. Group learning is essentially the collaborative learning strategies which develop the skills to compete for 21st-century global education. Pair learning emphasizes collaborative learning and develops the learners by sharing knowledge. In my observation, collaborative learning promotes the learners to be engaged in pair or group discussions which give students direct involvement in the meaning-making process. It also helps students to share knowledge and enhance students centered learning approach in teaching mathematics.

Collaborative Learning Promotes the Engaged Learning through Questioning

It was the event of day-10 of my field work (12th September 2021). After the discussion of some previous day's work, I turned towards them and said, *"Today we are going to draw parallelograms which are standing on the same base and between parallels"*. Consequently, one student asked me *"we can draw a parallelogram with their measurement of sides or angles but how can we draw two parallelograms which are standing on the same base and between parallels?"* I realized that they were freely questioning the teacher which we couldn't see in previous classes. After his question, I replied, *"Okay! First discuss with your colleagues in your respective group and try to construct parallelograms"* By saying this, I distributed worksheets (*See Annex- B*) to each of the groups. After that, I said, "*Everyone, let's focus on task-1 which is related to constructing parallelograms standing on the same base and between parallels*". I could see that they were starting to draw parallelograms but some groups were still in confusion. Pythagoras' group asked me in a single voice, *"Are we getting confused in drawing parallel lines*?" "*If the base of the parallelogram is AB and how can we draw a parallel line of AB*?" I found that every group had confusion on this matter. I gave them the basic concept of constructing parallel lines by using a set square. After that they felt it was very easy to draw parallel lines.

Now they were busy doing their work on worksheets. After some time, Thales Group asked, "*Sir, we supposed base as AB and its opposite sides as a CD on another parallel line then we can join corresponding sides as A and D, B and C respectively, we got ABCD is parallelogram but how can we construct next parallelogram which is standing on the same base and between parallels*?" I appreciated their work and let them discuss in the group and find the solution to whatever is questioned by the *Thales group*. They discussed in their respective groups and tried to sketch the rough figure without using the geometrical instrument. Still, they were questioning me in each and every step. Eventually, the Archimedes group added,*" Yes sir, we got the final construction of parallelograms here it is*". They showed the final construction and elaborated in the classroom. I felt that questioning habits is very fun in mathematics teaching and learning. At that moment, I was a little disappointed because sometimes students asked un-relevant questions but at the same time I was really happy that they started to take mathematics learning in a somewhat interesting manner.

After the group discussion, I told them to start the task as instructed in the worksheet. They were supposed to construct parallelograms standing on the same base and between parallels. Task-1(*See annex B*) was related to constructing parallelograms and Task-2 (*See annex B)* was related to showing the area of parallelograms is equal. I told them to construct altitudes of parallelograms separately. In the beginning they were more concerned about constructing right-angled between two parallels but confused about the altitude of the parallelogram. Euclid's group added," *How can we construct the altitude of the parallelogram*?" One student of this group added," *Distance between the parallel line which is perpendicular to the base might be the altitude, isn't it sir*?" I appreciated him. All the groups were working.

One remarkable thing was that they could easily construct the altitude of parallelograms by using a set square and even they were finding the area of parallelograms. They started to conclude their result now not only in words but in mathematical terminologies as the parallelograms standing on the same base and between parallels are equal in area. After they concluded their statement, they moved to the implementation statement into problem-solving. I could see that they were easily being able to find the solution to the assigned task. In the case of difficulty, they discussed within respective groups. They asked me for help.

In my context, the traditional teaching and learning approaches couldn't accumulate the students' questioning in the classroom teaching. In my experiences, I found that 'questioning' is the better choice for involvement of students' in teaching and learning process. The students were more alert when I asked them a question. They were obligated to think about the *'issues raised in the question'* and get involved in the learning process. On the 7th day (8 September 2021) of my field work, a boy surprised me by answering the question related to the congruency of the triangle by the RHS axiom. I have presented the topic upon congruent triangle by RHS axiom and given some group tasks. Suddenly, I asked the question, "*How can we use the RHS axiom in different cases*?" to the *Thales group* B. Deula stood in his place and shared his understanding. He added, "*If two triangles are right-angled triangles and they have equal hypotenuse and side then we can use the RHS axiom in that case"*. At the beginning he was one of disinterested students in mathematics learning who never interacted in class and a submissive receiver of knowledge by the teachers. Also, in my mind, he was a submissive student. He had never answered my questions since the beginning of the academic session. But, now he started to answer the question after carrying out a collaborative learning approach in the classroom teaching. This shows

that the teacher has to play a facilitating role for driving students to construct knowledge by motivating the learners to be engaged actively in questioning.

Similarly, students showed their interest and eagerness to learn mathematics through an engaging question and answer session. They were engaging themselves by inspiring their misunderstanding and fallacies with the means of questioning which is more effective for engaging students in the learning process (Caram & Davis, 2005). On the second day of field work (2 September, 2021), a student asked, *"Why graphical method couldn't be used in a real situation/ common situation?"* This opened up the space for thought processes in the classroom. All the groups were pondering inside them for sketching geometrical figures in the graph. They concluded that all the regular geometrical figures having an interior angle of 90^0 could easily find their area by using the graphical method. Other regular or irregular geometrical figures which have to vary in interior angle might be difficult to find the area. It showed that students were more curious to probe questions and share the answers in the classroom.

According to Caram and Devis (2005), there are two ways of generating questions- these are 'teacher generated questioning' and ' Students generating questioning'. In my field work, I adopted both questioning methods. At first, the teacher starts questioning the students and second students ask questions about their confusion about the subject matter. I realized that probing questions plays a vital role in learning through both parties' teacher and student. As Le and Huse (2016) stated, there are six different methods of Socratic questioning. These are questions that require clarification, question probing assumptions, question probing reasoning and evidence, questions probing perspective, question probing implications, and question about the question. I was influenced by the first question type (question for

clarification), third (questions probing reasoning and evidence) and sixth (question about the question). Basically, students used to generate questions like, '*How to find the answer to this question? How can we solve this problem? What to do next and how can I move ahead? I am getting confused, please teach me again?'* etc. However, during my fieldwork, the student started to ask questions in this manner, *'Is this the correct solution?, What should I find……(this)…..for……..(that)…..Isn't it?*' In my experience, students were mostly involved in copying teachers' procedural solutions and seeking the support of the teacher for solving problems furthermore. They are habitual to give-up whatever they have difficulties in problem-solving and wouldn't search the alternative ways of the solution. After implementing a collaborative teaching and learning approach, they tried to solve problems themselves and in the case of problem solving they tried to consult with their friend.

It might be the result of the collaborative teaching and learning approach which gave a platform to ask questions. It changes the ways of asking questions during class and after class as well. Previously students barely asked questions in the classroom because they dared to ask questions. But, now I found students started asking questions and reformed the nature of questioning. This might change my behavior in my classroom teaching such as cooperative behavior, creating an interactive environment and suggesting ideas. Before, I used hard control of the students and I didn't give any interactive space for the students in the classroom. Students were hard to control under the teachers so, they wouldn't get the opportunity to share their knowledge within friends circle. The class with *pin-drop silence* was regarded as the best class in my point of view. But, while applying a collaborative learning approach, such views of students have been changed due to active and interactive classroom teaching. "*At first sir told us to do work individually without*

discussion but now gave us a platform of questioning and interaction in problem solving that could be more essential for mathematics learning" (Interview, 16th Sep. 2021). I gave them open space for discussion in the class and enough time for sharing knowledge with their friend circle. It promoted the teaching and learning methods of both teachers and students.

According to Gardner (1993), Students' learning is better when they are actively engaged in the learning process. For that, questioning is one of the best ways of active engagement in the learning process. While students raise the questions, they are involved in active interaction with their colleagues and teachers. So, we claim that questioning is the dialogue between two different parties that convince both in the particular subject matter, and as a result, they can construct knowledge. At the same time, the self-awareness of students in constructing knowledge alerts them to the truth and falsity of the subject matter. Moreover, questioning forces learners to examine the subject matter and steer inside the mind for better choices. While the students were involved in discussion with their own questions that showed the positive and negative issues in the particular subject matter, the resulting learner got motivated for the right and better choices. In this regard, collaboration helps the students to engage in learning and encourages them to construct knowledge through questioning.

Collaborative Learning Promotes Meaningful Learning through Engagement in Learning

It was the day of September 2nd 2021, which was the second day of my field work. Students were assigned to draw different geometrical figures on a graph. All the group members were busy drawing different geometrical figures with group discussion. The first fifteen minutes was given for drawing figures and collecting some information about them. This was a planned activity. Students involved in it and

they constructed the knowledge. They were so excited because they were learning about the area of the regular geometrical figure by graphical method. While I moved round the group a student asked," *what should we do after drawing a figure on a graph*?" and another student asked," *How can we find the area of drawing a figure*?". This question also reflects how much the students were excited. Students had shown keen interest to learn mathematics using different techniques and methods which was possible through students' collaboration. As Habarmas (1972) states, when children get self-motivated then they could develop their own understanding and learn themselves. So, students should mentally and emotionally be prepared to learn mathematics. From there, they know the ways of knowledge construction.

After that, I gave them extra ten minutes to count the unit square rooms that were covered by different geometrical figures. I found most of the groups had constructed regular geometrical figures like rectangles, squares, parallelograms, triangles, rhombus, etc. All the students were busy counting unit square rooms that were covered by geometrical figures in their respective groups. I asked, "*Have you finished the task*?" In a single voice, everyone said "*Yes sir*". After that, they shared the collected information. On sharing, a member of the *Pythagoras Group* was talking about the area of rectangles and squares. And the conversation followed,

Presenter: "*It is a very easy way to find the area of a rectangle and square. First, we should count the unit square covered by a rectangle and square then how many square rooms are there which are represented by the area of the figure*".

Listening to the presentation a student of *Thales Group* asked, "*Why count square rooms represented by the area of the figure*?"

"*Unit square itself is an area and how many square rooms are covered by the geometrical figure that becomes an area of this particular*" another student from *Archimedes Group* replied.

Presenter: "*Yes you are right*". The presenter agreed on the shared knowledge of *Archimedes Group*.

A student from *Thales Group* asked, "*How to find the area of the figure which is partially covered, especially those that occur in triangles, parallelograms, rhombus, etc?"*

Presenter replied," *If a geometrical figure covered the square rooms partially, in that case we can collect the part of the square room and add two or more than two parts together and make it single then count as one*".

Similarly, the presenter continued his presentation. Again, a student from *Euclid Group* stood and said, "*I found that, while multiplying length and breadth of the rectangle, if total counting square rooms covered by the rectangle are equal, the area of a rectangle is LxB*". In this conversation, I realized that they got a clear sense of the derivation of the formula. Throughout the conversation, the students were constructing knowledge or making meaning through different means and engaging in the learning process. I could see that they had lots of conversations about that subject matter. Each and every group member was actively engaged in the question-answer session. After the class, I wondered about the conversation between the students from different groups. They shared their understanding.

Firstly, learners were constructing knowledge by thinking from different perspectives and dimensions of mathematical thinking. They were engaged in the process of making meaning through another dimension for subject matters. According to Hall and Greeno (2008), "Concepts and their meanings develop and evolve in

settings of practice and are maintained in practice" (p. 213). So, the students' involvement in interaction developed the concepts and ways of formula derivation in the particular cases making meaning to effective learning. The concept of the area of the geometrical figure had been observed by the graphical method. They derived the formula through the graphical method separately by two different lenses. A student claimed that the product of horizontal length and vertical length of constructed geometrical figures remains the same when we count the number of unit squares covered by it. That is the perspective of finding areas through the graphical method and other students viewed the same terminology from the lenses of formula derivation.

Secondly, students constructed knowledge by linking with their prior knowledge. They engaged themselves in the process of meaning-making relating the present ideas with their prior knowledge. As Harris, Marx, and Blumenfeld (2008) states, holding prior knowledge can be used to interpret learning experiences and construct new knowledge. It seems that students interpret and elaborate the present learning process through previous knowledge in the subject matter and help to construct new knowledge. In the conversation, students were more concerned about an area of the geometrical figure by plotting on a graph. They connected that process with prior knowledge. They knew, the area of the rectangle was LxB. They connected the prior knowledge with the present issues and developed a kind of understanding of subject matters.

Thirdly, the meaning making-process took place through the active engagement of students in group discussion. Most of the students from various groups actively participated in conjugal conversation and the rest of them were listening to them curiously. These types of events exhibited by the students during the course are

a better sign of participation in the meaning-making process. I implemented a collaborative learning approach in my class, and took an interview with a few key students. She admitted, "*Group interaction and sharing among the groups helped me to understand mathematics in meaningful ways*." (Interview, 8th September 2021).

According to Fredrick, Blumenfeld and Paris (2004), three dimensions are associated with student engagement. They are behavioral engagement, emotional engagement and cognitive engagement. Behavioral engagement refers to abiding by rules and norms; emotional engagement refers to students' reactions to classroom activities; and cognitive engagement refers to students investing in their learning and enjoying a challenge. In my observation, if a teacher can balance these three dimensions in their classroom teaching, then students have a positive sense of mathematics teaching and learning. Students are mostly engaged in the learning process to get mathematical knowledge in problem-solving through interaction and discussion. Students actively participate in the learning process through various means such as thinking about the topics, providing suggestions, asking questions, and sharing their understanding with other learners. Such a case helps the students to identify the capability and ability of fellow friends and develop shared understanding collaboratively. Habarmas (1972) stated, through active interactions students were engaged in the learning process and made aware of learning methodology. They tend to learn through deeper interaction with their colleagues in the *practical interest* of knowledge. In the above conversation, students were engaged collaboratively, creating a knowledge-building environment themselves. In my observation, I found that children enjoyed the class more than on previous days. So for meaningful learning, students should participate actively in collaborative activities.

This was my bitter experience that there were many challenges in applying the collaborative learning approach for meaningful learning. During my field work, I noticed that some of the members of the groups were not interested or were passive in learning about their friends' conversations. In this case, learning might not be meaningful as they were not engaged in the conversation. At the same time, some participants were left behind. Immediately after the conversation with the students, I observed a child who got puzzled. I asked a question related to the conversation but he couldn't answer question. It means that there might be some other children as well, who couldn't understand the ideas of subject matters in robust form. Despite that, students engaged in learning that obviously activated the subconscious mind of learners when they were engaged in learning.

Collaborative Learning Helps to Construct Shared Knowledge

It was the event of my field work day 18 (21st September 2021). When I moved towards the classroom, they were sitting in their respective groups. After setting the classroom environment, I asked them, *'Any confusion at the beginning task*?' They said, they didn't have any queries about the previous class. And then I distributed the worksheet and said, "*Everyone lets focus on Task-1*". Task-1(See Annex-B) was related to the cyclic quadrilateral where different four quadrilaterals were constructed within (inside or outside) the circle. I turned towards the groups and said," *All the groups! Let's discuss in your respective groups and share your understanding*". After assigning the task, students were more concentrated in the task. I could see all the group members busy in identifying whether the quadrilaterals are cyclic quadrilateral or not. I asked those groups who finished identification of cyclic quadrilaterals. They shared the understanding in class. A representative of the *Pythagoras group* stood and said," *We have concluded that figure-I might be a cyclic*

quadrilateral." I added, "*Why figure- I only be a cyclic quadrilateral*? A presenter tried to justify the question and added," *You can see here, all the vertices of quadrilateral lie on inside the circle and quadrilateral is covered by the circle*". Suddenly, the Euclids' group added," *No! No! There is not any geometrical argument. If we can draw a quadrilateral inside the circle it might be diverse shapes and sizes which could not hold any geometrical fact.*" I could see they had a very fruitful discussion on the subject matters. In that situation, the classroom was crowded and students were more engaged in making new concepts on the subject matter. I realized the importance of creating an interactive environment in class that gives a suitable platform to the students for constructing new knowledge. Eventually, *Archimedes' group* said," *sir, we have found the concept of it.*" I asked them, "how?" They said," *In a third figure, all the vertices lie on the circumference of the circle but in others that lie somewhere outside the circle or inside the circle so it is a cyclic quadrilateral.*`` With a little bit of a suspicious look, I turned towards other groups and asked, " *What do you think about this concept, that is shared by the Archimedes group?*" They looked at the figures once again and quickly added their response.

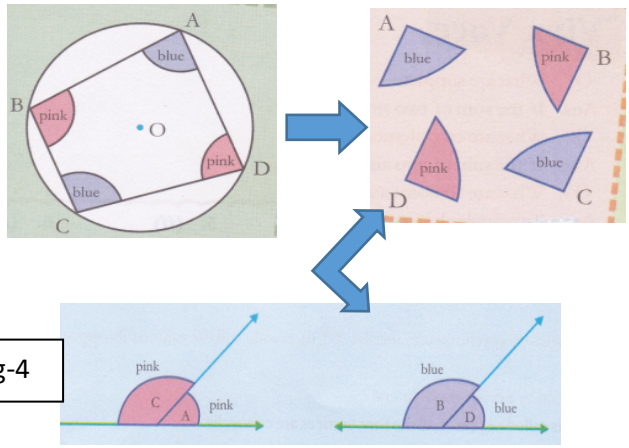

Fig-4

Pythagoras Group: "No sir! Still we got confused."

Thales Group: "*We are convinced with the statement of Archimedes group*". And again *Archimedes group* elaborated sequentially. They added" *All the vertices of a quadrilateral are placed on the circumference of a circle. If we construct a rectangle or square as a cyclic quadrilateral then we can see opposite sides and angles are*

equal and the sum will be 180^0." After that, all the groups constructed a cyclic quadrilateral in their exercise copy and tried to verify the condition of whatever *Archimedes group* had shared.

Pythagoras group: "*Yes sir! We found that the sum of opposite angles of a cyclic quadrilateral is 180^0 by paper cutting. You can see here opposite angles give us a semi-circle which is made in a straight line".*

Euclid group: "*Yes sir! We too found the same result*".

Thales Group: "*Yes, that's right*!"

I also felt very good to see their conversation. I said to the whole class, "*Yes, Archimedes group is absolutely right and could you please search for other properties of the cyclic quadrilateral*?" Some were saying," *Even we were thinking about this*". At the same time, a student stood and said," *Sir! I too have found one new thing*". I asked, "*What is that*?" He elaborated," *While we extend any sides of a cyclic quadrilateral then the exterior angle is equal to its opposite interior angles. Like ABCD be a cyclic quadrilateral while we extend the side AB to E then we got ∠ CBE as exterior angle and its opposite interior angle is ∠ADC which has equal sizes of angle*". When he was elaborating on such conditions, others were also checking their figures and they were saying, "*Yes! Absolutely correct that statement*."

The beauty of a heterogeneous class is that each individual is unique in their level of understanding. For a common instruction, each individual understanding and response to the same subject matter may act differently. Changing the track of traditional teaching to collaborative teaching, students should get the opportunity of group discussion with active participation. They exhibited different colors. The above conversation shows that the students are not only following the instruction but at the same time they are being deeply engaged in knowledge construction by examining the

results from their own cognitive level. Some of them started to show their active participation whereas some of them started to share their understanding. They were being interested in sharing their findings and at the same time others were cross-checking them and consensually constructing knowledge from them. The environment was so interesting and suitable that they were extracting new knowledge and practically examining and then finally concluding their innovative ideas in the group. Keeping aside my algorithmic process of a problem-solving approach, introducing collaborative teaching and learning approaches helped learners to *think beyond the box*.

Students were active participants in the learning process through different thoughts on subject matters, providing suggestions, raising questions and sharing their understanding among their colleagues. It is more helpful to the students' for identifying their capability and ability by sharing understanding collaboratively. "*Sharing our understanding among our circle gave me self-satisfaction and more insight into our mathematical problem solving"* (Interview, 16th Sep. 2021). In this scenario, students started to think beyond real classroom teaching. They started to connect their learning through the socio-culture perspective (Vygotsky, 1978). They helped their peers to solve the problem, actively engaged in group discussion, and shared an understanding which was helpful to the students' actual development by independently solving the mathematical problems. In this sense, the zone of proximal development (ZDP) helped the students to develop their ability for solving problems in collaborative ways. Vygotsky (1934) defined, " the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more peers' '(p.86). This concept enhances individual

learning. When students are working in a group with group collaboration, such collaboration endeavors help the students to develop new concepts and skills in mathematics learning. The main goal of education from Vygotsky's perspective is to keen learners in their own ZPDs as possible by giving them culturally meaningful learning and problem-solving task. It is quite difficult, if they do it alone. They will need to work in a group among their colleagues to finish the tasks. In this regard, when they work in a group, students could get the concept of problem-solving and then share their understanding in the classroom which gives authentic learning in mathematics teaching and learning. Sometimes, the task assigned to the students is beyond their understanding. Students cannot do that individually at first but with the guidance or help of others, they have the potential to accomplish their tasks. In my experience, when I introduced a mathematical problem in the classroom, they were actively engaged in discussion and able to share their understanding sequentially.

In the sense of a collaborative approach, students didn't only act as passive recipients but they became essential parts of the learning process as contributors. This made students construct the knowledge with the help of prior knowledge and experiences through group discussion and peer work. Such types of work create a knowledge-sharing environment during class. In this regard, if the students are given a chance to be exposed in an innovative learning environment rather than underestimating and doubting their ability and their innovative nature, they feel free to engage in knowledge construction and create shared understanding of the subject matter. Students are involved in group discussions and they share their understanding in the classroom that could be reformed in shared knowledge. Shared knowledge is an understanding and appreciation (Cooprider, 1993) among the students in the process of teaching and learning. Appreciation among the groups is characterized by the

frame of interpretation of the other group and knowledge resides primarily in the relationship between individuals and groups which connect with particular norms, attitudes, information flow, and ways of making decisions. In that sense, while students work in a group and are engaged in knowledge construction that gives opportunities for sharing knowledge in a group, it may changed to shared knowledge. So, the prior knowledge of students and their cultural background which leads to the development of new knowledge is crucial. They get involved in the learning process actively which led to exposure to shared knowledge. Knowledge is created as it is shared (Thijssen, et al, 2002). In this regard, students create an individual learning route according to their interests, abilities, and learning styles. If the students get an opportunity to share their views in the learning process, the learner will be more active for meaningful learning.

Students are more interactive during group discussions and arguing with each other during the process of sharing knowledge. Vygotsky (1978) believes in experiential learning that occurs from the prior knowledge of learners. Students are more interactive during group discussions. Utilizing their prior knowledge, they get opportunities to experience such a learning environment. That scenario creates a knowledge-sharing atmosphere for the students in their group and classroom. Classroom is a society. Learning begins in the social context which supports learners in the processes whereby they construct their own understandings (Whitebread, 2012). Constructed knowledge is shared on behalf of their meaning-making process. Shared knowledge could be more authentic for students during group interaction to get the mathematical concept.

Nevertheless, I compared my class before and after the implementation of collaborative approach. I experienced a vast difference. I used to stop them for all

kinds of extra thinking and sharing knowledge in the classroom. I used to think about whatever I taught. Instead of open discussion and letting them respond, they were largely instructed to go for rigorous practice and rote memorization. Due to this reason, though I used to tell them so many facts about the subject matter, they didn't pay attention to much or simply renounce them. But the collaborative learning approach gave them at least some space for their personal reaction to subject matters. In this way, collaborative learning created an interactive environment for sharing their understanding and arguing with each other during the process of constructing new knowledge.

Chapter Summary

In this chapter, I elaborated on the multiple themes generated by the data collection. I collected the data through real classroom teaching in a natural setting. The recorded data, students' reflection, my self-reflection and field notes were used to analyze the major themes and thoughts. I explained different themes in different headings, and observed them based on the literature and theories which I had mentioned in the previous chapter.

In the next chapter, I have presented the research findings and conclusions of my research by reviewing my research question based on the literature and a short description of my future plan in the research field. Also, I have figured the implications of the findings.

CHAPTER V

REFLECTING BACK AND SETTING CONCLUSIONS

This chapter reflects my own journey towards selecting the research agenda, formulating the research question, research paradigm and methodology for future directions, and possible implementations of the research conclusions. It also discusses the research conclusions based on the major themes extracted in the previous chapter.

In my context, I tried collaborative teaching and learning approaches in my natural classroom setting. The implementation of the collaborative approach as a new method of teaching and learning mathematics in my class was very fruitful and effective for my students and me as well. In my past teaching experiences, students had disinterest and might be dissatisfied with mathematics learning so the performance of the students wasn't satisfactory. Now, the new learning environment has encouraged the students to engage actively and has created a creative environment themselves. They have started to accept that they were an important member of the classroom and they have been playing a vital role to construct new knowledge.

As a practitioner-researcher, I reflected on my motivation factors for research, particularly how I got interested in research on this research agenda, and how the research was conducted. I reflected upon my own learning experiences and interests that I felt and observed during the study. This chapter mainly focuses on the interpretations, implementations and conclusions of the research study.

My Research Journey

I was attracted by the incident in my usual class as a student. He shared his aim to be a businessman but not a mathematics teacher. The reason behind it was that he felt it very difficult to teach geometry. That issue directly questions the present

teaching and learning process in the classroom. The question (*What is the use of geometry in my life*?) inspired and obliged me to reflect upon my teaching and learning practice in the classroom. Then, I consciously searched for the cause of such problems in my mathematics class.

Why does a learner feel bored learning mathematics?

Why do learners have little interest in learning mathematics?

How can I reform the perception of my students?

How can I create a learning environment for effective learning?

And how can I engage them in mathematics learning?

These questions troubled me continuously.

Suddenly, I remembered how my university classroom was set up. Professor divided us into different groups and assigned reading tasks of varied journal articles. We were given enough time to discuss in a group and then a platform to share understanding within the group as well as in the classroom. Professors used to talk about various teaching and learning methods for better understanding. Then, I realized the need for pedagogical change in my teaching of mathematics.

Initially, I planned for the collaborative teaching and learning approach in my class for teaching geometry. I planned an eighteen days action plan and these days were so busy and interesting for me. From the beginning to the end of the field work I tried to adopt various ways of learning mathematics such as presentation, pair-sharing, posing tasks and lived practical tasks to see the students' participation in the learning process. I observed my student's behaviors, enthusiasm, and participation in group task and pair collaboration. Still, I remember that students were waiting for me in the class with excitement on their faces, in their dialogues, and their hurriedness.

Students were excited, engaged, and curious to know *what comes next*. These all events made me feel proud of the collaborative learning approach.

I, myself being a teacher-researcher, the action plan developed was applied in the classroom and information was collected. At first, I studied the information collected and searched for the major ideas on all data/information. After that, I made four themes based on the entire information. And next, I consulted with my dissertation supervisor for a discussion on the major theme whatever I found.

In my experience of fieldwork and research journey, the collaborative teaching and learning was productive in the sense of engagement and meaning-making process of students. Students got more opportunities to relate mathematical concepts through group work and peer discussion while engaging in the process of constructing mathematical knowledge. Working in a collaborative learning environment has not lightened their workload but it has made them open up themselves to learn on the basis of their prior knowledge. During the learning, students got an opportunity to know each other more deeply. It leads them to acquire some transversal skills such as communication, presentation, team work, inter/intra personal skills, and leadership skills.

This research study has become very fruitful for me as a researcher or practitioner of teaching mathematics. It has changed my deep-seated beliefs and perception about students and their learning environment. Through this research, I came to realize the individual difference of my learners in depth. Previously, most of the students had misconceptions about learning mathematics. Later on, when I applied a collaborative approach in my teaching, it changed the students' behavior and their perception of mathematics learning. They started to work in groups, and had collaborative discussions with friends. It changed my mindset in teaching and

learning. Learning is a social process and being a social creature, learner gets the opportunity in the learning process through engagement. In my context, collaborative teaching and learning helped me and my students to know the actual meaning of mathematics learning and to seek how the knowledge is constructed through engaged learning in particular subject matters.

Formulation of the Research Question

The research question drives the researchers to the right path. In this way, the research question is taken as an essential component of research. I discussed in the earlier section the pain while giving birth to the research question. I moved through various modes of research in the beginning and finally landed on a question. At the time of developing the data analysis chapter, those tentative research questions got refreshed many times with multiple settings with research supervisors. This study has only one research question as mentioned below;

1. How can I engage learners in the meaning-making process while teaching and learning geometry through a collaborative approach?

I developed only one chapter for my research question to capture participations' beliefs and assess my practices on different constructs. I attempted to unfold my beliefs about learning mathematics by the student and its influences on classroom teaching. I have experienced the traditional teaching and learning of mathematics almost all the time during my schooling. It was more difficult for me to flip the traditional approach to the modern teaching approach in my practice. Traditional teaching approach couldn't easily welcome the notion of collaborative teaching and learning approach. The self-justificatory approach of proving mathematical facts and theorems couldn't take me outside the classroom.

The work of Norton (2018) in the field of action research and pedagogical implications opened my eyes to the planned base mathematics teaching. In the beginning, I encountered the problem that most of the students were not motivated to learn mathematics. That situation leads to passive participation in learning mathematics. The classroom behaviors of my students drove me to think about my teaching and learning practice in the real world. I questioned myself, "*Why are the students not participating in group discussions*?", "*Why is the child not capable of solving a mathematical problem in an understanding manner*?" Such questions popped into my mind. I searched for different pedagogies to make students' participate in classroom interaction. I decided to select and apply a collaborative teaching and learning approach in my class. In the beginning, I formulated the research question based on my research problems.

Reflecting on My Action Research Cycle

In the beginning, my thought was that 'mathematics can be learned by rote memorization and thorough practice'. Somehow these were reflected in my past schooling and after being a mathematics teacher in a private school. My position was- mathematics teachers should be strict and students should memorize the process of problem solving and precise practice of mathematical problems that bounce more authenticity in learning mathematics. I mostly focused on the algorithmic rigorous practice of mathematics problems. In my teaching career, these were my misconceptions about mathematics teaching and learning. I could search for new teaching methods for the betterment of students in their learning mathematics. But as the years passed by, my mathematics teaching in the classroom was being unfair to the students for their learning rights. It was because of the lack of interest and lack of pedagogical changes in mathematics teaching. At the same time, I used to describe

and teach as thoroughly as I could and even used to share how to get very good marks in mathematics. That could be useful to only students who were tagged by talented students in class. Rest of them had no concern about it. I remember that I used to prepare a plan, explanation, and demonstration in my regular class. Now, I can easily see that I was at the center of students' learning. The class seems more teacher-centered and my students were just passive receivers. I was applying just a lecturer method in my teaching and following algorithmic steps of mathematics to make my students 'talented' in mathematics subject matters. But amid this horrible situation, I got an opportunity to know the various ways of teaching and learning that were practiced in schools and colleges. Among the various approaches, my attention was "*why we could facilitate the students through student collaboration in the learning process*?" I adopted a collaborative teaching and learning approach in my teaching. So, I decided to research this approach in my classroom teaching.

At the beginning of my research, I reviewed different research, scholar's journal articles, and a number of dissertations from the KUSOED library. I made a common outline of my research. Normally, the outline was included with a background of the study, literature review, methodology (action plan, observation, and data interpretation and analysis), and at last the data analysis. This guideline always alerted me to move along the right track, being aware of my research agenda. As my proposal was accepted by the research committee, I made my precise action plan for Eighteen days and moved into the real classroom setting to implement my plan. I applied my plan in a real classroom setting among the students and through the direct observation of students' responses to new teaching and learning approaches. I recorded all the information as students' reflections, my daily field notes and self-reflection. For confirmation, I took interviews with four key students according to

their past performance in mathematics. In the end, I tried to extract the themes from the collected data and information which was precisely explained in chapter IV.

Conclusions

In a broader sense, my research agenda was to explore the pedagogical shift in the secondary level of mathematics teaching. In responding to research questions, I employed an interpretive paradigm with a focus on action research. I used multiple methods of data collection like participants' observation and interviews. I planned an eighteen days action plan and implemented it in my regular classroom setting. I analyzed and interpreted data based on my observation, students' reflection, daily notes and my reflection. Finally, I concluded my research agenda with multiple theories and drew the major as my insights after analysis and interpretation.

In doing so, I concluded that the conventional teaching approaches were ir/relevant for diverse learners. Contents and courses were directly focused on teacher centric approaches. In the present situation, due to rapid development of technology, modernism and perception of parents, students and teachers has been changed but our teaching method is directly influenced by teacher centric approaches. In that scenario, our teaching and learning process, student's minimum learning outcomes did not match due to lack of appropriate teaching approaches. So, students were taking mathematics as one of the most difficult subjects. They were studying mathematics because it was one of the compulsory subjects at the school level. They sometimes question themselves, *"who made this subject?", "why is it a compulsory subject?", "where are the uses of this subject*?" Moreover, they showed their disinterest and frustration towards mathematics learning and felt as if mathematics was a foreign subject (Luitel, 2009).

At the beginning, I went through a complete conventional teaching approach such as rote memorization, rigorous practice and *chalk and talk* method during my classroom teaching. It was more effective for those who can easily memorize the concepts and those who were more used to rigorous practice. And the rest of students were looked at as a passive listener. They were dependent upon other colleagues to complete their task. Sometimes I was tired of explaining concepts of mathematics. I could read their faces demanding alternative ways of learning strategies. We as a teacher had to go beyond the concept of "one size fit all" types strategy of teaching approach. I decided to implement a collaborative teaching and learning approach in my regular classroom setting. I purposely chose a chapter on the geometry of Grade X.

The existing pedagogy which was implemented in the Nepali classroom was not appropriate to teach mathematics. There was a huge gap between the students' perception and teaching method. The pedagogy used by teachers should be concerned with the lives and daily experience of diverse students. After implementing a collaborative teaching approach in my natural classroom setting, students linked their mathematical concepts beyond the four walls of the classroom. They were aware about the weakness and strength of their friends circle. They accepted their limitations and at the same time acted collaboratively. I claim that the collaborative teaching and learning approaches play a vital role to engage the students in mathematical learning through meaningful ways. The study proposed to shift the pedagogy from the traditional *chalk and talk* approach to collaboration learning. During the research, I came to know that collaboration helped the students to engage in mathematics learning in various ways such as questioning, pair/group discussion, sharing knowledge, and conceptual understanding through different activities. Students

actively participated in learning activities through various ways and it helped them to build up self-confidence, leadership, and sharing and constructing new knowledge.

This study helped to understand the effectiveness of implementing collaborative teaching and learning approaches to motivate and encourage the learners to actively participate in the learning process. Previously, while I adopted a lectured method in my mathematics classes, I had struggled to attract the students in the learning process. After implementing the collaborative teaching and learning approach, they showed generic interest to interact with their friends for constructing knowledge through active engagement and group discussion. Further, their discussion involved thinking about the related topic being taught. I have had the role of facilitator in the class and helped them to share knowledge through their understanding, and support them in need based. It has helped me to create a learning environment in classroom teaching. In this process, students benefited from their burden of mathematics and reduced the misconception about mathematics. They discussed the particular task and got the opportunities to interact/share their ideas during the class. Indeed, they were not limited inside the four walls of the school. They have begun to explore mathematics in the community/ society too.

This study has reflected my teaching and learning style and the nature of students in mathematics class. My study can be more helpful to novice teachers and teachers' educators for further development of educational practice and self-motivation in teaching and learning. I understood that every learner is unique and they have different abilities to grasp the knowledge. If teachers applied such pedagogy to recognizing the actual problems of the students, they could become more engaged and could catch the meaningful learning of mathematics. Although the selection and development of the suitable group task to overcome the existing mathematical

problems were rigorous for me, regular monitoring and updating the lesson plan helped me to know the effective ways of emerging for the students for meaningful learning through their curiosity. Nevertheless, this study made me aware about *'pedagogical turn'* in my teaching profession.

My Pedagogical Change

In my past experience, I was mostly focused on students' performance as good marks achiever in the exam. And at the beginning, I was called successful in some aspects as well. Years were passing. I practiced the same teaching and learning mechanism. I started to see the disinterest and dissatisfaction of my students towards mathematics learning. I was doing my best during class to make the 'robots' react to my instruction. But coming to the extreme point of my teaching when I was about to be worn out, I got an opportunity to try something new for my literally dying class. I introduced a collaborative teaching and learning approach in my teaching and learning. That situation completely changed my perception of teaching and learning as well as my role inside the classroom. Previously, I used to have full responsibility for classroom learning but now my students have responsibly in their learning. I used to move around from bench to bench and almost the whole time was spent for writing, solving problems, explaining and checking their work during class. I had a concept that mathematics should be well understood in the *pin-drop silent* class and involved in thorough explanation followed by rigorous practices. But now, my classroom structure has completely reformed.

Rather than being passive knowledge receivers, now my students have started to take part in the learning process as active learners. After applying the collaborative approach, I have found that they have been logically discussing the various subject matters in much a deeper manner. They are working together to construct knowledge,

put their views among their friends, share individual opinions on the particular subject matter and discuss various mathematical facts by themselves through different mathematical induction. I realized that for a long time I was tired of explaining the importance of mathematics and the uses of mathematics in their daily life. That time, I was a failure but now my students are trying to see mathematics around them. Those were the pedagogical changes of my teaching experiences. My students have understood that knowledge is not a thing that is achieved through competition only but it is a continuous process. We see the classroom as a society where the students of different ethnic groups and diverse cultural backgrounds gather under a common roof. In that sense, the collaborative teaching and learning approach helped the students to get knowledge through group discussion, peer collaboration and sharing knowledge among friends rather than competition.

Now, my focus and my pedagogical change in my teaching are providing an appropriate learning environment for students where they are engaged in logical discussion and classroom activities with collaboration. During the overall research time, I observed my students learning various mathematical concepts. I facilitated the basic ideas in particular subject matters. Sometimes, I acted as a learner and took part in their discussion rather than being part of them. In this regard, my pedagogical changes are very fruitful for my students as well as my teaching career too.

Implementation of the Study

This research study was intended to improve the teaching and learning practices of mathematics intending to engage learners actively in the meaning-making process through collaborative teaching and learning approach. It might be useful for teaching and learning mathematics as well as for implementing in my teaching practice. In my research, there were two roots: one was to explore the possible ways

that can be implemented in classroom teaching which could help the students to know the mathematical concepts in a meaningful manner and the next was to help them to build the construction of knowledge through collaboration by reducing the competitive attitude in knowledge gain. As I reflect back on the complete scenario of the research study, I have got the appropriate positive result by implementing the collaborative teaching and learning approach.

The research findings might be useful to the teacher educators, and my fellow teachers. The effectiveness and efficiency of research findings is based on different constraints such as academic grades, participants, subject matters, learners in the classroom, attitude and behavior of learners and their psychological feelings. In my view, the collaborative learning approach is applicable to a large number of other related situations and learning environments. In my context, this approach helped me to understand the diverse learning nature of my students. At the same time, I could understand how I should facilitate them to address their needs. Each student has their own belief in their learning which is determined by his/her society and cultural background. So, without understanding their cultural background, if we try to fill them with our knowledge, it might be like water in the sand. The research findings may not be applicable for all settings. But, if the readers are facing similar kinds of problems in their teaching profession such as less interest in learning mathematics, problems in getting conceptual understanding, disinterest in assignments etc., it might be more effective in such related fields.

Every teaching and learning practice is limited concerning participants, place, and time duration. There may be so many shortcomings and limitations. In a similar manner, the collaborative teaching and learning approach also has certain constraints. It cannot be generalized directly but other scholars can go through it and practice

transforming it in their practical context. The research findings may be more effective for some groups of participants and less effective for others. Likewise, it might be productive in one context but might not work in another case.

Collaborative learning is more beneficial for constructing a conceptual understanding of students rather than procedure. Students were more engaged in group discussion, peer interaction, presentation, and sharing of knowledge among the groups, which enhanced the conceptual understanding of the students. In that sense, I have observed that all the students were able to answer the question in a logical manner but unable to write algorithmic ways of solutions.

This research study can be applied to a small group of participants where various forms of activities can be handled easily and effectively. It can be a limitation of the research study in a large group. It is difficult to address the needs of all participants. In this approach, students should work in groups with not much number of students. In that situation, the large number of participants may not be manageable for different activities. And as this approach is based on learning through students' collaboration where participants actively take part in the learning process, it needs a separate classroom setting with enough infrastructures. Due to this reason, it cannot be practiced with a large number of students and at the same time, the rigid structure of the classroom is the main obstacle to implementing this approach.

However, I cannot deny that every student has a different capacity and capability for learning and thinking about mathematics. Learning might be affected by their interest and effectiveness. In my understanding, there might be some students who are left-over or who could not achieve the intended learning outcomes through a collaborative learning approach. In this regard, my research study may be helpful for those teachers and academicians who are facing such types of problems in classroom

teaching. This research study will be more useful as a reference for further research studies. Time estimation of course content allocated by CDC seems relevant. My suggestion to the policy makers is to incorporate or adapt such an approach in the planning of mathematics education in future.

Future Directions

This research study has been an excessive success for my teaching and learning in terms of pedagogical transformation. I got the golden opportunity to observe my students' sense of multiple perspectives such as their behavior in group work, initiation in leadership, peer collaboration and group collaboration for shared knowledge and socializing skills. Even though collaborative teaching and learning do not favor thorough practice, such an approach helped my students to construct conceptual knowledge in mathematics. The learning behaviors of students such as continuously working on their task, active participation and questioning in class, and colleague supportive nature have been observed during and after implementing the collaborative learning approach. I have started to cooperate with different other approaches in my teaching and learning which can support the diverse nature of my students. It helped me to develop a vigorous and friendly relationship with my students in my classroom teaching.

I got excited and motivated to use different pedagogies to engage my students in the construction of meaningful mathematical concepts. Implementation of collaborative approach provided me with a new technique to know my students in real situations and encourage them in knowledge construction. Therefore, this approach has raised my positive attitude toward mathematics teaching in my classroom teaching. I will continue using the collaborative method in my teaching practices and hence extend its further research in my further research.

Chapter Summary

This chapter is a more extended review of the whole research process and interpretation of research agenda. I reflected my present status as a teacher-researcher and my perception of my pedagogical shift in classroom teaching. I observed how my students started to construct knowledge by being actively engaged in their group tasks; how the research question had been formulated and which belief system guided my research study. I reflected on the present position as a researcher and most importantly, the pedagogical shift in my teaching and learning process. Finally, I ended the research by exploring my future directions and further destination in the research field under the collaborative teaching and learning approach.

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ANNEX A

Action Plan

S.N	Topics	Estimated Days (CDC)	Estimated days	Estimated Period	Remarks
1	Area of plane figure (Area of triangle, square, Rectangle, parallelogram, Rhombus, trapezium and)	2 Days	2days	1st and 2nd	September- 1 and 2
2	Similar triangles Axiom for similarity of triangles Relations of corresponding sides and angles of similar triangles	2 days	2days	3rd and 4th	September- 3 and 5
3	Congruent triangles Axioms for congruency of triangles Relations of	4 days	4 days	5th and 8th	September- 6, 7, 8 and 9

	corresponding sides and angles of congruent triangles				
4	Relation between the area of triangles and quadrilateral Related theorems - Diagonals of parallelogram bisect the parallelogram. - Parallelograms standing on the same base and between parallels are equal in area. - The area of a triangle is equal to half of the area of a parallelogram standing on the same base and between the same parallels. - Triangles on the	4 days	4days	9 th to 12th	September-10, 12,13 and 14

	same base and between the same parallels are equal in area.				
5	Geometry- circle Definition of term related to circle (Circumference, Semi-circle, Arc, Chord, Segment, Sector, concentric circle, intersection of circle, Tangent of circle, secant)	1day	1 day	13th	September-15
6	Theorem related to arcs and the angles subtended by them - If two arcs of a circle subtended equal angles at the center of a circle, the arcs are equal. - Equal arcs of a	5 days	5 days	14th to 18th	September-16,17,19,20 and 21.

	circle subtend equal angles at the center of the circle. - Equal chords of a circle from equal arcs in the circle. - The angle at the center of a circle is twice the angle at its circumference standing on the same arc. - The angle in the circumference of a semi-circle is one right angle. - Angles in the same segment of a circle are equal. - The opposite angles of a cyclic quadrilateral are supplementary.				

Action Plan I (Day 1, 2)

Area of Plane Figure

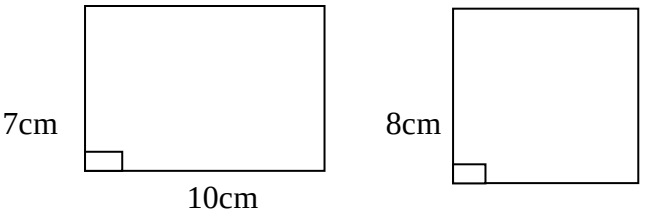
Learning Targets
Day 1 - Students can give the examples of plane figure with the help of real objects. - Students can differentiate the plane figure according to their sides. - Students can identify the relation among the sides and angles of plane figure. Day 2 - Students can solve the problem related to area of plane figure by using formulae.
Guiding Questions
Q.1. Can you give the examples of plane figure which is found in our surrounding? Q.2. Do the all plane figures are regular? How can you categorize the regular and irregular plane figure? Q.3. what are the process to find the area of plane figure?
Required Material
Various plane figures pieces of papers, geometrical construction and measurement tools, real objects (Desk, Benches, Door, windows, Books, copies etc.)

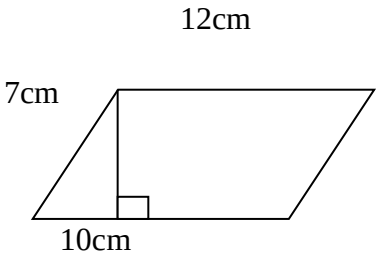
Sequence of plan	Estimated time	Differentiated Instruction
Day 1 Pre-assessment: (Look or observe) 1. Starting with the discussion about plane figure. Let them to collect the plane figure with the real	5 mins	

objects in respective groups. 2. Asking question to the students randomly about what are plane figure with real objects examples. 3. Asking them, what are the difference between plane figures and real objects? Main Activity: (Act) 1. Dividing students in to 4 groups and each group contain 4 students. 2. Discussion on group and tell them to collect real objects and categorize them. Interrelating real objects and let them to draw different plane figures by the help of construction instrument. 3. Let them to categorize these figures with their sides and interior angles. 4. Let them discuss and try to find the characteristic of different geometrical plane figures. Post-assessment: (Reflect) Q.1. How can we categorized plane figure? Q.2. what are the characteristics of different plane figure? Q.3. Is the plane figure is 2D? Why? Home-assignment: Work sheet:	30 mins 10 mins	

Day 2 Pre-assessment: (Look and observe) 1. Asking students randomly about previous day findings. 2. Can you differentiate between plane figure and real objects? 3. What are the characteristics of plane figure? 4. Do you have any idea to find area of plane figure? Main Activity: (Act) 1. Let them sit in their previous day's respective groups. 2. Provide the graph sheets to each group and tell them to construct different plane figures. 3. Let them to discuss in their respective group, what the area occupy by different plane figure. 4. After that let them find the area of parallelogram by the help of graph sheets then apply in other plane figures. Post-assessment: (Reflect) Q.1) How to find the area of different geometrical shapes.	5 mins 30 mins 10 mins	

Q.2) Find the area of given figures;





Home-assignment:

Work sheet:

Action Plan II (Day 3, 4)

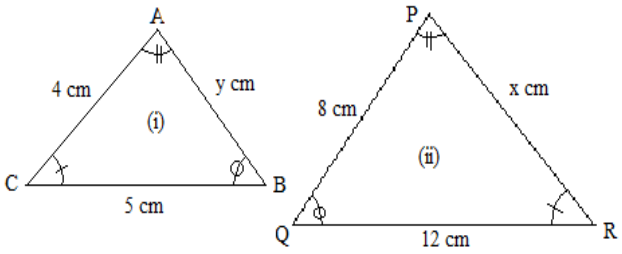
Similar Triangles

Learning Targets
Day 3 - Students can give the examples of various similar figures. - Students can identify the similar triangles. - Students can identify the relation among the sides and angles of the pair of similar triangles. Day 4 - Students can identify the relation among the sides of the pair of similar triangles. - Students can find the unknown sides from the pair of similar triangles.
Guiding Questions
Q.1. Can any pair of triangles be similar triangles? Q.2. Do the similar triangles have same shape and size? Q.3. Will there is any relation among sides and angles of similar triangles?
Required Material
Various triangular pieces of papers, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Day 3 Pre-assessment: (Look or observe)		

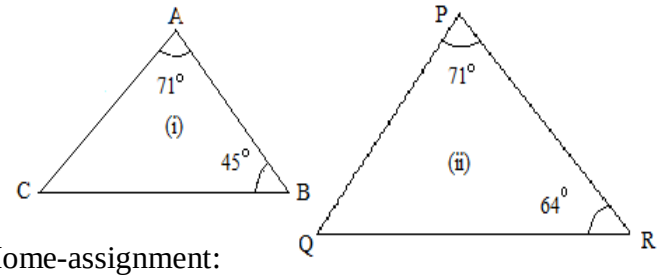
1. Starting with the discussion about similar objects. 2. Asking students randomly about what are similar objects with some examples. 3. Asking them about how certain things are similar. 4. How the triangles can be similar triangles? Main Activity: (Act) 1.Dividing students in to 4 groups (same as previous days) 2.Let them draw different two triangles with the same angles (example – 50^0, 70^0 and 60^0) 3. Let them write all angles and their corresponding sides and let them fill table with their measurements. 4. Let them discuss and try to find the relation among the sides of the similar triangles on the basis of measurement table. Post-assessment: (Reflect) Q.1. Is similarity related with shape or size? Q.2.By which axiom a pair of triangles can be similar? Q.3. If pair of triangles is similar then will there be any relation among sides of them? Home-assignment:	5 mins 30 mins 10 mins	

Work sheet: Day 4 Pre-assessment: (Look and observe) 1.Asking students randomly about previous day findings 2. If the triangles are similar, what is the relation among angles between the similar triangles? 3. What is the relation among the sides of the similar triangles? Main Activity: (Act) 1. Let them sit in their previous day's respective groups. 2. Providing them different sheets of papers where they will find out the length of unknown sides of various pair of similar triangles. 3. Let them write the relation among the sides of the similar triangles through discussion. 4.After that let them find the solution for some related Mathematical problems. Post-assessment: (Reflect) Q.1. Find the length of x and y if $\Delta ABC \sim \Delta PQR$.	5 mins 30 mins 10 mins	



Q.2. If ΔABC ~ ΔPQR then find the measure of

$\measuredangle ACB$ and $\measuredangle PQR$..



Home-assignment:

Work sheet

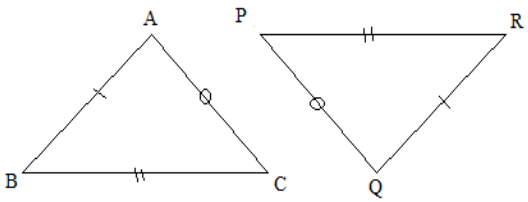
Teacher's comment

Action Plan III (Day 5)

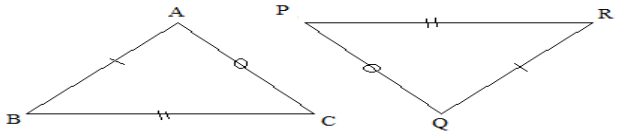
Congruent Triangles

Learning Targets
- Students can write the corresponding angles and sides of the congruent triangles which are congruent under S.S.S. axiom. - Students can obtain the solution for finding unknown angles and sides of the pair of the congruent triangles.
Guiding Questions
Q.1. Do the congruent triangles have same shape and size? Q.2. How S.S.S. axiom is tested and implemented in pair of triangles?
Required Material
Various triangular pieces of papers, geometrical construction and measurement tools.

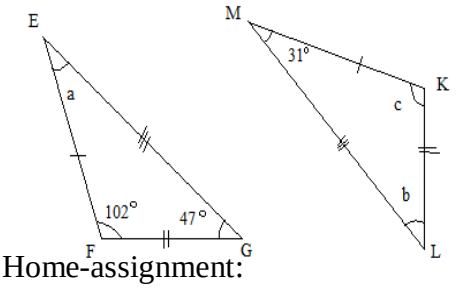
Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: Look and observe Q.1. What do you mean by congruency of triangles? Q.2. What is the symbol for congruency? Q.4. What do you mean by S.S.S. axiom? Main Activity: Act 1. Dividing students in their different groups.	5 mins	

2. Asking them to draw two triangles having equal shape and size. 3. If some of them they could draw such triangles then let them share their construction idea in front of class. 4. In case of then could not draw then asking them to draw two triangles of given sides lengths with the help of compass and pencil. (AB = 6 cm, BC = 8 cm and CA = 5 cm) 5. Now ask them to measure all the sides and Their corresponding angles. 6. Let them share their findings regarding sides and their corresponding angles. 7. Also asking them to find out the axiom for this type of congruency. 8. Now with discussion asking them to do calculation for the unknown sides of the given pair of congruent triangles. Post-assessment: Reflection Q.1. By which axiom ΔABC and ΔPQR are congruent?  	30 mins 10 mins	

Q.2. If $\Delta ABC \cong \Delta PQR$ then write the all pairs of equal corresponding angles.



Q.3. If $\Delta EFG \cong \Delta KLM$ then find the unknown angles.



Home-assignment:

Work sheet

Teacher's comment:

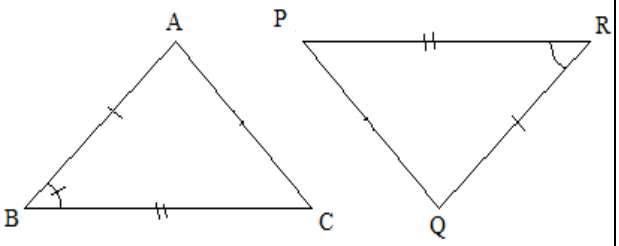
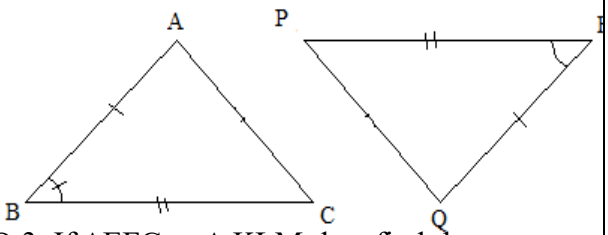
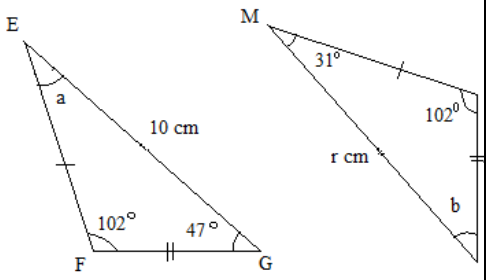
Action Plan III (Day 6)

Congruent Triangles

Learning Targets
- Students can write the corresponding angles and sides of the congruent triangles which are congruent under S.A.S. axiom. - Students can obtain the solution for finding unknown angles and sides of the pair of the congruent triangles.
Guiding Questions
How S.A.S. axiom is tested and implemented in pair of triangles?
Required Material
Various triangular pieces of papers, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: Look and Observe 1. Briefly revising previous day work. (Oral question) 2. What does S.S.S. axiom stand for? Main Activity: Act 1. Dividing students in their respective different groups. 2. Asking them to draw two triangles having	5 min	

equal shape with any two sides and angle between them equal. 3. If some of them they could draw such triangles then let them share their construction idea in front of class. 4. In case of then could not draw then asking them to draw two triangles of given sides lengths with the help of compass and pencil. (AB = 5 cm, BC = 6 cm and $\measuredangle B = 75^0$) 5. Now ask them to measure all the sides and their corresponding angles and fill the table. 6. Let them conclude their result and share their findings regarding sides and their corresponding angles. 7. Also asking them to find out the axiom for this type of congruency. 8. Now with discussion asking them to do calculation for according to S.A.S. axiom. 3. Let them find the corresponding angles, sides of given pair of congruent triangles. Post-assessment: Reflection Q.1. By which axiom ΔABC and ΔPQR are congruent?	30 min	

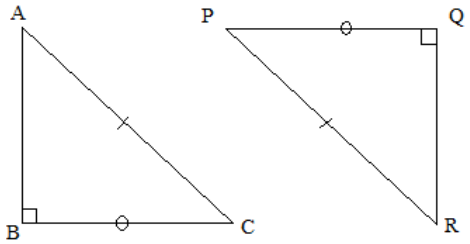
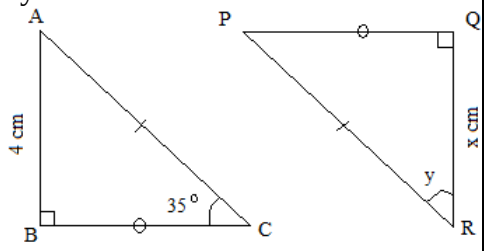
 Q.2. If ΔABC ≅ Δ PQR then write the all pair of equal corresponding angles.  Q.3. If ΔEFG ≅ Δ KLM then find the unknown side and angles.  Home-assignment: Work sheet Teacher's comment:	10 min	

Action Plan III (Day 7)

Congruent Triangles

Learning Targets
- Students can write the corresponding angles and sides of the congruent triangles which are congruent under R.H.S. axiom. - Students can obtain the solution for finding unknown angles and sides of the pair of the congruent triangles.
Guiding Questions
How R.H.S. axiom is tested and implemented in pair of triangles?
Required Material
Various triangular pieces of papers, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: 1. Previous day review Main Activity: 1. Introduction about R.H.S. axiom. 2. In group, discussing and plotting the triangles according to R.H.S. 3. Let them find the corresponding angles and conclude their answer. Post-assessment: Q.1. By which axiom ΔABC and Δ PQR	5 mins 30 mins	

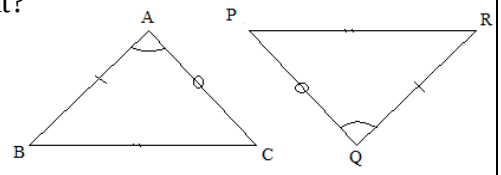
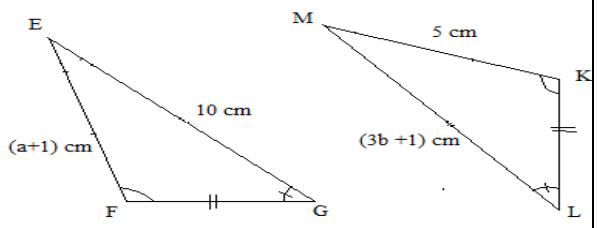
are congruent?  Q.2. If ΔABC ≅ Δ PQR then write the all pair of equal corresponding angles. Q.3. If ΔABC ≅ Δ PQR then find the value of x and y.  Home-assignment: Work sheet Teacher's comment:	10 mins	

Action Plan III (Day 8)

Congruent Triangles

Learning Targets
- Students can write the corresponding angles and sides of the congruent triangles which are congruent under A.S.A. axiom. - Students can obtain the solution for finding unknown angles and sides of the pair of the congruent triangles.
Guiding Questions
Q.1. Do the congruent triangles have same shape and size? Q.2. How A.S.A. axiom is tested and implemented in pair of triangles?
Required Material
Various triangular pieces of papers, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: 1. Previous day review Main Activity: (Act) 1. Introduction about A.S.A. axiom. 2. In group, discussing and plotting the triangles according to A.S.A. axiom. 3. Let them find the corresponding angles and conclude their answer.	5 mins 30 mins	

Post-assessment: Q.1. By which axiom, ΔABC and ΔPQR are congruent?  Q.2. If ΔABC ≅ Δ PQR then write all the corresponding equal sides and angles. Q.3. If ΔEFG ≅ Δ KLM then find the values of a and b.  Home-assignment: Work sheet Teacher's comment:	10 mins	

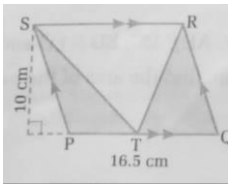
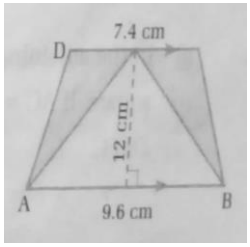
Action Plan IV (Day 9)

Relation between area of triangle and quadrilateral

(Diagonal of a parallelogram bisect the parallelogram)

Learning Targets
- Students can draw the parallelogram and recognized the diagonal of parallelogram. - Students can use the outcomes of the theorem in related problem solving.
Guiding Questions
Q.1. How can we draw the diagonals in the parallelogram? Q.2. what types of postulates and axiom can be used to prove the theorem? Q.3. How can you apply the theoretical outcomes in the related problem?
Required Material
Various parallelogram pieces of papers, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: 1. Review the prior knowledge of parallelogram. Main Activity: (Act) 1. Introduction parallelogram on the sides and interior angles. 2. In respective group, discussing and plotting	5 mins 30 mins	

the diagonals of parallelogram in the pieces of parallelogram paper. 3. Let them to discuss and point out the possible postulate and axiom can be use in theorem proving. 4. Prove the theorem on the basis of their appropriate ideas and understanding. Post-assessment: Q.1. Find the area shaded portion of following adjoining figures.   Home-assignment: Work sheet Teacher's comment:	10 minutes	

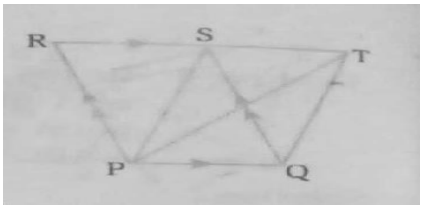
Action Plan IV (Day 10)

Relation between area of triangle and quadrilateral

(Parallelograms on the same base and between the same parallels are equal in area)

Learning Targets
- Students can recognize parallelograms standing on same base and between parallels. - Students can establish the relation of parallelogram which stands on same base and between parallels through the method of area of parallelogram and use of postulates and axiom.
Guiding Questions
Q.1. Why altitudes between two parallel lines are equal elsewhere? Q.2. how can you identify that two parallelograms are standing on same base and between parallels? Q.3. How can you apply the theoretical outcomes in the related problem?
Required Material
Various parallelogram pieces of papers, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: 1. Previous day review. Main Activity: (Act) 1. Introduction parallelogram and see the	5 mins	

drawing skills of each group then let them to present their understanding in group" how can we draw the parallelograms standing on same base and between parallels". 2. Ask them to draw altitudes between parallels and students can find the area of parallelogram by the help of area's formula and conduct group presentation to share the result and findings. 3. Prove the theorem on the basis of their appropriate postulates and axioms. Post-assessment: Q.1.Why the area of parallelograms is equal which standing on same base and between parallels? Write your understanding on the basis of findings. Q.2. In the given adjoining figure, PQRS and PQTS are two parallelograms standing on same base and between same parallels. If the area of ΔPQR is 50cm^2 then find the area of parallelogram PQRS.  	30 mins 10 minutes	

Home-assignment: Work sheet Teacher’s comment:		

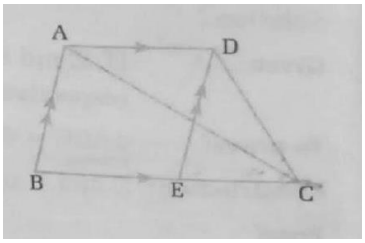
Action Plan IV (Day 11)

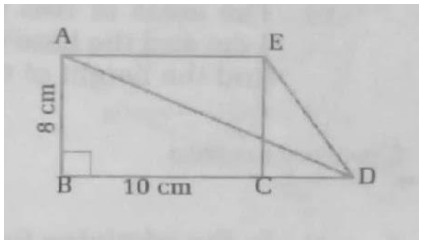
Relation between area of triangle and quadrilateral

(Area of a triangle is equal to half of the area of a parallelogram standing on the same base and between the same parallel)

Learning Targets
- Students can recognize parallelogram and triangle which is standing on same base and between parallels. - Students can establish the area of parallelogram is double of the area triangle which is standing on same base and between parallels through the method of area of plane geometrical figure and use of postulates and axiom.
Guiding Questions
Q.1. How can you identify that the parallelogram and triangle are standing on same base and between parallels? Q.2. How can you calculate the area of parallelogram and triangle? Q.3. How can you apply the theoretical outcomes of the theorem in your related problem?
Required Material
Various parallelogram pieces of papers, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: 1. Previous day review. Main Activity:	5 mins	

(Act) 1. Introduction parallelogram and triangle which is standing on same base and between parallels through their common understanding within group. 2. Let them to plotting triangle and parallelogram which is standing on same base and between parallels. 3. Individual groups share the ideas and understanding of above activities in a classroom. 4. Ask them to draw altitudes between parallels and students can find the area of parallelogram and triangle by the help of area's formula. 5. Prove the theorem on the basis of their appropriate postulates and axioms. Post-assessment: a) Q.1. In the adjoining figure, the area of trapezium ABCD is 100cm^2 and the area of ΔADC is 40 cm^2. Find the area of ΔDEC.  	30 mins 10 minutes	

b) In the given adjoining. If, ABCE is a rectangle, find the area of Δ ADE.  Home-assignment: Work sheet Teacher's comment:		

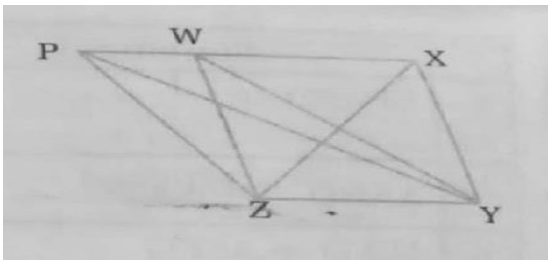
Action Plan IV (Day 12)

Relation between area of triangle and quadrilateral

(Triangles on the same base and between the same parallels are equal in area)

Learning Targets
- Students can recognize triangles which stand on same base and between parallels. - Students can establish the area of area triangles are equal which is standing on same base and between parallels through the method of area of plane geometrical figure and use of postulates and axiom.
Guiding Questions
Q.1. How can you identify that the triangles are standing on same base and between parallels? Q.2. How can you calculate the area of triangle? Q.3. How can you apply the theoretical outcomes of the theorem in your related problem?
Required Material
Various triangle pieces of papers, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: 1. Previous day review. Main Activity: (Act) 1. Introduction triangles which stand on same base and between parallels within their	5 mins 30 mins	

common understanding of individual group and let them to share common ideas in a classroom. 2. Let them to plotting triangle and parallelogram which is standing on same base and between parallels. 3. Ask them to draw altitudes between parallels and students can find the area of triangle by the help of area's formula and interrelate the result to the argument. 4. Prove the theorem on the basis of their appropriate postulates and axioms. Post-assessment: a) Q.1In the adjoining figure, WXYZ is a rhombus in which XW is produced to the point P. If, WY=10cm and XZ = 9cm, find the area of Δ PYZ.  Home-assignment: Work sheet Teacher's comment:	10 minutes	

Action Plan V (Day 13)

Geometry- circle

(Definition of term related to circle)

Learning Targets
- Students can define a circle and point out the center of circle and circumference of circle. - Students can recognize the term related to circle.
Guiding Questions
Q.1. How can you define the circle? Q.2. Why terms related to circle are important to us? Q.3. How can you identify that the center angle and circumference (Inscribed) angle by indicating center and circumference point?
Required Material
Various A4 size papers with drawing different measurement of circle, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: 1. Review the prior knowledge about circle. Main Activity: (Act) 1. Let them to draw a different circle in respective group. And tell them to discuss on radius of circle, diameter of circle.	5 mins 30 mins	

2. Students are involving in the activity which related to cut and paste. Let them to cut the pieces of paper by drawing circle. And then by the paper fold method, each group present their understanding about semi-circle, Segment and Sector. 3. Ask them to draw line segment in the circle then let them to categorize terms related to circle in the different formation (like chord, arc, secant and tangent) 4. Let them to draw a two or more circles in intersection form and relate the concept of intersection of circle and concentric circle. Post-assessment: Q.1. Explain and defined the terms related to circle and write down it in sequential form. ……. ……. ……. …… ……. …… Home-assignment: Work sheet Teacher's comment:	10 minutes	

Action Plan VI (Day 14)

Geometry- circle

(Relation between equal arc and subtended central angle)

Learning Targets
- Students can measure the central angle and its corresponding arc. - Students can identify that, two arcs of a circle subtended to equal central angles then measurement of two arcs are equal. - Students can identify that, two equal arcs of a circle subtend equal angles at the center of the circle.
Guiding Questions
Q.1. How can you measure the central angles and its corresponding arcs? Q.2. what is the result, when two arcs subtended to equal central angle? Q.3. what is the result, when the center angles standing on equal arc?
Required Material
Various A4 size papers with drawing different measurement of circle, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: 1. Review the prior knowledge about circle. Main Activity: (Act) 1. Let them to draw a different circle by	5 mins 30 mins	

using geometrical instrument in respective group. Students are marking equal arcs on the circumference of circle. Ask students to make center angle which subtended to equal arcs. 2. Ask them to prepare table for filling measurement of center angles. After filling table, conducting group presentation of collaborative work on the basis of result and findings. 3. Let them to draw a different circle by using geometrical instrument in respective group. Students are drawing equal central angles on the center of circle. Ask students to measure the arcs which corresponding to center angle. 4. Ask them to prepare table for filling measurement of arcs. After filling table, conducting group presentation of collaborative work on the basis of result and findings. Post-assessment: Q.1. Draw a two circle with different radii. And construct two equal arc in	10 minutes	

which O be a center of circle, AB and CD are equal arc whose corresponding central angles are ∡ AOB and ∡ COD respectively. Discuss in your group and fill the following table.

Figure	∡ AOB	∡ COD	Result	
I				
II				

Conclusion:-

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Home-assignment:

Work sheet

Teacher’s comment:

Action Plan VI (Day 15)

Geometry- circle

(Relation between corresponding arc and chord)

Learning Targets
- Students can measure the sizes chord and its corresponding arc. - Students can identify that, equal chords of a circle form equal arcs in the circle. - Students can identify that, the arcs formed by chords of a circle are equal then chords are equal.
Guiding Questions
Q.1. How to measure the chords and its corresponding arcs? Q.2. Is there any relation between equal chord and its corresponding arc?
Required Material
Various A4 size papers with drawing different measurement of circle, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: 1. Review the previous lesson. Main Activity: (Act) 1. Let them to draw three circles with different measurement of radii by using geometrical instrument in respective group. Students are marking equal chords on the circle.	5 mins 30 mins	

2. Ask them to prepare table for filling measurement of arc corresponding to equal chords. After filling table, conducting group presentation of collaborative work on the basis of result and findings.

3. Let them to draw three circles with different measurement of radii by using geometrical instrument in respective group. Students are marking equal arcs on the circumference of circle.

4. Ask them to prepare table for fill up the measurement of arc corresponding to equal arc. After filling table, conducting group presentation of collaborative work on the basis of result and findings.

10 minutes

Post-assessment:

Q.1. Draw three circles with different radii. And then, marked arcs on a circumference of circle as arc AB and arc BC in each of circle. Draw a segments by taking ends points of each arc. Then tabulate the result in following table;

Figure	Length of segment AB	Length of segment CD	Result

Home-assignment:

Work sheet

Teacher’s comment:

Action Plan VI (Day 16)

Geometry- circle

(Relation between central and inscribed angle)

Learning Targets
- Students can draw the center and inscribed angle which subtended to same arc. - Students can measure the central and inscribed angle by the help of geometrical instrument. - Verification experimentally proves, the central angle is double of inscribed angle subtended to same arc.
Guiding Questions
Q.1. How can you differentiate central and inscribed angle of a circle? Q.2. Is the measurement of central angle and inscribed angle is equal? Q.3. What are the process of experimental verify the above relation?
Required Material
Various A4 size papers with drawing different measurement of circle, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: 1. Review the previous lesson and adding some relative questions. Main Activity:	5 mins	

(Act)

1. Let them to draw three circles with different measurement of radii by using geometrical instrument in respective group. They are marking arc on the circumference of circle and drawing central, and inscribed angle by the help of geometrical instruments.

2. Ask them to measure the central and inscribed angle, and result are tabulated. After tabulating, conducting group presentation of collaborative work on the basis of result and findings.

3. Prove the above relation theoretically with the help of definite postulates and axioms

Post-assessment:

Q.1. Draw three circles with different radii. Marked the central angle as ∡AOB and inscribed angle as ∡ACB. ∡AOB and ∡ACB both are standing on the same arc AB. Tabulate the result in following table after group discussion.

30 mins

10 minutes

Figure	∡AOB	∡ACB	Result

Home-assignment:

Work sheet

Teacher's comment:

Action Plan VI (Day 17)

Geometry- circle

(Relation between inscribed angles on same arc)

Learning Targets
- Students can draw the inscribed angle which subtended to same arc. - Students can measure the inscribed angles by the help of geometrical instrument. - Verification experimentally proves, the inscribed angles are equal when it subtended to same arc.
Guiding Questions
Q.1. How can you draw inscribed angles of a circle on the same arc? Q.2. Is the measurement of inscribed angles is equal? Q.3. what are the process of experimental verify the above relation?
Required Material
Various A4 size papers with drawing different measurement of circle, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: 1. Review the previous lesson and adding some relative questions. Main Activity: (Act) 1. Let them to draw three circles with different measurement of radii by using geometrical	5 mins 30 mins	

instrument in respective group. They are marking arc on the circumference of circle and drawing inscribed angles which subtended to same arc by the help of geometrical instruments.

2. Ask them to measure the inscribed angles, and the result is tabulated. After tabulating, conducting group presentation of collaborative work on the basis of result and findings.

3. Prove the above relation theoretically with the help of definite postulates and axioms

Post-assessment:

Q.1. Draw three circles with different radii. Marked the inscribed angles as ∡ABC and ∡ADC. ∡ABC and ∡ADC both inscribed angles are standing on the same arc AC. Tabulate the result in the following table after group discussion.

10 minutes

Figure	∡ACB	∡ADC	Result

Conclusion:-

Home-assignment:

Work sheet

Teacher's comment:

Action Plan VI (Day 18)

Geometry- circle

(Opposite angle of cyclic quadrilateral is supplementary)

Learning Targets
- Students can draw the cyclic quadrilateral and marked the vertices. - Students can measure the measure the each vertices of cyclic quadrilateral. - Verification experimentally proves, opposite angles of cyclic quadrilateral is supplementary.
Guiding Questions
Q.1. How can you recognized cyclic quadrilateral? Q.2. Is there any difficulty to measure of vertices of quadrilateral? Q.3. Is the sum of opposite angles of cyclic quadrilateral is 180^0? Q.4. How can you interpret the cyclic quadrilateral on the basis of angular measurement of vertices?
Required Material
Various A4 size papers with drawing different measurement of circle, geometrical construction and measurement tools.

Sequence of plan	Estimated time	Differentiated Instruction
Pre-assessment: 1. Review the previous lesson and adding some relative questions. Main Activity: (Act)	5 mins	

<table>
<tr>
<td>1. Let them to draw three circles with different measurement of radii by using geometrical instrument in respective group. Ask students to draw a quadrilateral inside the circle and marking vertices of cyclic quadrilateral.
2. By the help of geometrical instrument let them to measure the vertices and the result is tabulated. After tabulating, conducting group presentation of collaborative work on the basis of result and findings.
3. Prove the above relation theoretically with the help of definite postulates and axioms
Post-assessment:
Q.1. Draw three circles with different radii. Draw a cyclic quadrilateral ABCD and mark the opposite angle. Tabulate the result in the following table after group discussion.
<table>
<tr><td>Figure</td><td>∡A</td><td>∡B</td><td>∡C</td><td>∡D</td><td>Result</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
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</table>
Conclusion:-
Home-assignment:
Work sheet</td>
<td>30 mins

10 minutes</td>
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</table>

Teacher’s comment:		

ANNEX B

Day To Day Schedule Work Sheets

Work sheet – Day 1

Q.1) Collect the name of real objects which consider following geometry shapes.

a) Rectangle Shape:-…….

b) Square Shape:- ……

c) Triangular shape:-……

d) Parallelogram shape:- ……

e) Quadrilateral Shape:- ……..

Q.2) what are differentiating between real objects and plane figures? (Let's discuss on your respective group and mentioned the common understanding)

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Q.3) Is the plane figures are 2D? If, yes write the characteristics of 2D figure.

……

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Task 1:- Fill the given table. (Let's discuss on the group and fill the following table)

S.N	Plane figure	Real objects(at least 2)	Characteristics of plane figure(at least 2)
1			
2			

Task 2:- Conclude your result by group discussion on the basis of your findings.

Work sheet – Day 2

Q. 1) what do you understand by area of plane figure? What are the different ways to find the area of plane figure? (Let's discuss in your respective group then write the common understand)

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Task -1, Draw the following figure in a graph sheets according to their dimension which mentioned below.

a) Area of rectangle whose length is 5units and breadth 3units.

b) Area of square whose length is 6units.

c) Area of parallelogram whose base is 6units and altitude 4 units.

d) Area of triangle whose base is 7units and altitude is 5units.

Task – 2, Conclude your result of task-2, by group discussion on the basis of your findings

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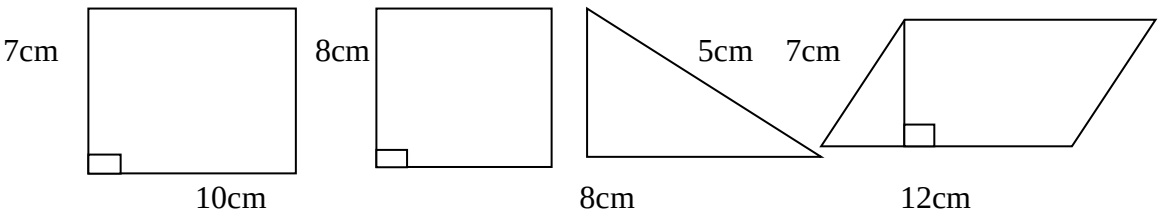
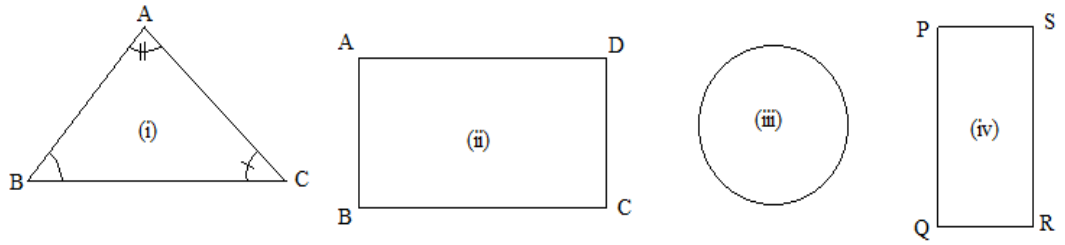
Task -3, Find the area of following plane figures.



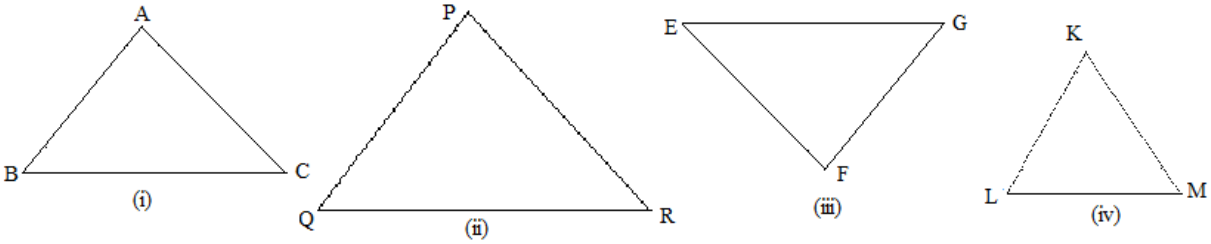
Work sheet – Day 3

Q.1) which of the following figure are considered as similar figures?



a) (i) and (iii) b) (ii) and (iii) c) (i) and (iv) d) (ii) and (iv)

Q.2. which of the following figures are similar?



a) (i), (ii) and (iii) b) (i), (iii) and (iv) c) All of above d) None of the above

Q.3. which one of the following symbols represent the similarity?

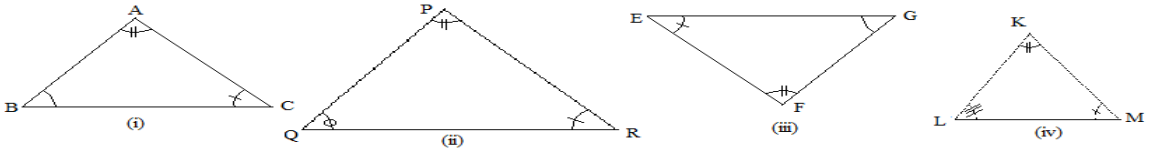
a) $\approx$ b) $\cong$ c)$\sim$ d) $=$

Q.5. which of the property belongs to the similar triangles?

a) Same Shape and same size. b) Same shape and different size

c) Different shape but same size. d) Neither shape nor size are same.

Q.4. which of the following triangles are similar triangles?



a) (i) and (iii) b) (ii) and (iii) c) (iii) and (iv) d) (i) and (ii)

Q.5. Draw any two triangles Δ ABC and Δ PQR with the help of protector whose angles are ($\measuredangle A = \measuredangle Q = 60^0$, $\measuredangle B = \measuredangle R = 75^0$, $\measuredangle C = \measuredangle P = 45^0$) then fill the following table:

Task 1 : Fill the given table:

Name of Triangle	Measurement of angles			Measurement of sides		
Δ ABC	$\measuredangle A$ =	$\measuredangle B$ =	$\measuredangle$ C =	AB =	BC =	CA =
Δ PQR	$\measuredangle$ A =	$\measuredangle$ B =	$\measuredangle$ C =	PQ =	QR =	RP =

Task 2 : Fill in the blanks:

1. Opposite side of $\measuredangle A$ = …… 2. Opposite side of $\measuredangle P$ = ……

3. Opposite side of $\measuredangle B$ = …… 4. Opposite side of $\measuredangle Q$ = ……

5. Opposite side of $\measuredangle C$ = …… 6. Opposite side of $\measuredangle R$ = ……

Task 3 : Fill in the blanks:

A. $\frac{AB}{QR}$ = — = — , $\frac{BC}{PR}$ = — = — , $\frac{AC}{PQ}$ = — = —

Task 4: Conclude your result by group discussion on the basis of your findings.

……

……

……

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Work sheet – Day 4

Q.1. By which axiom a pair of triangles will be similar?

a) S.S.S. axiom b) S.A.S. axiom c) A.A.S. axiom d) R.H.S axiom

What is the reason?

……

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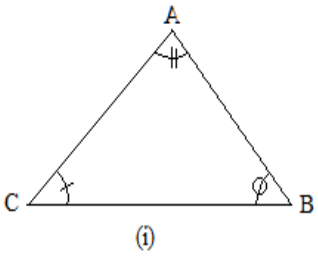
Q.2. If ΔABC and ΔPQR are similar triangles then (Discuss in your group and give the common understanding)

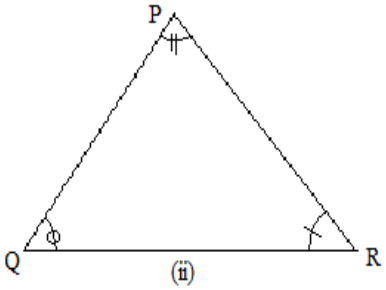
a) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$

b) $\frac{AB}{QR} = \frac{BC}{PQ} = \frac{CA}{RP}$

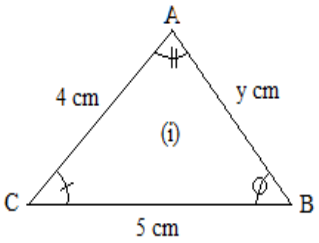
c) $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RP}{AC}$

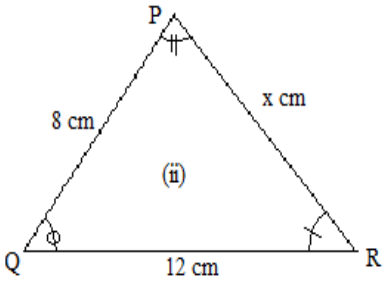
d) $\frac{PQ}{AC} = \frac{QR}{BC} = \frac{RP}{AB}$



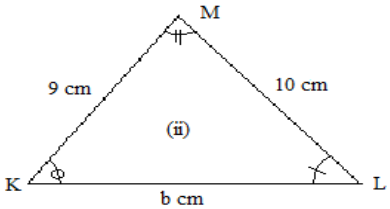


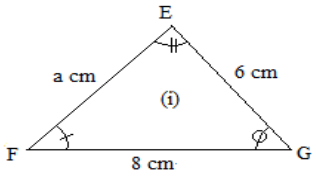
Q.3. If ΔABC and ΔPQR are similar triangles, what are the values of x and y?





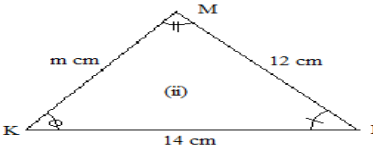
Q.4. If ΔKLM ~ΔEFG, what are the values of a and b?

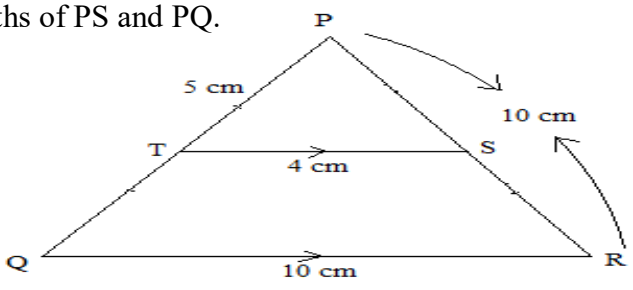




Q.5. prove that ΔKLM ~ΔEFG and also find the values of m and n.

Q.7. If ΔPQR ~ΔPTS then find the lengths of PS and PQ.





Work sheet – Day 5

Q.1. what is the symbol is used for the congruency of triangles?

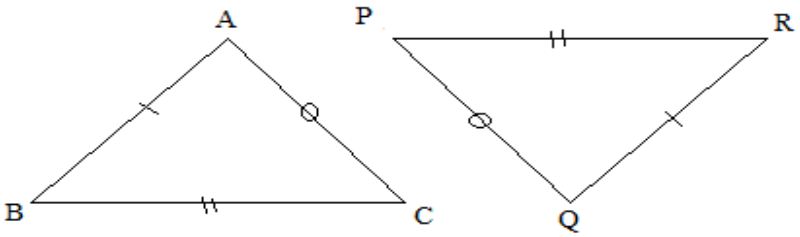
a) $\approx$ b) $\cong$ c) $=$ d) $\equiv$

Q.2. what is the property of congruent triangles?

a) Same shape and same size b) Different shape but same size

c) Same shape but different size d) Different shape and different size

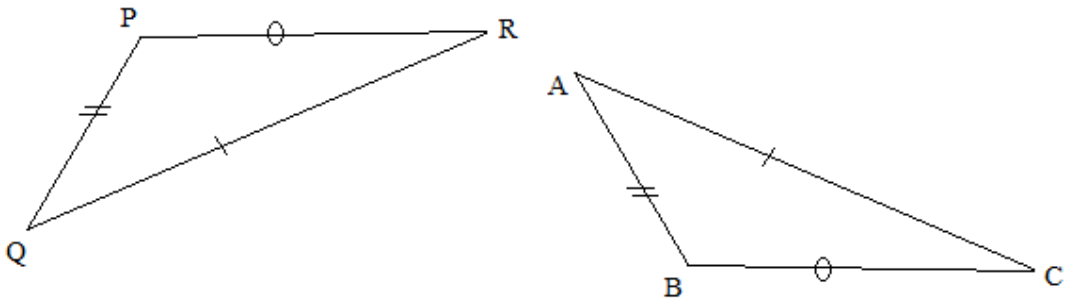
Q.3. By which axiom the following pair of triangles is congruent?



a) S.A.S. axiom b) S.S.A. axiom c) S.S.S. axiom d) A.A.S. axiom

What is the reason?(Discuss in the group and present in the class)

Q.4. If $\Delta ABC \cong \Delta PQR$ then which one of the following is true?



a) $\measuredangle P = \measuredangle A$ b) $\measuredangle Q = \measuredangle C$ c) $\measuredangle R = \measuredangle B$) None of the above.

What is the reason?

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Q.5. If the triangles are congruent then

a) Ratios of the corresponding sides are equal.

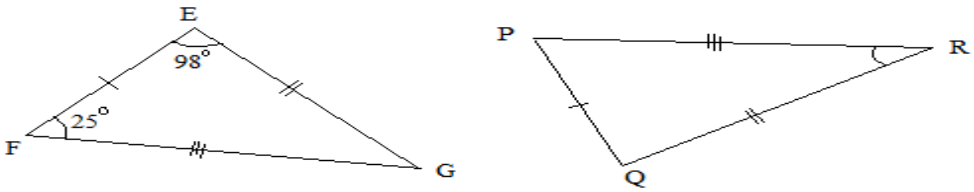
b) Corresponding sides are different.

c) Corresponding sides are equal.

d) Difference of the corresponding sides is equal.

Content Related Task-

a) If ΔEFG ≅ ΔPQR then what is the measure of ∡PRQ.



b) If any two triangles are congruent then what will the relations among the angles of them? (Let's discuss in the group and write the common understanding)

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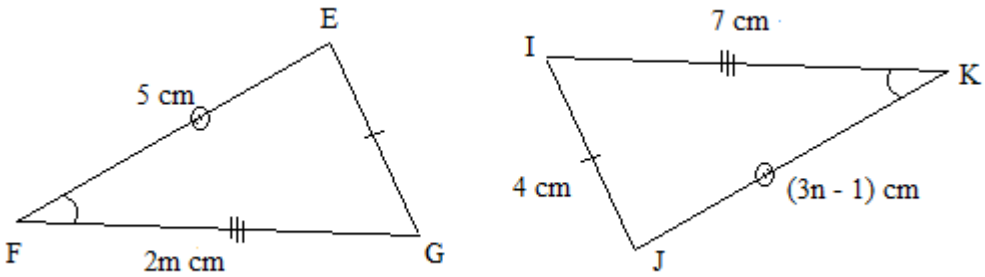
c.) If ΔEFG ≅ ΔIJK then what is the value of 'm' and 'n'?



Reflective Task – Conclude your result by group discussion on the basis of your findings of above content task.

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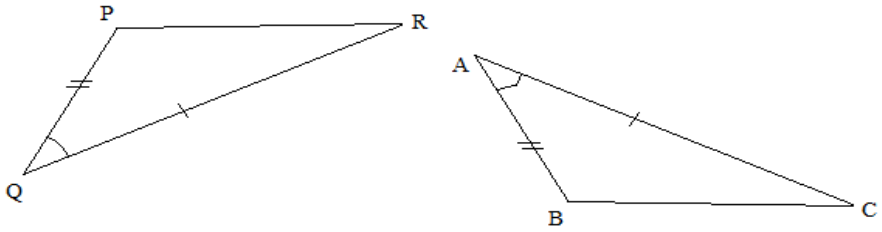
Work sheet – Day 6

Q.1. By which axiom the following pair of triangles is congruent?



a) S.A.S. axiom b) S.S.A. axiom c) S.S.S. axiom d) A.A.S. axiom

Give reason

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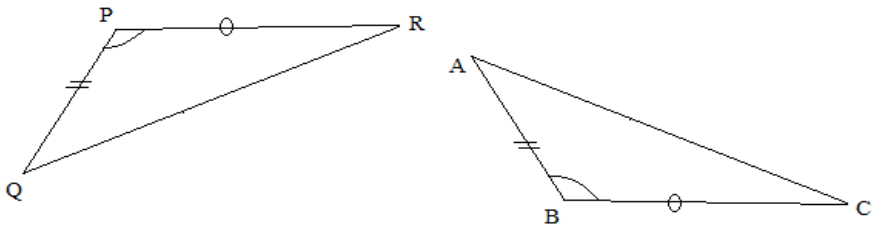
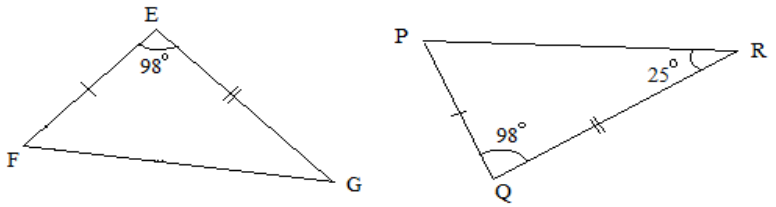
Q.2. If ΔABC ≅ ΔPQR then write remaining pairs of equal sides and angles. Present the findings of individual group in the class room.

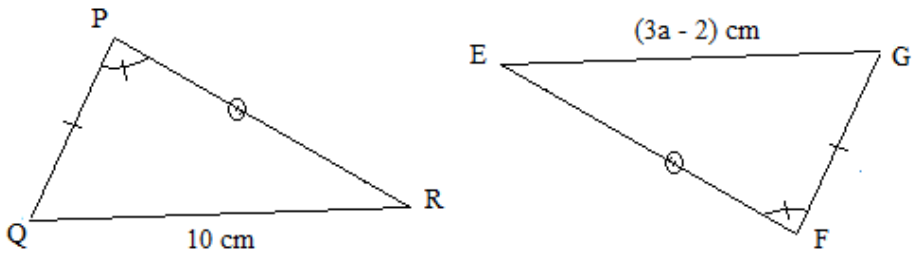


Content Related Task

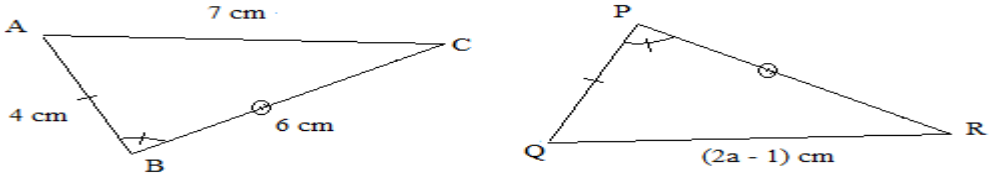
a) If ΔEFG ≅ ΔPQR then what is the measure of ∡EFG.



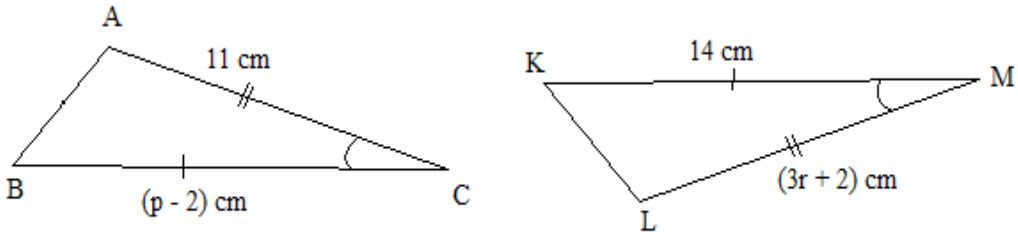
b) Prove that ΔPQR ≅ ΔEFG then find the value of 'a'.



c) If ΔPQR ≅ ΔABC then what is the value of 'a'?



d) If $\Delta ABC \cong \Delta KLM$ then what is the value of 'p' and 'r'?



Reflective Task – Conclude your result by group discussion on the basis of your findings of above content task.

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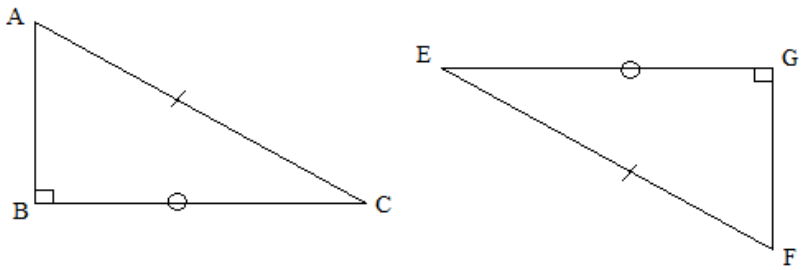
Work Sheet – Day 7

Q.1. By which axiom the following pair of triangles is congruent?



a) R.H.S. axiom b) S.S.R. axiom c) S.A.S. axiom d) S.A.S. axiom

Give reason

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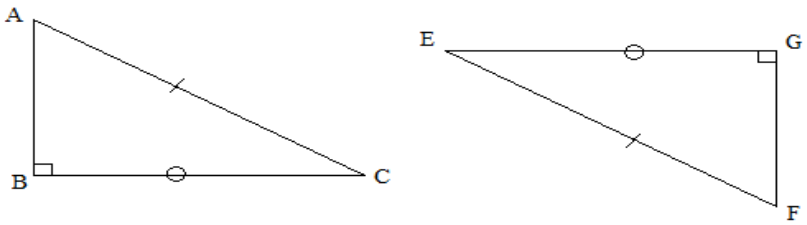
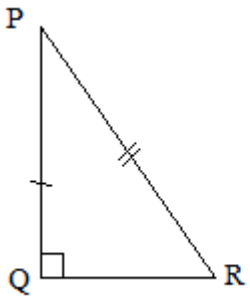
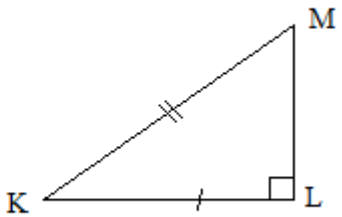
Q.2. Prove that the following pair of triangles are congruent triangles.

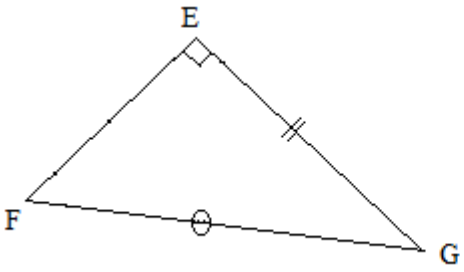


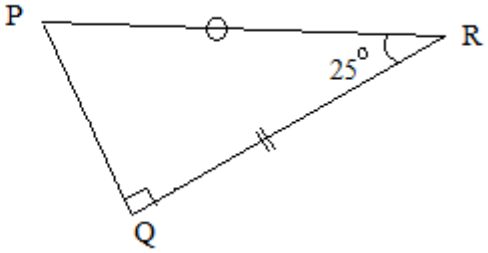
Q.3. If ΔPQR ≅ ΔKLM then write all the remaining pair of equal sides and angles.



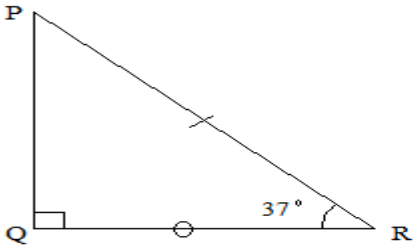


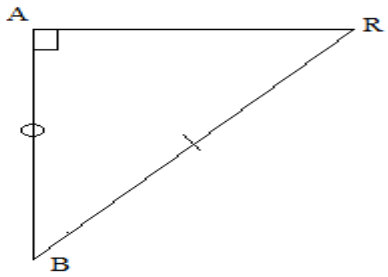
Q.4. If ΔEFG ≅ ΔPQR then what is the measure of ∡EFG.



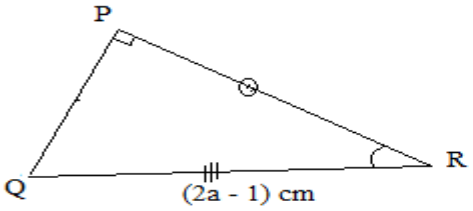


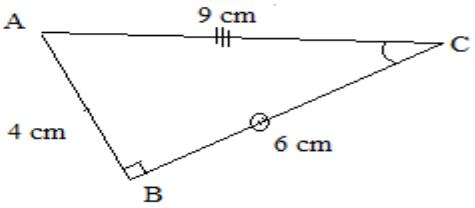
Q.5. If ΔPQR ≅ ΔABR then what are the values $\measuredangle ABR$ and $\measuredangle ARB$.



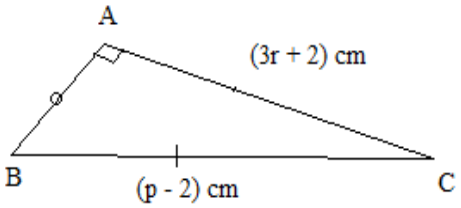


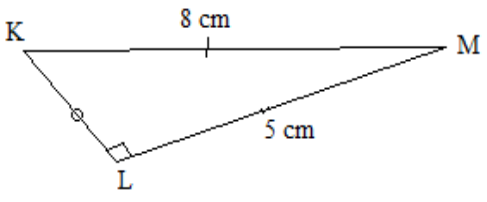
Q.6. If ΔPQR ≅ ΔABC then what is the value of 'a'?





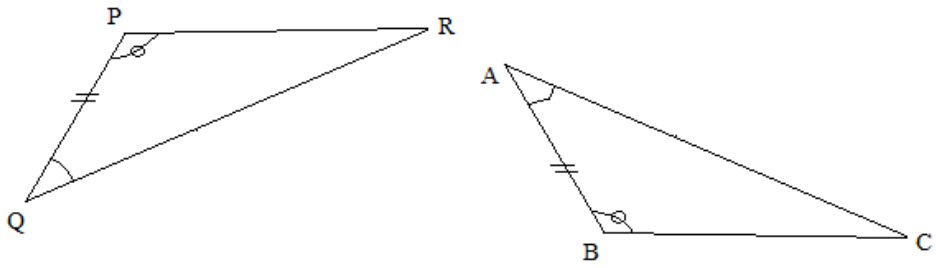
Q.7. If ΔABC ≅ ΔKLM then what is the value of 'p' and 'r'?





Work Sheet – Day 8

Q.1. By which axiom the following pair of triangles is congruent?



a) A.A.S. axiom b) A.S.A. axiom c) A.A.S. axiom d) S.A.S. axiom

Give reason

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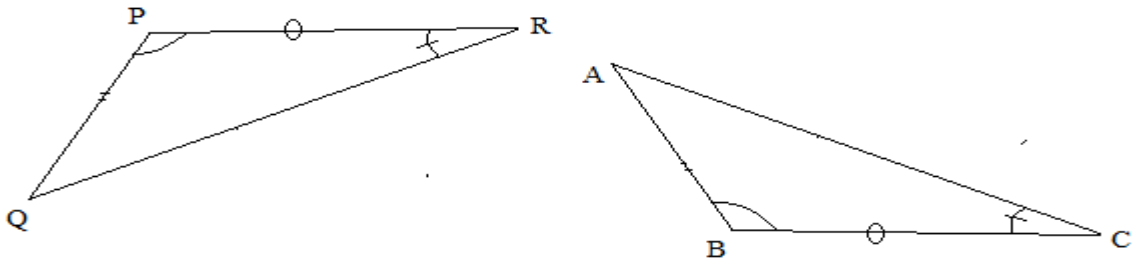
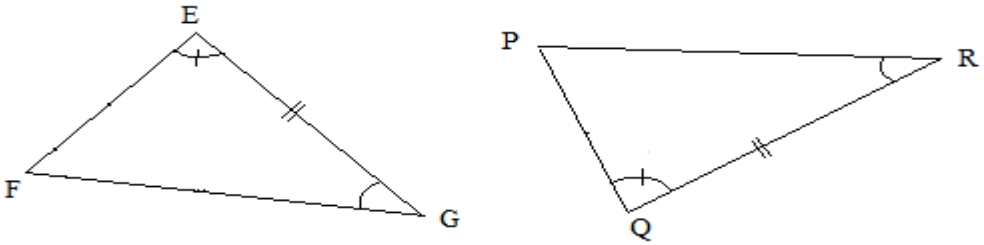
Q.2. If ΔABC ≅ ΔPQR then which one of the following is true?



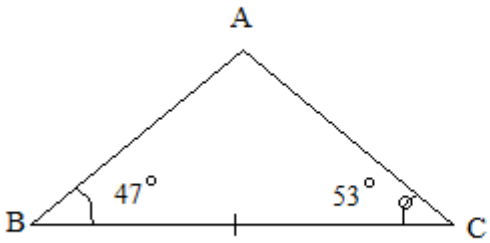
a) ∡ Q = ∡ C b) PQ = BC c) PQ = AB d) None of the above.

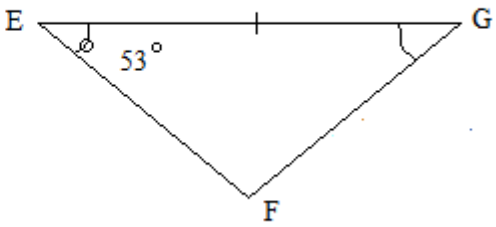
Give reason

Q.3. Prove that the following pairs of triangles are congruent triangles and also write a pair of equal sides and equal angles.

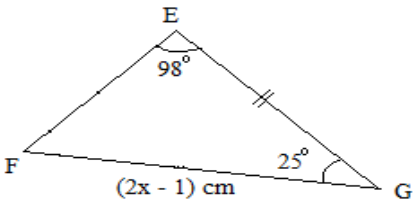


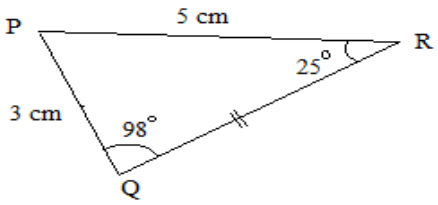
Q.4. If ΔEFG and ΔEFG are congruent triangles then find the measure of ∡EFG and ∡EGF.





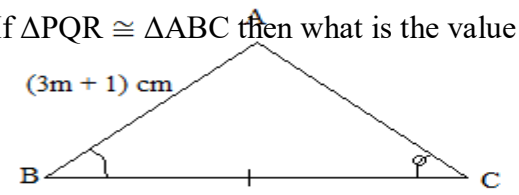
Q.5. If ΔEFG ≅ ΔPQR then what is the value of x.

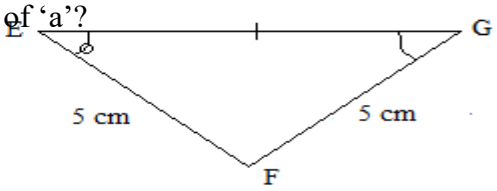


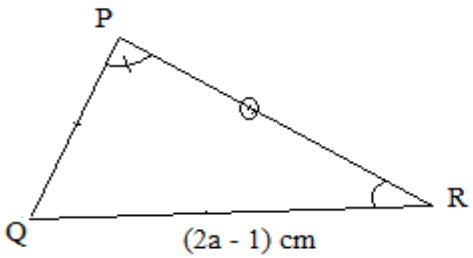


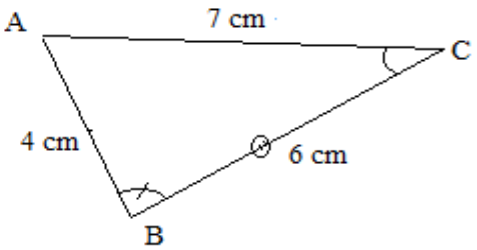
Q.6. If ΔABC ≅ ΔEFG then what is the value 'm'.

Q.7. If ΔPQR ≅ ΔABC then what is the value of 'a'?

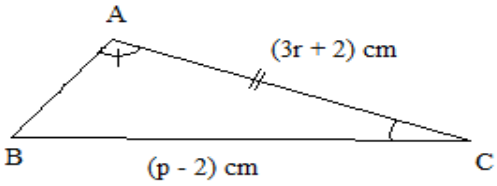


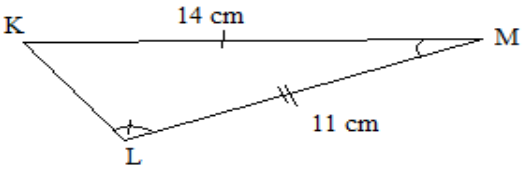






Q.8. If ΔABC ≅ ΔKLM then what is the value of 'p' and 'r'?

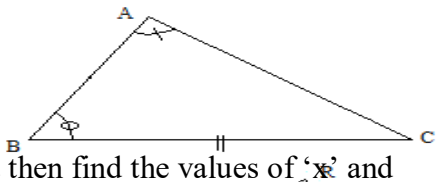




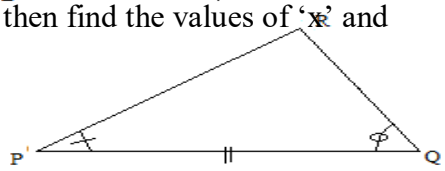
Work Sheet – Day 9

Revision Task:

Task 1: Are the following triangles are congruent with each other? Why or why not? Give reason.



Task 2 : If the following triangles are congruent then find the values of 'x' and 'a'.



Beginning Task:

Task 1: Draw the Parallelograms under the following measurement.

Group A Task:- Draw a parallelogram PQRS in which PQ =6cm and PR = 7cm.

Group B Task:- Draw a parallelogram WXYZ in which XY =8cm and XZ = 9cm.

Group C Task: - Draw a parallelogram ABCD in which AB =6cm and $\measuredangle A$= 60^0.

Group D Task:- Draw a parallelogram OPQR in which OP =6cm and $\measuredangle P$= 45^0.

Task 2: Discuss in your respective group and share the result and findings of above group task in the classroom.

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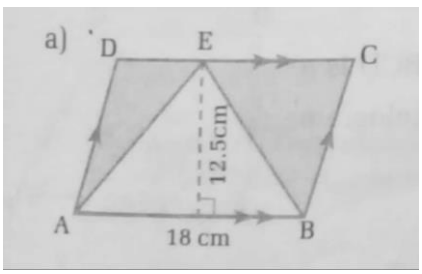
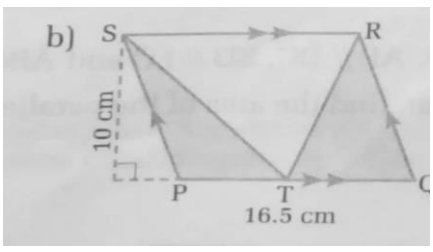
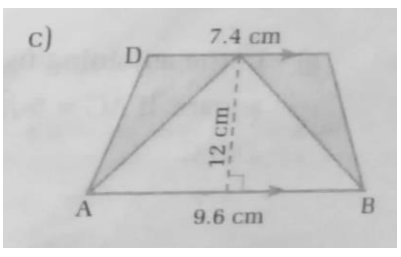
Task 3:- Find the area of shaded portion of following adjoining figures.







Work Sheet – Day 10

Revision Task:-

What types of postulates and axiom can be used to prove 'the diagonal bisect the parallelogram'?

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Group Task -1

Tell them to draw parallel lines. Let them to construct parallelograms between parallel lines which has common base under the following measurement.

a) Construct a Parallelograms PQRS and PQTU where PQ is a common base.
b) Construct a Parallelograms WXYZ and XYUV where XY is a common base.
c) Construct a Parallelograms ABCD and ABEF where AB is a common base.
d) Construct a Parallelograms OPQR and OPST where OP is a common base.

Task -2

Conclude your result by group discussion on the basis of your findings.

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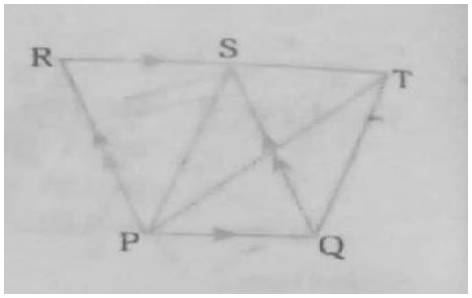
Task -3

In the given adjoining figure, PQRS and PQTS are two parallelograms standing on same base and between same parallels. If the area of ΔPQR is 50cm^2 then find the area of parallelogram PQRS.



Work Sheet – Day 11

Revision Task:-

What types of postulates and axiom can be used to prove 'the area of parallelograms is standing on same base and between parallels'?

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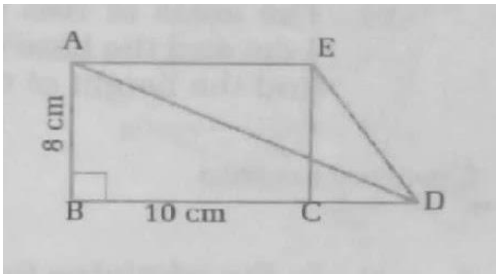
Group Task:-

Draw a triangle and parallelogram which is standing on same base and between same parallels. By drawing altitude, find the area of triangle and parallelogram. Then discuss in your group and share the ideas and findings of group task.



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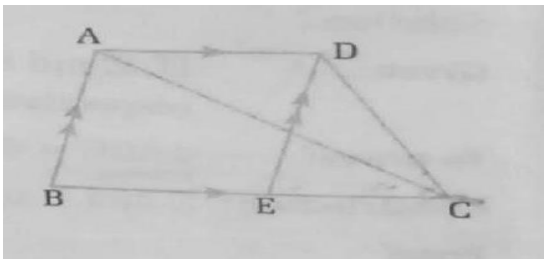
Content Based Task:-

c) In the adjoining figure, the area of trapezium ABCD is $100cm^2$ and the area of ΔADC is 40 cm^2. Find the area of ΔDEC.

d) In the given adjoining. If, ABCE is a rectangle, find the area of Δ ADE.



Day – to – Day Schedule of worksheet

Work Sheet – Day 12

Revision Task:-

What types of postulates and axiom can be used to prove 'the area of triangle is half of the area parallelogram which is standing on same base and between parallels'?

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Begging Task:-

Draw triangles which are standing on same base and between same parallels. By drawing altitude, find the area of triangle. Then discuss in your group and share the ideas and findings of group task.

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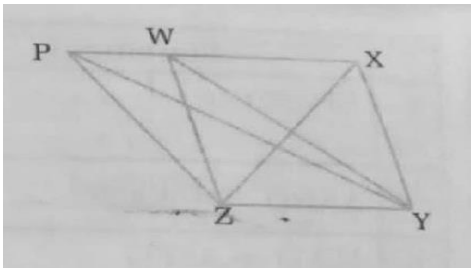
Content Based Task: -

b) In the adjoining figure, WXYZ is a rhombus in which XW is produced to the point P. If, WY=10cm and XZ = 9cm, find the area of Δ PYZ.



Day – to – Day Schedule of worksheet

Worksheet of Day:-13

Name of Group:- …….....

Beginning Task

a. Write the terms related to circle(Please discuss in group and try to find the term related to circle)

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b. Draw a circle on the basis of following measurement.(By using geometrical instrument)

i) Radius = 3cm ii) Radius = 5.5 cm iii) Radius = 4.4cm

iv) Diameter = 8cm v) Diameter = 10.6cm

Task- 1, Draw a big circle (any size of radius) in A4 size paper. And cut the circle from the paper and described it by showing the semi-circle, segment and sector. (Discuss in group write the common understanding of your group in a given dialogue box.

Task -2 , Draw three different circles with different measurements and mention the following terms related to the circle. (Chord, arc, secant, tangent, central angle and inscribed angle)

Explain and define the above terminology of a circle within your respective group and write the common understanding in the given dialogue box.

Day – to – Day Schedule of worksheet

Worksheet of Day-14

Name Group:-

Beginning Task,

a. Draw three different circles with different radii. Fill the blanks according to angular terms of central angle and measurement of corresponding arc.

   i) Central Angle = ……. and Length of arc = ……

   ii) Central Angle = ……. and Length of arc = ……

   iii) Central Angle = ……. and Length of arc = ……

Task -1, Draw a two circles with different radii. And construct two equal central angles in which O be a center of circle, ∠ AOB and ∠ COD are equal central angles whose corresponding arcs are AB and CD respectively. Discuss in your group and fill the following table.

Figure	Length of Arc AB	Length of Arc CD	Result
I			
II			

Write the findings and result of the above table in the following dialogue box.

Task-2, Draw two circles with different radii. And construct two equal arc in which O be a center of circle, AB and CD are equal arc whose corresponding central angles are ∡ AOB and ∡ COD respectively. Discuss in your group and fill the following table.

Figure	∡ AOB	∡ COD	Result
I			
II			

Write the findings and results of the above table in the following dialogue box.

Worksheet Day -15

Beginning Task

a) Draw a circle with radius 6cm and mark the arc as AB, CD, EF and GH on the circumference of the circle. Also draw respective segments of the arcs. Discuss the group and mention the findings and results.

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Task-1, Draw three circles with different radii. And then, marked arcs on a circumference of the circle as arc AB and arc BC in each circle. Draw a

segment by taking the end points of each arc. Then tabulate the result in following table;

Figure	Length of segment AB	Length of segment CD	Result

Write the findings and results of the above table in the following dialogue box.

Task-2, Draw three circles with different radii. And then draw an equal line segment on the circle as AB and CD. Then tabulate the result in following table;

Figure	Arc length of AB	Arc length of CD	Result

Write the findings and results of the above table in the following dialogue box.

Day – to – Day Schedule of worksheet

Worksheets of Day-16

Begging Task,

a. Draw two different circles with different radii. Fill the blanks according to angular terms of central angle and inscribed angle subtend into the same arc.

iv) Central Angle = ……. and Inscribed angle = ……

v) Central Angle = ……. and Inscribed angle = ……

vi) Central Angle = ……. and Inscribed angle = ……

Task-1, Draw three circles with different radii. Marked the central angle as ∡AOB and inscribed angle as ∡ACB. ∡AOB and ∡ACB both are standing on the same arc AB. Tabulate the result in the following table after group discussion.

Figure	∡AOB	∡ACB	Result

Write the findings and results of the above table in the following dialogue box.

a. Draw two different circles with different radii. Draw inscribed angles in a circle which stands on the same arc. Fill the blanks according to angular terms of inscribed angles subtended into the same arc.

   i) Inscribed angles = ……..

   ii) Inscribed angles = ……..

   iii) Inscribed angles = ……..

Task-1, Draw three circles with different radii. Marked the inscribed angles as ∡ABC and ∡ADC. ∡ABC and ∡ADC both inscribed angles are standing on the same arc AC. Tabulate the result in the following table after group discussion.

Figure	∡ACB	∡ADC	Result

Write the findings and results of the above table in the following dialogue box.

Begging Task,

a. Draw two different circles with different radii. Mention the four points in a circle then join the adjacent points respectively. Then discuss in group which figure could be drawn and marked the opposite angles of quadrilateral.

   i) First figure = Opposite angles are (…… and ……. / ……. and ……)

ii) Second figure = Opposite angles are (…… and ……./ ……. and ……)

Task-1, Draw three circles with different radii. Draw a cyclic quadrilateral ABCD and mark the opposite angle. Tabulate the result in the following table after group discussion.

Figure	∡A	∡B	∡C	∡D	Result

Write the findings and results of the above table in the following dialogue box.

ANNEX C

Personal Interview Questionnaire before Plan Implementation

Q.1. Do you complete and submit your Mathematics assignments regularly?

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Q.2. Do you ask questions to your teachers?

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Q.3, How the teachers facilitate you?

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Q.4. Which is your favorite subject and why?

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Q.4. Do you have any one to help you for doing assignment?

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Q.5. Do your class friends help you for learning?

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Q.6. Do you feel any alternative ways adopt by the teacher?

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PERSONAL INTERVIEW QUESTIONNAIRE AFTER PLAN IMPLEMENTATION

Q.1. Do you discuss with your friends about subject matter?

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Q.2. Do you answer the questions asked by your teachers regularly?

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Q.3. How do your friends respond when you put your opinion about anything?

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Q.4. how do you take mathematics subject now?

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