

TEACHERS' UNDERSTANDING AND USES OF VIRTUAL MANIPULATIVES  
IN TEACHING MATHEMATICS: A NARRATIVE INQUIRY

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A Dissertation

in Partial Fulfilment of the Requirements for Degree of  
Master of Mathematics Education

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## DECLARATION

I at this moment, declare that this dissertation has not been submitted earlier for the candidature of any other degree.

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## DEDICATION


This dissertation is dedicated to my late Grandfather and Grandmother, Mr. Bal Krishna Rajak and Mrs. Santu Maya Rajak. It also goes out to my father Mr. Prakash Rajak—my greatest source of inspiration, my mother, Mrs. Subarna Rajak who showers me with love every day. Above and beyond all other considerations, I dedicate this dissertation to my elder brother Mr. Prashant Rajak whose footsteps I have followed throughout my life. I would also like to dedicate this dissertation to my sister-in-law, Mrs. Diya Rajak Shrestha whose charming personality has lightened up our house. Last but not least, my eight-year-old nephew, Mr. Prasiddha Rajak is equally praiseworthy as he was constantly checking on me and concerned about whether I finished my dissertation on time.

*Master of Education in Mathematics*, dissertation of *Siddhartha Rajak*, entitled  
“*Teachers’ Understanding and Uses of Virtual Manipulatives in Teaching  
Mathematics: A Narrative Inquiry*” was presented on October 16, 2022.

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I understand and agree that my dissertation will be a part of the permanent  
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## AN ABSTRACT

Of the dissertation of *Siddhartha Rajak* for the degree of *Master of Education in Mathematics* presented on October 16, 2022 at Kathmandu University School of Education,

Title: *Teachers' Understanding and Uses of Virtual Manipulatives in Teaching Mathematics: A Narrative Inquiry*

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By the end of the second decade of the 21<sup>st</sup> century, COVID-19 had rocked the whole world and affected most sectors across the globe. In these trying times, the education sector could not remain unscathed as the physical distancing was the only preventive measure that all were applying. The drumbeats of the COVID-19 pandemic escorted education into the virtual mode as physical distancing was imposed by governments of almost all the countries. In the context of Nepal, the majority of schools were implementing virtual teaching and learning for the very first time due to which the need to know the understanding of mathematics teachers on virtual manipulatives was essential. From the eyes of a mathematics teacher, I wanted to know the extent of mathematics teachers' understanding when they opted to teach mathematics through the use of virtual manipulatives.

In order to know about the contemporary practices of virtual mathematics teaching in Nepal, I have chosen narrative inquiry. In this research, I have explored the understanding of different mathematics teachers on their use of the virtual manipulatives using narrative research design.

The purpose of my research is to explore teachers' understanding and uses of virtual manipulatives in teaching mathematics by probing deep into the teachers' environment, reliving the teachers' gain, pain, dilemmas and ecstasies. So, to find the answer to my research question, I have put together the narrations of three mathematics teachers from the Kathmandu Valley and made meaning of the narrations there forth. Among the three mathematics teachers, the interview of two of them was possible physically and one had to be done virtually via google meet. The audio and video recordings of the conversations were done to maintain the ethical standards of the research. Narratives were then transcribed with the help of the recordings and discussed based on my experience and literature. I have used social constructivism theory as my theoretical referent. The paradigm of interpretivism helped me to understand and interpret the narrations of the mathematics teachers by prolonged engagement with them in the process of rich narrative generation.

After having generated the narratives, I explored that the virtual manipulatives helped the students and teachers on many fronts. The virtual manipulatives helped the students understand mathematics conceptually in a meaningful fashion. Virtual manipulatives made the teachers' work easier and boosted them professionally. Virtual manipulatives also had a telling contribution in promoting classroom interaction in the mathematics class.

This research study is beneficial to the mathematics teachers who are using virtual manipulatives or those who want to use virtual manipulatives in the future to teach mathematics.

October 16, 2022

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Lastly, it would not be fair if I did not thank my batch mate, Mr. Shailendra Sapkota, who has harmonized with me perfectly throughout my two years at Kathmandu University. His critical feedback and instantaneous help have always been with me. I have realized that I have learnt so much with him and would like to acknowledge him.

Siddhartha Rajak, Degree Candidate

## ABBREVIATIONS

ANCOVA	:	Analysis of Covariance
APA	:	American Psychological Association
B.A.	:	Bachelor of Arts
BLE	:	Basic Level Examination
B.S.	:	Bikram Sambat
B.Sc.	:	Bachelor of Science
COVID-19	:	Corona Virus Disease 2019
ICT	:	Information and Communications Technology
KU	:	Kathmandu University
M.Ed.	:	Master of Education
M.Phil.	:	Master of Philosophy
Ph.D.	:	Doctor of Philosophy
PTM	:	Parent Teacher Meeting
SEE	:	Secondary Education Examination
ZPD	:	Zone of Proximal Development

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## CHAPTER I

### INTRODUCTION

In this chapter, I have presented the background of the study accumulated through my experiences as a learner and a teacher. Furthermore, from my experiences as a learner and a teacher, I have come up with the statement of problem.

Subsequently the purpose of the research is generated in this section followed up by the research question. Ultimately, the significance and delimitation of the study are incorporated to complete this section.

#### **Background of the Study**

In retrospect to my experience, I believe that the students in the mathematics class learn best when they are engaged to their fullest. The mathematics teacher should constantly find new ways to better the student engagement in mathematics class. In the quest for engagement and interaction in the virtual class, teachers can adopt and use virtual manipulatives for effective teaching of mathematics which in turn fosters student engagement.

#### **My Experience as a Learner**

I started my schooling at the very young age of three in a nearby school named Anmol Jyoti School. I do not reminisce much about those days, but as far as I can remember, I had a very sound relationship with Mathematics from a very young age. I had a direct correlation with numbers. I was fascinated by the mathematical calculations and those were the things I loved to do in school. I whole heartedly used to play with dice and found mathematics out of the class even more interesting. As the days went by, I slowly discovered my orientation on what I liked in Mathematics. I started enjoying playing with building blocks, dice and cards. As much as I found

mathematics fun-filled outside the classroom, I used to feel lethargic when the teacher compelled us to stay inside the class and taught mathematics by writing on the blackboard. The 40-minute classes were a complete pandemonium since we had to stay the whole time. The monotony of the class had longstanding repercussions as my muscle memory had already appreciated mathematics as a subject which is foreign to the real world and is always instructed within the classroom by the use of equipment such as pencil and paper.

Slowly and steadily, there was a slight turn of events as my new mathematics teacher in Grade III had a philosophy like none other. She used to believe that students learn better when they are taken outside the class. She used to take us outside the class but always gave us clear instructions before she led us outside. I gradually started to find zeal and enthusiasm in learning mathematics outside the classroom, playing with concrete objects rather than calming myself down inside the class on what the teachers had to say within the four walls.

When I set foot in my teens, my learning of Mathematics was highly text book oriented. Our mathematics teacher focused on the routine mathematics problem that the book had prescribed. The class was highly repetitious where my teacher used to do the first problem on the board and follow that up with the last problem. The leftover problems in between were flooded at our direction, and we did not have much time to swivel and save ourselves. Our teachers did not use any materials to increase our conceptual understanding. Our classes were unidirectional and always had the same pace. The content was the prime focus, and the role of the teacher was to deliver the content. The ways of delivering did not vary and was always pretty much the same. However, I experienced a change in the scenario when I took upon teaching as my profession.



## **My Experience as a Teacher**

The footprints of my journey as a teacher educator were completely different from my learning experience. When I started my teaching career, I was fortunate enough to have the support of my elder brother, who was in the education sector as an educator and also an author. He marshalled me in the early stages of my journey as an educator. His role was instrumental in getting me established as an educator. He always told me to use materials in the class as he was aware that the education sector in Nepal needed a huge shift in paradigm and the shift has to start from us. He guided and helped me prepare materials, which played an important role in manoeuvring me in conducting a riveting class. He also used to tell me that the first impression we showcase in the class as a teacher will have a knock-on effect on the way the whole class picturizes us. So, he helped me prepare a full-fledged plan with a well-organized activity and that is how I entered the class for the very first time. The well-planned class helped me establish trust and build rapport amongst the students. The first impression that I set in the class had huge repercussions as the students felt that I am a fun-filled teacher who takes students outside the class and connects the learning inside with the learning outside, which resonates with the line, literacy requires reading the word and the world simultaneously (Freire, 1987 as cited in Tutak et al., 2011). I always used to feel the disengagement in the students when I made them do routine mathematical problems, but the moment, I make them do activities that connect their understanding with the real world, the enthusiasm shot up to newer heights every time. The procedural understanding that teachers inject in the students has to be backed up by understanding the concepts. Procedures are algorithms, sequence of actions that will lead to the correct answer when executed correctly (Rittle-Johnson & Scheneider, 2015) but that does not always produce a conscious mind which can enter

the real world and solve real-life problems. The students are not getting the correct/enough exposure if they cannot solve the problems that they face in their life. So, I was motivated to indulge my students in activities that simulate the real world.

### **The Corona Virus: “A Blessing in Disguise”**

In the physical setting, as a teacher, I used to subtly balance my class by using materials to trigger the students’ conceptual understanding, and in other classes, the routine text book problems were also given emphasis as they were also equally important from the examination point of view. The procedural understanding of mathematical concepts also needed to be focused on for the students’ advancement for the board examinations (BLE/SEE).

However, even if I had enough time and opportunities to get exposed to the virtual manipulatives that can be used in teaching-learning mathematics, I never found the enthusiasm to learn and get used to it explicitly. I was not really fond of using the virtual manipulatives unless the pandemic (COVID-19) struck the whole world. Due to the change in times, I came to use virtual manipulatives while teaching and learning Mathematics as it was the only passage that led to light at the end of the tunnel. The progressive schools were slowly coming up, but the pandemic was a huge threat to the change in mindset that the schools had just started to acclimatize. When I started to use the virtual manipulatives in teaching-learning mathematics, I felt the difference. The shift while teaching Mathematics through the use of virtual manipulatives was colossal. The enthusiasm of the students escalated at the rate of knots as they were able to visualize Mathematics in a crystal-clear manner. In a span of two weeks or so, I noticed a change in the students’ conceptual understanding. Students started to pose inquisitive questions about Mathematics and were desperate

to connect it to their own experiences. The students started to visualize mathematics better through the use of virtual manipulatives.

This is a dialogue between a student from Grade X and myself conversing.

Student: Sir, Can I ask you a question about Mathematics if you are free?

Me: Yes, sure. I will be glad if I can help you with that.

Student: Sir, since the weekend, I was wondering, if trigonometry has a role to play in measuring the heights of towers and mountains.

Me: Wow, I am glad you thought about connecting Trigonometry outside the classroom as well. The use of the right-angled triangle and the trigonometric ratios play a vital role in measuring the height of objects that are manually very difficult to measure.

Student: Can you explain to me how it is measured?

Me: Sure, let's move to the computer lab. I will help you understand it through the use of the computer. I simulated and explained how the reference angle in a right-angled triangle can be used to find out the length of the perpendicular (Assuming the mountain or the tower to be perpendicular to the ground),

Student: Amazing! I am so intrigued to learn more about the practical applications of Trigonometry.

Me; Now, you should research and find out more, as you know it is easier to be committed once you are passionate.

The conversation with this student took me back to time where I made my students solve algorithmic problems and called it Maths. I did not give enough exposure to my students to critically think and connect Mathematics to the outside world and their lived experiences. The use of virtual manipulatives is an eye opener not only for the students but also for a teacher/educator like me who wants to instil

conceptual knowledge in the students by making them understand the Maths around us straight from the horse's mouth which reverberates Mark Twain's quote "If you catch a cat by its tail, you learn something you cannot learn in any other way".

In my teaching-learning experience till date, I have found that the use of virtual manipulatives is beneficial for inculcating conceptual understanding in the students, especially in this challenging time where the pandemic rocked the world. Virtual learning comes to the fore as a blessing in disguise at the very moment. I found the use of virtual manipulatives useful in my context and want to know the understanding of other teachers in a different setting while using virtual manipulatives in teaching mathematics.

### **Problem Statement**

In the later stages of the second decade of the 21<sup>st</sup> century, people from very few sectors have remained unscathed from the longstanding and devastating scars caused by the COVID-19 pandemic. The instantaneous drumbeats of COVID-19 have substantially affected the education sector. Teaching learning has had many stops and starts in the last couple of years in Nepal. In my eyes, quality education has also been at a halt in most of places, not only in Nepal but all over the globe. At these unprecedented and challenging times, many have compromised on teaching learning due to the physical distancing protocol imposed by the government. A study in Australia-New Zealand glorifies the occurrence of unexpected disruptions to the anatomical educational practices since the turn of the decade because the physical distancing regulations that need to be followed resulted into the loss of hands-on experience, which is instrumental for effective teaching and learning (Pather et.al., 2020). Owing to this fact, it is necessary for educators to find ways to regulate mathematical concepts fluently, even when the mode of learning is virtual. Hence,

there is an urgency to understand the effectiveness of the use of virtual manipulatives in teaching-learning mathematics as teacher educators and the whole teaching-learning fraternity will face huge repercussions if the virtual manipulatives do not cover up for the physical class of the students in developing a sound understanding of mathematics. In the context of Nepal, we have gone into the online mode only due to the pandemic, which is a blessing in disguise but hearing the word of mouth of many teachers are still struggling to get up to par in using manipulatives in teaching and learning mathematics. Research has shown that 70% of teachers need more professional development training in integrating technology in instruction and improving critical thinking skills (Mullis et al., 2021). So, the need to research on the level of understanding of teachers in using virtual manipulatives is clearly seen not only to know on what ground we stand as teachers but also to provide help to the teachers in the effective organization and management of virtual resources.

As the pandemic took its toll, I have felt many instances of students' disengagement in my mathematics class. My students' have found the prolonged use of pen and exercise book to solve routine mathematical sums tedious. Samur (2012) researched to measure the effects of educational games and virtual manipulatives on three domains of engagement (behavior, cognition, and emotion). The researcher designed an engagement survey that examines each domain in isolation along with its sub-domains while teaching fractions. In this pre-test and post-test quasi-experimental design, three experimental groups and one control group were made. The first experimental group was made to play an educational game called Candy Factory. The second experimental group also played another educational game called Pearl Diver for eight consecutive days. The third experimental group was made to perform activities with virtual manipulatives. However, the control group was made to do

paper-and-pencil drills for the same duration. The results of the analysis of covariance (ANCOVA) showed that the experimental group students' engagement scores were found significantly higher than control group students. The students who did activities with virtual manipulatives showed a significantly higher engagement than those who did paper-and-pencil drills in the control group. Hence, to find a solution to the disengagement of students in the virtual scheme of things in the context of Nepal, is the need of the hour. So, the intent of my research has established as I have experienced disengagement in virtual learning.

### **Purpose of the Study**

The major purpose of the study is to explore teachers' understanding and uses of virtual manipulatives in teaching mathematics.

### **Research Question**

My study revolved round the use of virtual manipulatives and attempted to seek the answer to the following research question.

- How do mathematics teachers narrate their understanding and uses of virtual manipulatives in teaching mathematics?

### **Significance of the Study**

This research is functional to those mathematics teacher educators who are integrating technology in teaching and learning mathematics. Teachers get to know about many virtual manipulatives that can be used in the teaching-learning setting for teaching mathematics as there is extensive talk on the manipulatives that can be used in teaching mathematics, while conversing with the teachers. The teachers' understanding of virtual manipulatives used in mathematics effervesces common problems in using virtual manipulatives. The research I have carried out has deep meaning and importance for mathematics teachers as they will reflect on their

professional practices. The importance of the teacher development programmes is felt as research has shown that 70% of teachers need more professional development training in integrating technology (Mullis et.al., 2021). The experiences of the teachers are showcased in my research which will help the teacher trainers to modify their planning for teacher development programmes in the days to come. This helps to bridge the gap of what teachers are struggling to implement in the virtual setting. The research may also be beneficial to the curriculum developers to focus on online activities that is comfortable for teachers to implement.

### **Delimitation**

The use of manipulatives in teaching and learning is often discussed in this study as the use of manipulatives has overpowered education this century. However, the prime focus of my study revolves around virtual manipulatives in teaching and learning mathematics. I have focused on the teachers' understanding of virtual manipulatives while they share their experiences of using virtual manipulatives in their teaching practices. My central idea revolves around the aforementioned forte.

## CHAPTER II

### LITERATURE REVIEW

Academic research is about standing on the shoulders of giants, and it is essential to know who the giants are and what the state of the argument is. A literature review is a summary of a subject field that supports the identification of specific research questions (Rowley & Slack, 2004). This section of literature review underpins the legitimacy of my research. I reviewed literature related to the “manipulatives” and “virtual manipulatives”. Moreover, I reviewed social constructivist theory as a referent. Finally, I commemorated past studies related to the teachers’ understanding and uses of virtual manipulatives in teaching and learning mathematics so as to realize the research gap which I projected to bridge at the end of my research.

#### **Thematic Review**

This section contains the major themes of this research study. The research question explored the understanding of teachers while they used virtual manipulatives in teaching mathematics as a result of which two themes ‘Manipulatives’ and ‘Virtual manipulatives’ have been reviewed. This section contains the review of the above-mentioned sub-headings, which helps us build more of an idea about the research study.

#### **Manipulatives**

Manipulatives in the context of education, are the physical tools of teaching which are used to visualize the abstract concepts with the help of physical object like coins, algebraic tiles, fraction bars, pattern blocks, graphs, etc. Jean Piaget (1971) also suggests in his stage of cognitive development that child between the ages of 7 to 11



is the concrete operational stage. At this stage child can think logically because they could manipulate concrete materials. In mathematics, manipulatives help student to understand important mathematical concept which ultimately help to breakdown the abstract concept of mathematics. Manipulative tools with basic direction allow student to explore the tools which make student actively engaged in discovery learning.

Over the years, as teaching-learning has evolved with time, the chalk and talk method of teaching has slowly gone out of fashion. Most of the learnings back in the days were highly lecture-based. However, the mindset of students has changed a lot in the present. Students have started to find the traditional chalk and talk method of learning monotonous and that has resulted in discontent. This has slowly become the root cause to change in the teaching-learning style.

In my experience, a considerable number of teachers are still clinging to lecture as the only method of instruction in teaching mathematics. However, even in the ancient times, people of several different civilizations used physical objects to solve every-day mathematics problems. Counting boards were used by the ancient civilizations of Southwest Asia where they would draw symbols in the sand to tally inventory or anything they may need to count. The ancient Romans created the first abacus based on a counting board. The abacus was made of stones which moved in grooves in sand or on tables of wood, stone, or metal (Boggan et al., 2010).

Students have also found the use of manipulatives and activity-based instruction engaging and better. A study of college students' learning experiences at a mid-sized college of arts and technology found active learning better than lecture (Kay et.al., 2019). Furner and Worrell (2017) outline that mathematics teachers can use manipulatives assuredly to work on mathematical concepts, rather than relying on

just worksheets and consequently, students can make connections between concrete and abstract, also learning the concepts of mathematics in an enjoyable way.

As I have used manipulatives regularly at the secondary level for the past 4-5 years, I found that students are understanding the mathematical content much better. The students seem motivated in the mathematics class. Durmus and Karakirik (2006) claimed that usage of manipulatives not only increase students' conceptual understanding and problem-solving skills but also promotes their positive attitudes towards mathematics since they supposedly provide 'concrete experiences' that focus on attention and increase motivation.

### **Virtual Manipulatives**

According to Bouck and Flanagan (2010) "Virtual manipulatives are web-based images on a computer monitor that permitted students to manipulate a visual model in three different dimensions." This visualization helps students to gain insight of mathematical concepts. Virtual manipulatives not only provide interactive environment but it also allows student to pose and solve mathematical problem. This kind of practice help student to connect mathematical concepts where they get immediate feedback which help to reflect on their conceptualization (Durmus & Karakirik, 2006). By the use of new tools and technology, student can get easily access with virtual manipulatives on computer, laptop, tablet and smartphone.

Since the turn of the decade, virtual manipulatives have been used ubiquitously as the physical classes have been at a halt. As a blessing in disguise, teachers started to use virtual manipulatives in their classes due to the longstanding interruption due to the pandemic. While doing so, I have personally found the use of virtual manipulatives better in teaching various concepts of mathematics. Reimer and Moyer (2005) outline that the virtual manipulatives helped third grade students to

learn more about fractions by providing immediate and specific feedback, which also enhanced students' enjoyment.

In the time, I have been in close proximity with the virtual manipulatives, I have noticed that virtual manipulatives have been a great help for the low-achievers. The students who did not understand various mathematical concepts have been elevated and boosted by the use of virtual manipulatives in teaching and learning. A two week-long study in a fourth-fifth grade classroom while teaching fraction addition has shown a significant gain for the low-achieving group when they used virtual manipulatives in the teaching-learning process (Moyer-Packenham & Suh, 2012).

### **Theoretical Review**

Theoretical review is the process of examining a bunch of theories gathered regarding an issue, concept or phenomenon. Theoretical review is performed to set up what theories are already in existence, the relationship between them and to what extent they have been scrutinized or looked into. In this section, I explain the theories that support my research to show on what ground my work has been established. Social constructivism is my theoretical referent that I reviewed in this section.

### **Social Constructivism**

Constructivism is a learning theory where knowledge is believed to be actively constructed by learners in their attempt to make sense of their experiences. In constructivist pedagogy, learning is expected to be done in context, where students identify and pursue their own learning goals. Fosnot (2013) outlines that knowledge does not and cannot have the purpose of representations of an independent reality, but instead has an adaptive function. Learners should be engaged in meaningful activities and should apply their knowledge. Students are viewed as active constructors of

knowledge and not as passive receivers. The role of the teacher takes a turn in the constructivist class as the teacher shifts their role from 'the sage on the stage to the guide on the side'. In the traditional classroom, the teachers behave in a didactic manner disseminating information to learners whereas in the constructivist class, the teacher is a facilitator who mediates the environment for the students.

Humans are social beings. They cannot remain isolated from society for a long time. The society and the culture they follow have the utmost importance in their individual development. For holistic development in a student, the social front is instrumental. Every function in the child's development appears twice: first on the social level and later, on the individual level: first between people and then inside the child (Vygotsky, 1978). As per Vygotsky, social interaction is significant to cognitive development in children.

The theory of social constructivism aligns with my research issue, which explores the understanding of mathematics teachers on the use of virtual manipulatives in teaching and learning. In order to achieve the needs of today's students in a digital or gadget world environment, instructional scaffolding needs to be embedded in the learning process. Scaffolding is a key strategy in cognitive apprenticeship, in which students can think critically and construct knowledge by working creatively on a social level (Ling & Harun, 2014). Vygotsky believed that adults (more knowledgeable other) are an important source of knowledge amongst the students. This understanding of teaching bestows scaffolding to the students in the zone of proximal development (ZPD)- the place where the students need support. In my study, I researched on the understanding of mathematics teachers as well as the use of virtual manipulatives in teaching and learning. I researched thoroughly whether

the virtual manipulatives are used as a scaffold, and helps to bridge the gap created by the lack of conceptual understanding in the students.

### **Empirical Review**

Universities do not want us to replicate the things that have been done a million times, so we do an empirical review to understand the state of our research and to see if there is a gap in the research so that we can establish the need for our specific research. In this review of literature, I have reviewed the past studies that come close to the research which I am currently performing at the very moment. This sort of review is going to help the researcher to find out what has already been done in the past in the field of research and what needs to be done. This helps the researcher to identify the research gap.

Pham (2015) investigated how elementary school teachers use manipulatives to help their students understand mathematics. This research stated that virtual manipulatives are capable of doing things that are simply not possible with physical manipulatives. The researcher also encouraged the teachers to use the virtual manipulatives for better teaching and learning.

Virtual Manipulatives provide a compelling and promising tool for teaching and learning mathematics. Young (2006) outlined that the effectiveness on the use of virtual manipulatives is reliant on two areas. The first one being the teachers and the second being the instructional design. He stated more research should be done on that front, which creates the need for my research as my study is on the understanding of teachers in the use of virtual manipulatives in teaching and learning mathematics. In addition, more literature and research are required in the areas of virtual manipulative development and implementation in online and secondary education.

### **Research Gap**

The research studies done by the researchers above were done in their own context. They were directly related to the teachers' professional development but not on the understanding of mathematics teachers on their professional development and the uses of virtual manipulatives in the context of Nepal. After going through the empirical review, I planned to research on the understanding of mathematics teachers on using virtual manipulatives in teaching mathematics. Lots of research studies have scrutinized teachers' understanding on the use of virtual manipulatives in teaching and learning mathematics. However, I am projecting to conduct my study on the understanding of teachers in teaching and learning mathematics in the context of Nepal.

After reviewing the aforementioned research studies, my study is on the teachers' understanding and use of virtual manipulatives but Pham (2015) has performed research on how the school teachers use manipulatives to help their students. Even though my research issue is similar to the reviewed research studies, my research study is different in terms of context. Young (2006) researched on the effectiveness of virtual manipulatives whereas my study is going to probe to find the understanding of the mathematics teachers and uses of virtual manipulatives in general.

## CHAPTER III

### RESEARCH METHODOLOGY

In this chapter, I have included my preliminary idea of the go about of qualitative research. I have also articulated my world view and furthermore showcased my values that have a strong say in my research. I have attempted to illustrate the data collection procedures as well as the ways in which they are going to be analysed. Finally, I have included the criteria that help establish quality in my research.

#### **Philosophical Considerations**

Philosophy is a belief about how one should collect information regarding certain research. Philosophy consists of three major parts, ontology, epistemology and axiology. Ontology is one's world view. One has the power to choose their world view, which affects how they see and understand the world. Epistemology is the theory of knowledge, whereas axiology is the study of value. In this section, I have discussed these three philosophical considerations based on my research.

#### **My Ontological Considerations**

In an educational sense, ontology refers to the researcher's world view. The researcher has the power to choose their world view, which affects how they see and understand the world. The belief system of the researcher plays an instrumental role in carrying out good research. My ontological orientation leans towards relativism where I accept multiple realities. I believe in the idea of subjectivity rather than objectivity.

As I glance outside my balcony on a Saturday morning, a drizzle might seem romantic and pleasing; however, on the other days of the week when I need to drive myself to work on a motorbike, the same sort of drizzle will be a nuisance. Hence

reality that an individual experiences is contextual and depends on the time and situation. Even among different individuals, this ontology of change can be experienced. An individual residing in the Terai region of Nepal feels 25°C temperature as moderate, whereas the individual residing in the mountain region feels 25°C temperature as extremely hot. Thereupon, we can say that the thought processes of people change with context and time and my belief system is also embedded likewise.

### **My Epistemological Considerations**

Epistemology is the branch of philosophy that deals with the theory of knowledge. There are two epistemological viewpoints. The first one is that knowledge is out there, waiting for someone to discover it and the second one is that knowledge is based on one's interpretation and experience. The first school of thought gives a sense of objective nature of knowledge, whereas the second gives a sense of subjective nature of knowledge. In my point of view, knowledge is a subject to interpretation and experience. If knowledge was not a subject to interpretation and experience, every student that goes to the same school should be getting the same knowledge; however, that is not the reality. Khera (2018) outlines that schools are like fountains of knowledge; some come to drink, some come to sip and few just come to gargle. I believe that knowledge gain depends on the individual and has many factors that can affect it. My epistemology reinforces the subjective nature of knowledge.

### **My Axiological Considerations**

Axiology came from the Greek words 'Axios', which means value and 'Logos' which means study. So, the philosophical study of values and how the values come about in the society is called axiology. Axiology when applied to human actions is called ethics and when applied to objects is called aesthetics. Ethics deals with the



rightness and wrongness of human actions and aesthetics deals with beauty.

Whenever research is conducted, I will make sure that I am ethical in performing all the processes. There should not be any unjust to any participant and I ensured that all participant's rights are restored and intact. In my research, whatever the researcher thinks or values will affect the overall interpretation of the research. Hence my axiology is value bound.

### **Research Paradigm**

A paradigm is a comprehensive belief system, world view, or framework that guides research and practice in a field (Taylor & Medina, 2011). My research is guided by the interpretive paradigm. Interpretive paradigm allows the researcher to stand in the shoes of the participants. The researcher is an insider as the individual investigates by getting inside the field of research. My research issue is to explore teachers' understanding and uses of virtual manipulatives in teaching and learning mathematics. While interviewing my participants, I had to go to their workplace(school) to take their interview. I entered inside their working environment to understand their pain and gain, I needed to be inside their environment to understand their context. So, to enter inside their environment, to hear their stories, pain and pleasure, interpretive paradigm was a best fit.

### **Research Method**

My research is qualitative research, where I have employed interpretivism to explore the understanding of teachers on the use of virtual manipulatives in teaching and learning mathematics. The method I have applied in my research is the narrative research design. In narrative research design, researchers describe the lives of individuals, collect and tell stories about people's lives, and write narratives of individual experiences (Connelly & Clandinin, 1990). Likewise, in my research I was

in close proximity with my participant mathematics teachers, creating an environment for them to share their stories and experiences regarding the use of virtual manipulatives while teaching and learning mathematics.

I went to the research site and spent a lot of time there in order to collect the narratives of my participants. I did multiple rounds of interviewing with my participants in the research site. I interacted with them rigorously to get to know what kind of a person they were and also to understand them through and through out. As I wanted to explore my participants' experiences and stories, I employed the narrative research design as it aims to explore and conceptualize human experiences.

### **Research Area and Research Participants**

I performed research that demands prolonged engagement with my participant mathematics teachers. In order to reach out to them constantly, it would be a whole lot swifter if the research area was inside the Kathmandu Valley as a result of which I chose three mathematics teachers from two schools within the Kathmandu Valley who fit into my research agenda, i.e., they were teachers who used virtual manipulatives in teaching mathematics. I chose the participants by purposive sampling from the pool of mathematics teachers I knew. I looked for female mathematics teacher as a participant to maintain gender balance but even after a lot of hardships I could not find a female mathematics teacher who fits into my research agenda. Hence, I had to go with all males.

### **Data Collection Tools and Approaches**

In collecting qualitative data, I prepared semi-structured questionnaires to interview respondents where I have a set of predetermined questions but I was responsive to the answers that the respondents gave me. While collecting the data, I spent three days in the school premises where I interviewed the teachers with the help

of the semi-structured questionnaires. I collected the data/information from them and transcribed them every time into my data base on my computer. Before I went for the interview, I received the informed consent from the principal of the school and the teachers.

### **Data Analysis Strategies**

The transcribed narrative from the above interviewing process is then brought together. The paraphrases that give similar meanings are skipped and non-relevant sentences are ignored. The paraphrased texts or quoted texts are then mentioned. Then from the lens of certain theories, the texts are analysed. Then my perspective as a researcher came to the fore where I interpreted the information gathered which reflected- my emotions, beliefs and values.

### **Quality Standards**

In any kind of research, researchers need to be aware of maintaining a certain bench mark for the research. The study should meet a set of criteria as a result of which research attains quality and sufficiency. These criterions are collectively called quality standards. In a more general sense, the commonly agreed criteria which are used to judge research is the quality standard. Educational researches have widespread intentions and different approaches are taken to fulfil those intentions. Qualitative researchers consider that dependability, credibility, transferability and confirmability as trustworthiness criteria ensure the rigor of qualitative findings (Guba, 1981; Schwandt, Lincoln, & Guba, 2007). The quality standards in qualitative research are regulated by standards such as trustworthiness, authenticity, critical reflexivity, praxis, pedagogical thoughtfulness, verisimilitude, etc.

My inquiry explores the teachers' understanding on the use of virtual manipulatives in teaching-learning mathematics. The criteria that ensure quality

standards include trustworthiness (credibility, transferability, dependability and confirmability) and authenticity.

### **Credibility**

Credibility is the extent to which research is believable. I ensured credibility by linking the research's findings with reality in order to demonstrate the truth of the findings. As I researched, I used the narratives of my participants and linked them to real-life instances in my class. I took their words and gave voice to them by connecting them to my experiences in my class.

### **Transferability**

Transferability is the assurance that a research finding can be applied in different contexts and situations. I provided evidence as a result of which my research is applicable to other situations as well.

### **Dependability**

Dependability is the consistency of the research findings. It refers to having sufficient details and documentation of the methods employed so that the study can be scrutinized and replicated. I ensured dependability in my research by the means of an external audit. I have an outside researcher conduct an inquiry on my research. In my research, I collected all the details of my interview through audio and video recordings which makes my research dependable.

### **Confirmability**

Confirmability verifies that the findings of the study are the ramification of the experiences of the respondents rather than the preferences of the researcher. The research would not acknowledge any personal bias. I ensured confirmability in my research by making my data transparent. I endorse that the research findings are completely deduced from the respondents and it does not instil my bias. I made my

interview dates transparent to my supervisor and also provided the audio and video recordings of the interview to him to ensure confirmability.

These four criteria establish trustworthiness in my research.

Authenticity means that one's research is genuine and contributes to the related field. This can be adjudged by the following parameters. Research has to be fair to its respondents. More emphasis should be given to the needs of the respondents rather than the objective of the study. The researcher should understand the respondents and the society, which has been a bench mark in establishing quality. I performed the same stance in my research to establish authenticity.

### **Ethical Standards**

Ethics is the set of moral values a researcher should consider before conducting research. Ethics provides rules that specify appropriate and inappropriate behavior during the conduct of research and the application of findings (Resnik, 2020). While conducting research, it is instrumental to understanding that the things we do to complete a study must not harm anyone some way or the other.

My research issue explores teachers' understanding and uses of virtual manipulatives in teaching and learning mathematics. Whenever I chose my research participants, I made sure that the decision that they make to join my research is completely voluntary. I maintained confidentiality of all my participants so that their participation would not affect them in their day-to-day activities as personnel. Respecting the stance of the respondents is of utmost importance and getting them out of any kind of intervention will be an ethic that should be followed in the research. I have used pseudo-names of the participants to maintain anonymity to clear them of any possible interventions in the future. I got the consent of the participants whenever I have to take their responses which develop a sense of ownership among the participants.

Whenever the participants are taken into consideration, there should not be any bias and everyone should be treated equally. There should be no discrimination by colour, caste, race, ethnic groups, etc. Every participant should be treated from the same eye, which will be a strong step forward in maintaining just in the society. I incorporated these standards in my research to make my research ethically sound.

## CHAPTER IV

### NARRATIVE GENERATION

In this chapter, I have put forward the narratives of three different mathematics teachers' who experienced the transition from in-person school to online classes and have agreed to participate in my research. I have dispensed the narratives of my three participants whose ideas and experiences I have gathered and made meaning of in the chapters to follow. In an attempt to do so, I have also tried to use my words as a source to give my participants' stories a voice. However, I have also endeavoured to bring in as many authentic real-life narratives as I could. In addition to accumulating stories straight from the horse's mouth, I have engendered a number of various themes based on the accounts of my participants, and to maintain anonymity and ensure their right to privacy, I have employed aliases, namely; Shyam, Aamir and Bishnu.

#### **Discourse with Shyam**

Changunarayan Municipality's ward number one is where Mr. Shyam currently resides. As a mathematics instructor, Shyam has been catering to the secondary level students at the New Zenith English Model School since 2073 B.S. After having completed his schooling from a Public English School in the year 2063 B.S, Shyam continued his +2 studies in the science stream, graduating from Caspian Valley College in the year 2066 B.S. In order to pursue his further education, he chose to study in Tri Chandra Multiple Campus. Shyam completed his Bachelor in Physics as his major subject whilst also choosing Statistics and Mathematics as a part of his subject combination. Since the completion of his Bachelor, he has been engaged in

teaching, and specifically in the case of mathematics, Shyam has been involved in this field for the past seven years or so.

### **Interview with Shyam**

The conversations I had with Shyam revealed that he has been involved as an Optional mathematics teacher for the past seven years. And like other teachers in the country, Shyam has also been engaged in traditional approaches to teach mathematics to his students. In the recent light of the COVID-19 pandemic, however, the teachers had very limited options other than learning to conduct classes virtually through various means such as zoom, google meet, teams, etc. Amongst other teachers in the nation, slowly and steadily, through trial and error, Shyam also used virtual manipulatives in his online classes to make the learning process more engaging and easier for his students. When I asked him how did he feel while starting to use the virtual manipulatives for the first time, Shyam said; “Initially, the hassle of implementing a new pedagogy took its toll on me. I was not accustomed to the virtual mode of teaching and learning. I did not find the implementation of such manipulations to be very simple. With time, however, I gradually became accustomed to teaching on platforms such as Teams and Zoom, as I acclimatized to the changes in approaches in the teaching and learning setting due to COVID-19.

Shyam added; “In my attempt to look for alternatives to engage my students like I used to in in-person classes, I made up my mind to diligently surf the internet and research vigorously to find options that may be able to fill the void that the pandemic had created”.

I had a few minor setbacks while looking for virtual manipulatives even though I conquered it later. For example, I discovered quite a few resources that looked to be quite promising and useful for my students. However, the applications I



found were only available in paid versions on the internet. I reminisce using ‘Braining Camp’.

Braining Camp is a form of a digital math manipulative created by David Brown that makes it simple for instructors to provide engaging math lessons and produce entertaining math games for their students in order to support their learning. Shyam used ‘Braining Camp’ in order to teach various concepts of basic algebra to students in the lower grades whenever he substituted for other teachers. But, since the content was only valid for the period of one month, Shyam was only able to use this useful resource for the first month that was free of cost.

Shyam mentioned; I used ‘algebraic tiles the most as I found this application intriguing, especially while explaining the concept of zero pair to my students. In the figure shown alongside, the purple square represents a square of area  $x^2$  and the red square represents a square of area  $-x^2$ . When the purple and red squares are brought together, both the squares disappear.

Shyam said; “The students have to understand the concept of zero pairs from the outset. I believe that those students who do not understand the concept of zero pairs will almost certainly

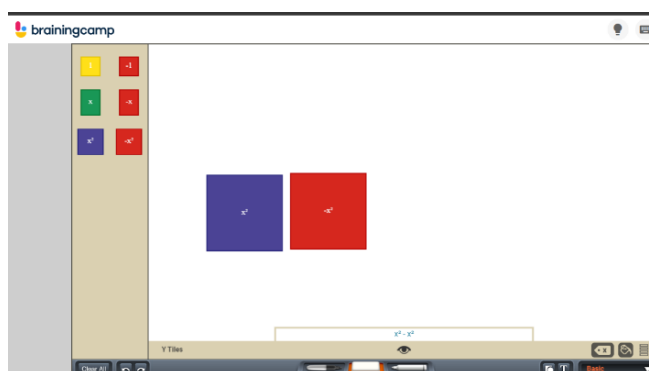
struggle in the higher grades

when they factorize by

completing the square.

While factoring  $x^2+6x+8$  by

completing the squares



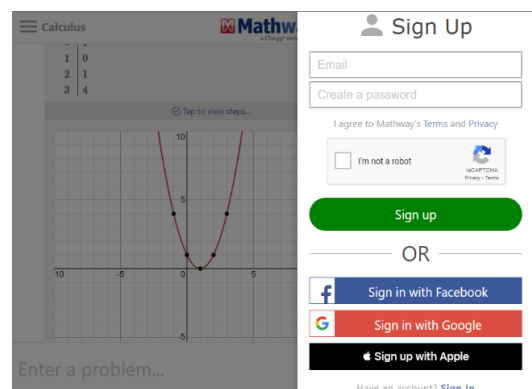
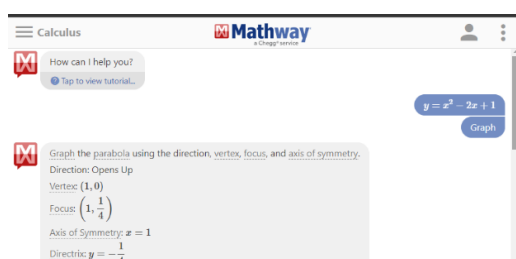
$$\begin{aligned}
 & x^2 + 6x + 8 \\
 &= x^2 + 2 \cdot x \cdot 3 + 3^2 - 3^2 + 8 \\
 &= x^2 + 2 \cdot x \cdot 3 + 3^2 - 1 \\
 &= (x+3)^2 - 1^2 \\
 &= (x+3+1)(x+3-1) \\
 &= (x+4)(x+2)
 \end{aligned}$$

Shyam believed that the students would not be able to comprehend why  $3^2$  is added and  $3^2$  is subtracted if the students are not aware of the concept of zero pairs. If the students can visualize the reason behind why we complete squares, then the concept of completing squares will not be merely a matter of procedural retention for the students.

Similar to Braining Camp, I also used 'Mathway' Shyam said. In contrast to the first program, however, I used this specific application in order to check the answer and get access to detailed

solutions to mathematical problems.

Like the programs I had previously utilized, this application also had a



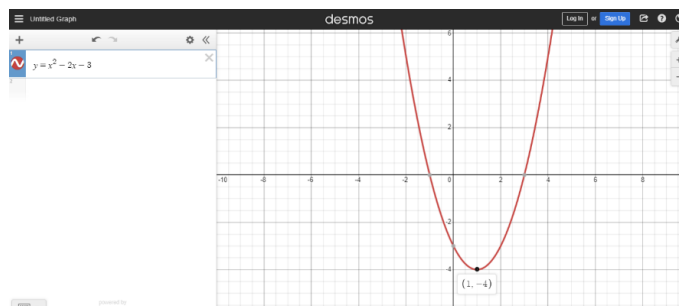
limitation: not revealing the complete

answer. Because of this, I could not access

the solutions in case I did not buy the paid version of this application.

Unlike Braining camp, and Mathway, I also discovered applications like ‘Geogebra’ and ‘Desmos’ as I continued to explore more on the internet to find out fitting virtual materials to support my class and make it interactive. I made the most of these applications by using them rigorously in a relatively short period of time. Since I teach Optional Mathematics—where graphical representation is of the essence of the subject—units like linear programming, solving quadratic equations graphically and transformation needs the extensive use of graphs. While teaching the units mentioned above online, I realized that the students could properly conceptualize the core ideas and theories underlined in those units. Even though I was found wanting using these programs at the earlier stages, however, I was slowly able to seek and find help. I found immense help from the YouTube as well.

Shyam said; “He used Desmos while teaching how to graphically find the solution of quadratic equations. I demonstrated the graphs of quadratic equations using Desmos and from the figure itself, I explained what the solution of a quadratic equation means and how one could interpret it geometrically. In the physical setting, I struggled to make the students understand the concept that the solution is actually the intersection of the parabola with the  $X$ -axis ( $y=0$ ); however, I could find the cutting edge in making the students understand this concept figuratively through animations in Desmos. I felt I could trigger the students’ concepts with the help of Desmos”.



In addition to spending a majority of my time in self-researching with vigour to receive the necessary scaffold from the internet—which has helped me a lot in teaching and learning mathematics virtually, I also took help from my friends who were involved in teaching mathematics.

However, in stark contrast, I did not receive any direct help in this area of adjusting to the new methods of teaching from the school in the form of teacher training but I did not let this demotivate me. Rather than relying on the school, I took it upon myself to learn new methods to teach to ensure that the pandemic did not bring any additional hurdles to my students' education. To do this, I surfed the internet with intent and made the maximum use of materials found on the web.

I believed that the proper use of virtual manipulatives would reinforce the concept of various mathematical ideas. Oftentimes, teachers and students, in an attempt to complete the curriculum in time, tend to focus heavily on the process and inadvertently neglect the concept in the mathematics teaching and learning process. However, the use of virtual manipulatives with precision will be a strand to uplift conceptual learning.

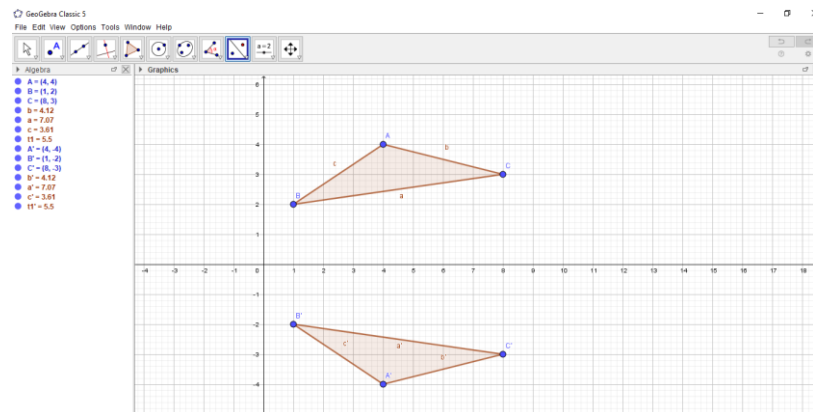
Similar to Desmos, I used the Geogebra application to teach the concept of reflection, rotation, translation, enlargement and reduction to my students. During in-person classes at school, it was tiresome for me to do activities on transformation as he had to make materials for each and every group of students, but once I started using Geogebra, I breathed a huge sigh of relief as I could use this platform to demonstrate any isometric as well as non-isometric transformation with ease. The excitement of the students shot up to newer heights when they were able to use the application hands-on. My students mastered the application much faster than I did.

Students learn concepts related to gadgets rapidly as they are already used to the working of apps which they come across while playing online games.

Students could clearly understand the concept of reflection, rotation, translation, enlargement and reduction as well as mathematically interpret them. In the physical setting, I was fed up with students not being able to conceptualize reflection, rotation, translation, enlargement and reduction. The students always memorized the formula of these transformations and were hesitant to link it up with the phenomenon that goes on in the real world. But to my surprise, the virtual class did wonders to

improve this situation.

The students were able to see how various polygons can

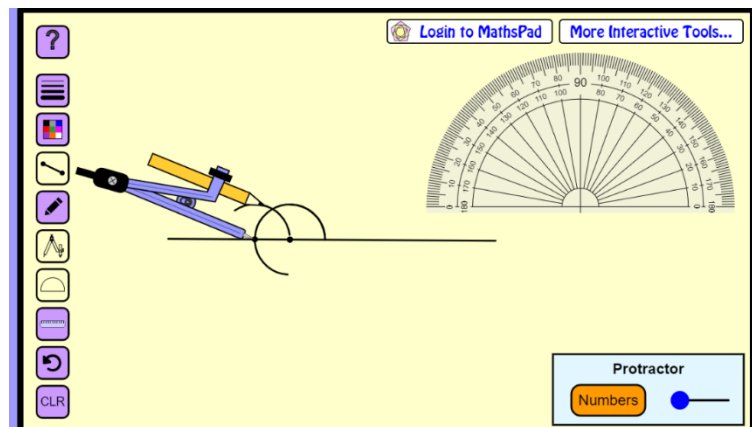


change their position and size when they are subjected to different transformations.

After performing all these different transformations, the students were also able to connect these phenomena with real-life scenarios and displayed a comprehension of where these take place in the real world.

Even in a comparatively easier unit like 'Transformation', I never found the performance of the students to be overwhelming in the physical setting. However, after the use of the virtual manipulative, the students started performing well in the tests as well which was really pleasing for me.

Another application that Shyam used was 'Mathspad'. He claimed to have specifically used this application in order to help students learn the



construction of triangles, parallelograms etc. when he took classes in other grades to fill in for absentees. Shyam said; Mathspad did not serve me as I am an Optional mathematics teacher and Mathspad is ideally suited to teaching geometry and more specifically construction but I utilized it in the substitution class. I made the most of this app as I felt that it was an opportunity for me to explore and apply apps like these.

I was more comfortable using this app in the substitution classes, where there is a lot less pressure to execute things to perfection. I took this as an opportunity to master apps that I was not confident to go through with, in the class because of my lack of exposure and experience in it.

As I compare the physical and virtual setting in the context of teaching and learning mathematics, I reflect on a few fronts. In the physical setting, when I made the students indulge in various activities, some of the materials that I used were ready-made materials. Wooden shapes of various objects, number lines, sextant, clinometer, geoboard etc. were materials the school bought directly from providers/shops, which I could directly use in the class. Nevertheless, there were materials that the teacher should make themselves according to the need of the class activity that the teacher

designs. I struggled to manage time in making physical materials for each class because of the time constraint. After taking six periods a day, I did not find the enthusiasm to work for making materials for the activities as I would already be lethargic. However, when I switched to the virtual mode, and used virtual manipulatives, I could use the same activity in different sections using the same link, which saved up a lot of my precious time. In a section of 35+ students, I had to make materials for at least six different groups per section but in the virtual scheme of things, once I designed one activity, I could easily reproduce the material for all of the other sections. The reproducibility of the virtual manipulatives was a paramount in making my work easier. By having the option of reusing the materials for all of my sections in one grade, my preparation time for classes was cut down by more than half. In a busy schedule, the reproducibility of virtual manipulatives speaks volumes and that is what I found significant. In hindsight, I found virtual manipulatives easier and more efficient. The time management for a busy teacher is instrumental in the teaching-learning setting.

Shyam extensively talked about student engagement when he used virtual manipulatives. I believe that the students who seldom interact in the physical class, have come to interact in the online class because some students are comfortable voicing their opinions when they are far away from the mass. Some students are fearful of the teacher and also may experience social anxiety, where they have a hard time communicating to a mass of their friends; because of which they do not speak in front of the physical class. However, while conducting activities virtually, I noticed a decrease in such behavior—as students may feel that they are physically very far from the teacher and their friends. Through my observations during online classes, I also

came to the conclusion that students feel more confident when they are physically distant from the teacher and their friends.

I reminisce a story of a student who did not utter a word when I taught him physically but as soon as the online classes started, he tried to interact through the chat in the zoom and slowly spoke up when questioned in the virtual setting. This is another positive aspect of virtual learning. More specifically, virtual manipulatives help in better understanding of varied mathematical concepts. Virtual manipulative articulates detailed explanations of mathematical concepts which students generally do not think of. When I tried to make the students clear about the slope of a line and the angle of inclination of the line with the X-axis, the students understood that as the angle of inclination increases, the slope increases until it reaches a vertical alignment. I demonstrated this to the students by using the slider (animation) in Geogebra.

While the students were rigorously using the virtual manipulatives in their class regularly, the students exhibited creativity in some of their works. The students learnt the concept of linear programming where milk and raisins were used in different proportions to make two different chocolates A and B.

*“Chocolate A requires 1 unit of milk and 3 units of raisins whereas Chocolate B requires 1 unit of milk and 2 units of raisins. If the company has a total of 500 units of milk and 1200 units of raisins. Additionally, if the company makes a profit of Rs 6 per unit A sold and Rs 5 per unit B sold, in order to maximize the profit. How many units of A and B should be produced?”* The students were taught to formulate a real-world problem by using linear programming. I triggered the students’ understanding by using Desmos to highlight the feasible region of the inequality after the formation of the inequality. This helped students understand the real concept of linear programming. Once the students formulated the mathematical model for the real-



world problem for chocolate production, one of my students thought about the problem meticulously and asked me how can he maximize the production of his father's ice-cream factory. Overnight, the student had already asked his father about what materials they use in making ice-cream in their ice-cream factory. He had listed two materials i.e., whipped cream and Choco chips as major ingredients. The student then realized that according to the stock in their ice-cream factory, he can help them maximize the profit by the help of linear programming. This is how the use of virtual manipulatives helped my students connect mathematics with the real-world and also helped make mathematics meaningful. The use of virtual manipulatives was not only an eye opener for my students but for myself as well. I felt that I was heightened as a teacher once I got used to teaching mathematics by using modern ICT tools. Before COVID-19, I was fearful that my hesitancy in using virtual manipulatives would certainly come back to haunt me in my professional development. This was a matter of concern to me before COVID-19, but it actually ended up being a blessing in disguise as online teaching and learning was imposed due to the nationwide lockdown. Because of this, I got the opportunity to work on my weakness due to which I feel more empowered now. In fact, I have even grown professionally as a teacher, and I am more confident in my skills to teach the students about the subject I love.

When I used virtual manipulatives, I self-reflected on my teaching and learning a lot because of which I had realized a lot of things. I remember an incident where I realized that  $y = \sin x$  and  $y = \sin^{-1} x$  are reflections of each other on the line  $y=x$ . I used to teach to graph the function  $y = \sin x$  in Grades 9 and 10 whereas I used to teach the inverse circular function in +2 where graphing  $y = \sin^{-1} x$  is of the essence. Since I taught both the concepts in isolation, I never realized the relationship between

the two. But one day, when I was graphing both the graphs together in Desmos, I suddenly came to realize this relationship between  $y = \sin x$  and  $y = \sin^{-1}x$ .

My understanding of mathematics has widened once I came in contact with the virtual manipulatives. My way of thinking has been altered in a positive manner. My horizons are breached and I understand mathematics a whole lot differently.

### Discourse with Aamir

Aamir is currently residing in the Chandragiri Municipality, ward number 15.

He has been working

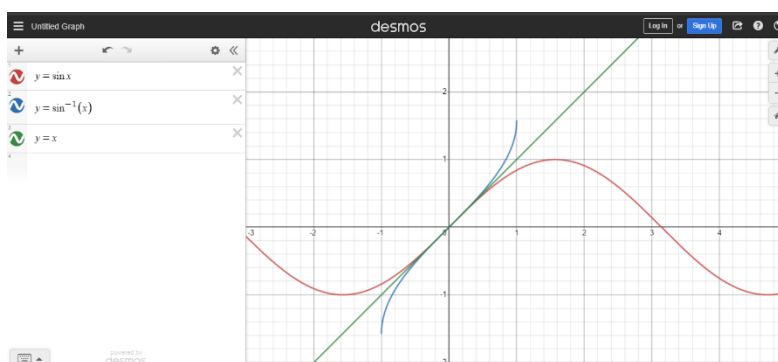
in the New Zenith

English Model School

since 2072 B.S as a

mathematics in

charge. He has been



looking after the mathematics department and takes classes in the secondary level. He had completed his schooling from Prime English School in the year 2063 B.S. He accomplished his +2 in science, in the year 2066 B.S from Acme Engineering College. He then chose Tri Chandra Multiple Campus to pursue his further education. He did his Bachelor choosing Physics as his major subject whilst Statistics and Mathematics were included in his subject combinations. He has been in the teaching field and involved with mathematics for 12 years or so.

### Interview with Aamir

Aamir teaches compulsory mathematics in Grade 10 and basic mathematics in grades 11 and 12. Online teaching and learning was not a smooth sail for Aamir as he did not have a taste of this mode of teaching and learning before the pandemic. This mode of teaching and learning was a rude awakening for him as he was not

comfortable at all while taking classes online, let alone making students do interactive activities to grasp the concept at hand. Aamir said; I was reluctant to use the ICT tools in the process of teaching and learning in the initial phase of the COVID-19 pandemic. I used to merely post assignments to the students in the starting month of the pandemic and I was not confident at all to use the new online platforms.

Ploddingly, I started to use Zoom and was able to demonstrate sums to the students on the Whiteboard present in Zoom. It took me more than a year to be able to apply the virtual manipulatives in the online class. In the first year of the COVID-19 pandemic, I never used the virtual manipulatives because I lacked the assurance to do so. As a perfectionist, I felt skittish about applying something I had not yet mastered in the classroom. So, I plunged into the traditional ways of demonstrating sums on the whiteboard. However, with assistance from my cousin who was in the IT sector for a long time, I started to understand and use virtual manipulatives. My cousin also conducted an online workshop for the teachers of his school where he demonstrated platforms like Desmos and Geogebra which were instrumental in boosting my self-confidence. For the first year of the COVID-19 pandemic, I learnt using virtual applications such as Desmos, Geogebra, Mathspad, etc. But it was only the second year of the pandemic when I started to apply them in the class in the teaching and learning process.

I reminisce the day when I took the risk of using the virtual manipulatives for the first time in class. I had to teach construction in grade X. I had bitter experiences while teaching construction to the students even physically. Students tend to ask a lot of questions while doing construction. A complete havoc will arise if the students miss out a step as the teacher demonstrates on the board. However, when I demonstrated construction from Mathspad in the virtual class, I could undo the step I

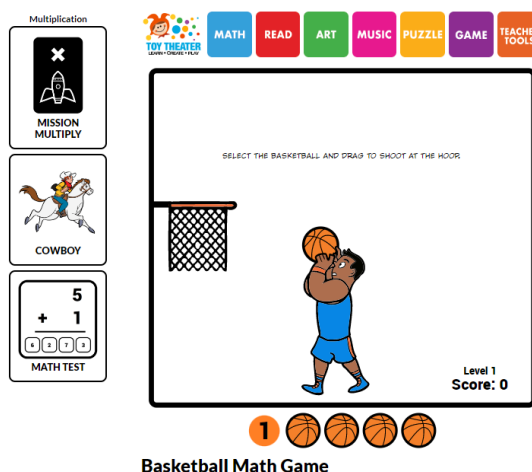
just did and redo it again in order to make the students understand what has been done. I felt that construction's virtual tool has made teaching construction much simpler. Using a big compass and a ruler on the white board was a tiring task for the teacher. After having used Mathspad, even in the physical class while teaching construction I have been using the same manipulative because I find this manipulative easy to use and feels that students understand construction the best when I demonstrate using this manipulative. Even in the physical setting, I take the students to the audio-visual room where they have a projector and demonstrate construction using this virtual manipulator.

I used Desmos to solve optimization problems while teaching linear programming. In my experience, I felt that to demonstrate graphical problems on the board is inconvenient from the perspective of the teacher. So is plotting inequalities in the graph, and then finding out the points of intersection of those inequalities to find the common space. Then using the coordinates of the points of intersection to optimize the given function is a colossal challenge to perform accurately. While graphing on the whiteboard, it is very difficult to accurately find the points of intersection. However, this is no longer a problem in his class as the graphing software gets rid of this hassle. Whenever I used Desmos to plot the inequalities, there was no confusion in the points of intersection which led to clarity of the concept amongst the students. Most of the time, in my teaching experience, I have had difficulties explaining the concepts of linear programming. However, using the likes of Desmos and Geogebra, I felt the shift. I, who would be physically exhausted after teaching linear programming am now calm and serene after using the virtual manipulatives. I used to try a lot to make the students understand the concept in the

physical class and would stress myself out in the process, but the use of virtual manipulatives has made my teaching process smoother.

I had the added responsibility of looking after the teachers in the department of mathematics as I was in-charge of mathematics at the New Zenith English Model School. I had to go for classroom

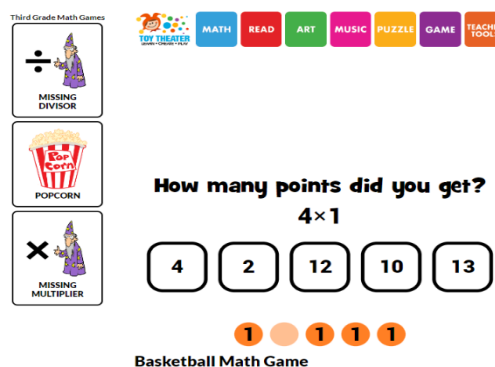
observations and provide critical feedback to the teachers in order to improve the quality of their classes. In fact, in the first year of the pandemic, I along with my co-teachers were ecstatic to have conducted the online



class. We were glad that our classes did not come to a standstill. However, as this pandemic had long-standing effects, we could not be content with merely being able to just conduct the online classes in the second year as well. I frequently visited other classes in order to see if teachers were able to use interactive manipulatives to teach mathematics in the class. I also played a hand when I visited other classes for observation. I helped the teachers with ideas on how they could make their class better. Additionally, when the teachers were on leave, I would happily take their classes. I did not want the students to miss out on learning mathematics even for a day. That’s how passionate I am for mathematics.

When I took substitution classes at the primary level, I made the students play mathematical games using the

manipulator named ‘Toy Theater’. In this virtual manipulator, the students got to play interactive games which

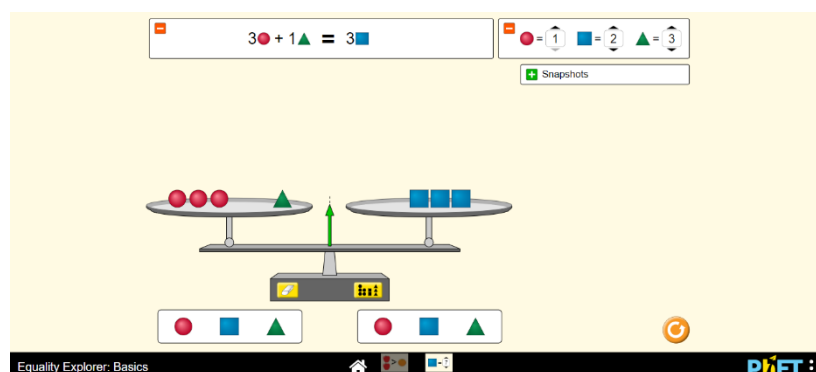


eventually focused on mathematical operations like addition, subtraction, multiplication, etc. In one of my substitution classes in Grade II, I made the students play 'Basketball' using Toy Theater'. The students played basketball and tried to get the ball in the ring. Most of the students were accompanied by their parents alongside in the virtual class who helped them play the game. The game had five levels. At every level, they would get five throws of the ball. In level 1, each score that they make will give them one point. After the end of five throws of the ball, the student will be asked a question where they need to calculate their total score at the end of the level. If the students get the multiplication correct, they will proceed to the next round and if they get it wrong, then they have to replay the same level. Once they proceed to round 2, the points for each score they make will be 2 and the same process continues up to level 5. The students will learn interactive ways of multiplication of numbers up to 5 by the application of this virtual manipulator.

The students, along with their parents, were extremely upbeat about this activity as virtual games provide a direct route to the students' hearts. The students enjoyed and made most of the class and the effervescence could be seen in their faces. I could feel the change when the students were completely indulged in their activities. I realized that day that the students concentrate better when the class is run using virtual manipulatives. In the physical classes students from grade 2 create absolute chaos when they are made to do activities using concrete materials. However, when the kids were taught using the virtual manipulatives, they concentrated without any distraction and completed the task given to them. The students in the modern-day are able to stick to their gadgets for a long time and if productive activities are designed to help them learn, then the students are going to learn them regardless of what the activity is.

I also used another manipulator where the simulation of various concepts of mathematics could be done in order to strengthen the mathematical abilities of the students. This virtual tool is known as ‘PhET Interactive Simulations’. In this virtual manipulative, real-life simulation of various activities is done, which helps the students to understand concepts better. When I went to middle school to take substitution classes, to make the students understand the concept of equation, I used ‘Equality Explorer’. This is one program in the ‘PhET Interactive Simulations’ where students put different shapes in the weighing balance and check whether the balance stays in equilibrium or not. If the level of the weighing balance is maintained, the students understand that both sides of the balance have equal weight in it. Likewise, this idea can be carried on to clarify the concepts of equations. The variables act like the shapes and if the equation has to be balanced, equal weight (value) needs to be present on both sides. I believed that students tend to always know how to solve equations but when the teacher probes into the understanding of the equations, students struggle to showcase their clear understanding. This application helped me to inculcate the concept of equations into the students. There are many other programs that PhET has to offer; one of them is simulations on median where students tend to kick footballs

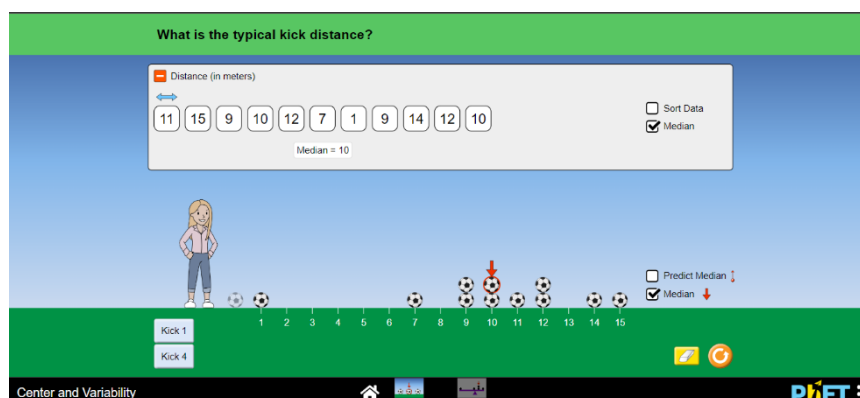
which end up in a number line with natural numbers.



The students then predict the median of the numbers where the football ends up. The students can kick as many footballs as they want. The football lands randomly on any number in the number line. The students can check the median at any time, kick a few more footballs and again check the median. This exercise helped students to understand median from a different perspective.

I noticed the rise in enthusiasm in the students once I started to teach the students using the virtual manipulatives. Students were keener than ever in his class, asked inquisitive questions and the interaction between the students and myself increased at the rate of knots. I used to get texts on Viber asking about the manipulatives I used in the class. Some used to text me late in the evening, when they faced some difficulty working with the manipulatives. Clearly, the students felt more motivated. In

my personal view as well, I found virtual manipulative much better,



looking at it from various perspectives. I found virtual manipulatives saved a lot of my coveted time. As an in-charge, I had my classes, class observations and many other works related to academics. So, I always had to stay overtime to plan and make concrete materials ready for my next class. However, as a blessing in disguise, virtual manipulatives saved a lot of my time due to its reproducibility. Additionally, virtual manipulatives are cost effective. Whenever concrete materials are made, lots of resources are used, which increase the cost but virtual manipulatives do not cost much. Also, in the physical class while doing activities with the students, there is a



high probability of noise. However, while doing activities with virtual manipulatives, students can make channelized interactions by unmuting when they have to speak.

I also reflected on my approach as a teacher before and after using the virtual manipulatives. Usually, I focused on the process of solving problems in mathematics before I used virtual manipulatives. This was because of the overloaded curriculum as well. I had the responsibility of completing all the chapters on time. This was an absolute must as the students had to appear in the board examination conducted by the government and compete with everyone else. I was always in a hurry to finish the course and never gave enough focus on concepts. I made the students practice sums repeatedly and believed the students would do better in the exams. But I realized that when the students do not understand the concept, the retention of problems is very short-term. After using virtual manipulatives in the class, I focused on the concepts more. I believed that students have to grasp the concepts first, which is fundamental in my eyes now. And after the students understand the concept, they can practice problems which will help them to gain accuracy. I also perceived a professional upswing as I learnt a lot of new things myself. My computer skills has ominously changed for the better and I feel empowered and a more complete mathematics teacher now.

### **Discourse with Bishnu**

Kathmandu Municipality's ward number 15 is where Mr. Bishnu currently resides. As a mathematics instructor, Bishnu has been catering to the lower secondary level students at the Excelsior School since 2072 B.S. After completing his schooling at Ananda Kuti Vidyapeeth in the year 2059 B.S, Bishnu continued his +2 studies in the management stream, graduating from Geeta Mata Higher Secondary School in the year 2062 B.S. For his further education, he chose to study in Tri Chandra Multiple

Campus and did B.A. Since the completion of his Bachelor, he has worked in the teaching field catering students in mathematics for the past 14 years or so.

### **Interview with Bishnu**

The discourse I had with Bishnu has released that he has been involved as a compulsory mathematics teacher for the past 14 years. Bishnu teaches compulsory mathematics at the lower secondary level and has been catering to sixth graders for the past 8-9 years. Mr. Bishnu has been enthusiastically working in the teaching field and he is quite passionate about his profession, which is why he chose this profession ahead of any other. Bishnu said; “Before the COVID-19 pandemic, I had already established myself as a good teacher and was well aware of the content I needed to teach, and has all the activities of the entire units sorted out. Since I had been working consistently as a mathematics teacher in grade 6 for 8-9 years, I had all the bases covered. After years of hardship, I finally thought that I had become a stable teacher, but to my bewilderment, the COVID-19 pandemic had other plans. Lockdown was imposed nationwide as a result of which physical classes needed an alternative. I was concerned about how the pandemic is going to affect teaching and learning. I believed that the main concern at the beginning of the pandemic was ‘How can teachers reach the students?’ I began to doubt myself whether I would be able to teach the students virtually or not. Slowly with time, I started the online teaching and learning via Zoom. The school I was teaching in, was progressive from the start of the pandemic as the school bought paid version of Zoom, which helped us to extend our classes to more than 40 minutes. Initially, I faced a lot of problems in the circulation of the online class. I did not have stable internet in my house, so the internet troubled me a lot. During the classes, I used to take an internet pack in the phone from the service provider as a backup so that my classes were not disturbed. I did not have a laptop

from the outset because before the pandemic, I was not into gadgets extensively. I never felt the need to have one as well. But due to the pandemic, I had to get one for myself in order to run the online classes. In the initial phase of the online class, I used the whiteboard feature in

Zoom to deliver content to the students. I used this feature to do sums which was similar to a whiteboard in the physical class. However, I urged myself to learn



more about interactive ways of virtual teaching and learning as I never wanted to be a traditional teacher, even in a virtual setting. Steadily I started to search for online tools that would help me make my class engaging. While teaching the arithmetic portion in grade VI, I used a website called visnos.com. In this website, I predominantly used the sieve of Eratosthenes which is the process of separation of prime numbers from a given set of numbers. In the ancient times, this process was done manually by cancelling multiples of 2, 3, 4..... and so on one after another, whereas the use of this virtual manipulative has made this task much easier. In this program, one can change the colour of the multiples of 2, 3, 4 and move until we have only prime numbers remaining. In the physical setting, it will take us a lot of time to start removing all the multiples of 2, 3, 4 and so on. But using this virtual manipulative, this could be executed in a click. The number '1' is to be left out and all the multiples of 2, 3, 4 and so on will be selected automatically. As shown in the figure alongside, the multiples of 2 are coloured in red, multiples of 3 are coloured in yellow, multiples

of 4 are coloured in red because all multiples of 4 are also multiples of 2. So likewise, all the multiples of numbers are coloured which gives us an idea that they are not prime. So, all the leftover numbers excluding '1' are prime numbers. This activity helped my students to find out the prime numbers between 1 and 100. This process could be done for any set of numbers from this virtual manipulative.

I found the sieve of Eratosthenes activity done through this virtual manipulative boost my student's conceptual knowledge. In my experience sixth graders more often than not, had misconceptions on prime and composite numbers. Some students believed that odd numbers are prime. However, after this activity students could see many odd numbers coloured when the multiples of 3, 5, 7, 9 came up. In the figure alongside, students could observe that odd numbers like 9, 15, 21 etc. are coloured. This helped the students overcome the misconception that odd numbers are prime. Once the physical classes got going when COVID-19 cases decreased to some extent, I talked about prime and composite numbers in the revision class. While doing so, I observed that students were clear about prime and composite numbers. My main objective of doing this activity from visnos.com was to make students clear about the idea of prime and composite numbers. However, this was not the only fruit I could bear from this activity. After the end of the class where he made the students do the sieve of Eratosthenes activity, I

had a long hard look at the sieve and realized many numbers were iridescent. I thought that this could be an opportunity to teach the students about common multiples.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

On the next day, I resumed the class from the same interface of the activity where I

talked to the students about multicoloured numbers in the sieve of Eratosthenes. Number 30 was tricoloured in red, yellow and green colours. I asked his students why the number 30 is tricoloured. After giving the question a little thought, the students were able to say that 30 is a multiple of 2, 3 and 5, which is the primary reason for 30 to be tricoloured. I put forward inquisitive questions to check and boost various concepts of mathematics. After using these virtual manipulatives, students were able to answer my questions as the students have become conceptually sound.

While teaching the geometry portion, I used a website named [mathspad.co.uk](http://mathspad.co.uk). By the help of this website, I was able to teach the students about lines, angles, shapes, etc. This is a platform where one can draw lines, angles and many other geometrical shapes. This website is mainly used for construction. The students can construct various geometrical shapes and observe the properties those shapes exhibit. I used to demonstrate construction in the online class by sharing my screen on Zoom. The students used to observe the process while I demonstrated and later, I let the students explore and make various shapes themselves which made my class engaging.

In the first instance, I did not have much idea about [mathspad.co.uk](http://mathspad.co.uk). The instructional supervisor for mathematics in my school had a big hand in helping me use this website. I gathered the preliminary idea of [mathspad.co.uk](http://mathspad.co.uk), but still, did not know the depth of the website. In the earlier phase I found this website difficult to use. I did not know how to erase already made lines and was unaware of the functions of this website. But slowly, with the help of my instructional supervisor, I was able to learn how this website functions.

When I started using such websites in the class, it was intriguing for the students as well. The students were glued to their screens. The students got to experience something different. They were used to playing with concrete objects in

the school but when introduced to virtual manipulatives, the excitement of the students shot up to newer heights. However, this was not the case for all the students. I experienced a great deal of trouble catering to students who came to classes occasionally. It was arduous to handle those sporadic students virtually. I felt that the students who were keen on learning from the outset became more engaged in activities in the virtual class using virtual manipulatives and those students who were disinterested became even more astray. I feel that for students of the lower secondary level, virtual teaching and learning can be challenging at times. I believe that physical classes are better for the age group of students I teach in. In school, students are able to touch and feel the concrete materials and learn various concepts of mathematics, but in virtual learning, it is very difficult to nudge the students who are disinterested.

We have a math lab in the school I teach in. I make the students go to the math lab and plan activities for them in the lab while the physical classes run. Students can touch the concrete objects and have a hands-on experience of the subject matter that was taught which is why I compare both the modes of learning and feel that physical classes are better than the virtual ones. I feel this also because in the virtual class, even if I try a lot, the students are not able to reach the intended learning outcomes, but in the physical classes if I feel that the students are not getting the concepts, I can always take time, give personal attention and modify the activity which is very difficult to execute in the virtual setting as the body language of the students cannot be seen. I feel if the students are a little grown up, then in such cases, virtual learning can be more beneficial.

I have pointed out that for virtual learning to happen smoothly, all students should have access to good internet and a suitable device. If the internet that they have is not strong, then the learning might not be smooth. Oftentimes, the video and the

device lag, as a result of which there is a hindrance in the construction of knowledge in the students. However, virtual teaching and learning has its own benefits as well. From the perspective of a teacher, I felt relief while readying the virtual materials as it was time saving for me. In school, when I made concrete materials for teaching a concept, I had to make it for 36 students. This took up a lot of time, but on the contrary, as I went virtual, one virtual manipulative was enough for all the 36 students. This cut my working hours by a lot which was wonderful from the teacher's point of view. This was not the scenario in the earlier phase, as everything was new to me. Each day I took 2-3 hours to make a slide for a class, but as time rolled on, I picked up the pace and became nimbler. I was in a dilemma on how to teach virtually at the start of the pandemic but now I see a huge shift in my confidence and attitude. I feel that I have taken huge strides forward and have grown professionally. I felt that I improved as a teacher when I started to use the virtual manipulatives in teaching mathematics. I acknowledged the fact that this dimension of teaching mathematics was not there in my teaching prior to the pandemic. When the pandemic occurred and the way of teaching shifted into a different mode, I too had to learn and then incorporate teaching through the virtual mode, which empowered me and made me more confident as a mathematics teacher.

I feel that the use of virtual manipulatives helped students to link mathematics with the real-world. When I taught the students about factors and multiples by using the sieve of Eratosthenes, I also mentioned to the students to write down numbers in rectangular form so as to get the factors of any number. I clearly instructed them to change a given number into rows and columns, which when multiplied with each other will give the original number. I asked the students to connect this phenomenon with a real-life example. A student looking at the chess board he had in his home told

me that since there are 8 square boxes row wise and 8 square boxes column-wise as a result of which the total number of square boxes in the whole chessboard is 64. The student also explained that 8 and 8 are factors of 64. I was impressed by the connection of the phenomenon to the real-life scenario by a 6<sup>th</sup> grader. I believe that the way in which students understand mathematics has altered in a positive sense by the use of virtual manipulatives. The students are connecting real-life scenarios to mathematics which pleases me.



## CHAPTER V

### FINDINGS AND DISCUSSIONS

In this chapter, I have discussed and interpreted the narratives of my three research participants, Shyam, Aamir and Bishnu which were incorporated in Chapter IV. Moreover, I have constructed some themes on the basis of the narratives of my research participants. Thorough discussions on those themes have been made in this chapter with the support of relevant literature.

#### **Virtual Manipulatives Contributing to Meaningful Mathematics Learning**

I would like to start the discussion with a story that Shyam shared where he taught the tenth graders the concept of linear programming. While doing so, he used the virtual manipulator called Desmos. He used Desmos to plot the inequalities and found out the feasible region from the graph showcased in Desmos. He then helped the students solve optimization problems. In doing so, students understood the materials that are required to prepare a certain product, the quantity of the materials used to prepare the product and in what ratio different materials are used in order to make different products. This helped students understand the real concept of linear programming. One student thought about the problem vigilantly and asked Shyam how he helped his father maximize the production of their ice-cream factory. Overnight, the student had already asked his father about what materials they use in making ice-cream in their ice-cream factory. He listed two materials i.e., whipped cream and Choco chips as major ingredients. The student then realized that according to the stock in their ice-cream factory, he could help them maximize the profit with the help of linear programming. This is how the use of virtual

manipulatives helped Shyam's students connect mathematics with the real-world and also helped make mathematics meaningful.

Durmus and Karakirik (2006) demarcated that virtual manipulatives have come up as viable computer applications which help students appreciate the meaningful applications of mathematics to solve real-world problems. In my experience as a learner, I have felt the importance of the connection of mathematics with the real world. When I was in school, most of the mathematics we studied felt inconsequential, which led many of my friends to believe that this was the last subject they wanted to pursue in their higher studies. Even as a teacher now, I feel most of the students do not want to choose mathematics. This is because we have not connected mathematics enough with real-life scenarios. I believe that students find mathematics fun-filled if the teachers are going to connect their mathematical experiences with the happenings in the real-world. Arthur et.al., (2018) outlined that teaching of mathematics is made interesting to the students when the teachers are able to connect the mathematical concepts to real-life problems.

Shyam narrated that in the physical class, his students were hesitant to link up transformation with the phenomenon that goes on in the real world. However, in the virtual class, when he used the virtual manipulative 'Geogebra' to demonstrate this phenomenon, the students could visualize polygons changing their position and size when they were subjected to different transformations. After this experience, the students could link the reflection that they studied in the chapter with the reflection of the sun in water i.e., the students were also able to connect these phenomena with real-life scenarios and displayed comprehension of where these took place in the real world.

Additionally, Shyam added that the performance of the students in the tests when they had the physical classes had not set the world alight. However, after the use of the virtual manipulative, the students started performing better in the tests as well. Rakes et al., (2010) stated the difficulty in connecting Mathematics to real- life problems have contributed to the failing and bad performance of students.

I have also realized the same situation in my career as an Optional Mathematics teacher when I taught the students the most feared topic ‘Trigonometry’. In my experience, tenth graders always found trigonometry arduous and burdensome. The results also supported this fact. When I took the unit test of trigonometry, the students had a very difficult time. Most of the students were only able to do the question from ‘Height and Distance’ and the questions from remaining exercises were properly done only by a few students. In the SEE, trigonometry consisted 20% of the whole Optional mathematics course. In the slight interactions I had with the students, I mostly heard students talking about passing the Optional Maths exam without attempting any of the questions from trigonometry except ‘Height and Distance’. After hearing this from the students, I talked to them about their problems in trigonometry. The students could only tell me that they are not able to conceptualize trigonometry. A few students told me that; they were not sure about which formula to use while proving the identities. When I asked students about the easiest exercise, they found in trigonometry, the unanimous answer was ‘Height and Distance’. When I probed deeper into the reason, I could understand that the students could link height and distance with the real-world scenario. They could feel that this part of trigonometry is not abstract, whereas other exercises went over the students’ mind as they could not link them with real-world scenarios. Hence, from my experience of

teaching trigonometry I have also realized that meaningful learning can only happen when the concepts are linked with real-life scenarios.

One can only gain knowledge which is meaningful if they have the clarity of the concepts/ideas taught to them. Hence as a teacher, a substantial amount of time and effort should be given to clarifying concepts which will only make mathematics learning meaningful and worthwhile. Stella and Fleming (2011) outlined that clarity in mathematics instruction is essential, because students cling to many misconceptions and it is necessary to obtain cognitive empathy to determine what goes on inside their heads as they think through mathematical problems.

Aamir taught construction to the students by using a virtual manipulator named Mathspad. While doing so, the idea of construction, along with the steps required to complete it was clear to the students. In the physical class, Aamir had previously experienced that the students tend to get mixed up with the steps of construction but when he used this virtual manipulative, the understanding of construction was crystal clear. The clarity of the idea of construction could be seen through the work of their students.

Aamir also used Desmos to plot the inequalities and to find out the common/feasible region after multiple inequalities were plotted. When those inequalities were plotted on the board in the physical setting, the students had doubts always in the point of the intersection. However, after the use of Desmos, students were no longer confused about the intersection of the inequalities.

Aamir also made the second graders play a virtual basket-ball game through a virtual manipulator called the 'Toy Theatre' at the end of which the students were made to calculate the total points they received. The students received 1 point for a score they made in the first round, 2 points for a score they made in the second round

and so on. They got five throws of the ball in every round. The round ends with a culminating question where the students are made to calculate the total points they received at the end of the round in the form of multiplication. If they multiply correctly, only then they can proceed to the next round or else they have to repeat the same round again. The students will only be able to answer the questions correctly if they are clear in multiplication.

Moreover, Aamir made the students clear about the concept of equations by using the virtual manipulator called PhET interactive simulations. In this real-life simulation, the students put different shapes in the weighing balance and checked whether the balance stayed in equilibrium or not. If the level of the weighing balance was maintained, the students understood that both sides of the balance had equal weight in it. This fundamental idea was used to clarify the concept of equations which was done with the help of this virtual manipulator.

In the case of Bishnu, the clarity in making students understand the concept of prime and composite numbers has been supported by the use of the virtual manipulative called visnos.com. Bishnu used visnos.com to demonstrate the sieve of Eratosthenes, to make the students distinguish prime and composite numbers. In this virtual application, numbers between 1 to 100 are displayed. When one presses the start button, the multiples of 2 starts highlighting. The numbers that are the multiples of 2 are coloured with a certain colour. Then same happens with the multiples of 3, multiples of 5 and so on. The process is completed in around 30-40 seconds. The numbers that are left uncoloured after the end of this process other than 1 are the prime numbers and the numbers which are coloured with some colours are composite numbers. In previous years, Bishnu's students had the misconception that odd numbers were also prime numbers. But after having completed the sieve of

Eratosthenes activity using visnos.com, the students do not have this misconception anymore. This is how virtual manipulatives helped to bring clarity of ideas in teaching mathematics hence contributing to meaningful mathematics learning.

### **Virtual Manipulatives in Promoting Classroom Interaction in the Mathematics Class**

In the traditional mathematics teaching and learning practices, mathematics was taught in a unidirectional fashion. The teacher was supposed to be the only source of information and the interaction between students and teachers was insubstantial. However, in the contemporary practices of teaching and learning mathematics, classroom interaction is instrumental.

Shyam extensively talks about classroom interaction when he uses virtual manipulatives. He believes that students who seldom interact in the physical class, have come to interact about the subject matter in the virtual class. The reason behind this might be because the students are comfortable to participate in the class when they are far away from the mass. Shyam also believes that few students are scared to interact when the teacher is near them and few have a hard time communicating with a mass of their friends as they do not usually speak in front of the physical class. Whenever Shyam made the students perform activities virtually, students who usually do not speak up in the physical classes started interacting. Shyam recalls a story of a student who did not utter a word when he taught him physically but as soon as the online classes started, he tried to interact through the chat in the zoom and gradually opened himself up in the virtual class. This is another positive aspect of virtual learning that Shyam emphasizes. Wang and Newlin (2001) outlined that a virtual classroom is a place where participants are identified by screen names, and their expressed ideas are judged solely on the basis of their merit. It is a non-intimidating

environment where the playing field is level. The relative anonymity of students and the fact that they access the chat room from comfortable surroundings engenders a comfort level that is not found in the live classroom.

In the case of Bishnu, he used to demonstrate the virtual activity in the class and let the students observe the whole process. However, once the students observed the whole process, he made the students explore and indulge them into the same activity in the class itself which made his class more engaging. While teaching the geometry portion, Bishnu used a website called mathspad.co.uk where he taught the students about lines, angles, shapes and further proceeded into construction of geometrical shapes. Bishnu demonstrated the activity and thereafter made the students do it in the class by sharing their screen. This fostered a good classroom environment and promoted classroom interaction.

Aamir has experienced classroom interaction in his days of teaching mathematics by using the virtual manipulatives. When Aamir made the students of grade II play the virtual basketball game by the manipulative 'Toy Theatre', the students were ecstatic and participated in the activity with utmost enthusiasm. Most students were accompanied by their parents since this was an activity done in the strict lockdown period where most of the parents were home. The students along with the parents enjoyed the interactive activity and the interest of students shot up to newer heights. Aamir could feel the change in the mood of the students as they were able to connect through this wonderful activity. Aamir also got very positive feedback at the end of the year in the PTM where few parents acknowledged the good work he was doing as a mathematics in-charge at school.

Moreover, when Aamir used 'PhET interactive simulations' to make the students understand the concept of median by making them kick virtual footballs

which ended up falling on a number line, this activity was able to generate a lot of interest in the students. The students had a lot of inquisitiveness in them and they constantly kept on messaging Aamir to ask him more about this activity. Aamir realized that this was only due to the use of virtual manipulatives.

In my involvement with the students for the past 10 years, I have come to realize that new students take around 2-3 months to get comfortable interacting in my class. The students gradually get used to me, my appearance and my nature in the physical class in around that time period. However, when the pandemic occurred, I had to deal with a new set of students and that too virtually from the outset. I was wondering how I would be able to connect myself with the students in a completely foreign mode of teaching. Slowly as the classes commenced and I started to implement virtual tools in the class to trigger the students' mathematical interests, the participation of the students was unprecedented. They started to interact and pose questions in the class from the first day itself. Students found the virtual manipulatives gripping and that helped them connect and interact in the maths class.

### **Virtual Manipulatives Making Teachers' Work Easier**

Teacher's job is really time consuming. They have a lot of things to prepare before they go into the class. From correcting the copies to making lesson plans to management of the materials for the activities, teachers have a lot to do. Mathematics teachers have more work than other teachers as they have a lot more exercises correction.

Shyam said when he had to make concrete materials for each student, a lot of his coveted time went into managing/preparing materials. However, when he started using virtual manipulatives in teaching mathematics, that saved a lot of his time which was because one link/material could be used for the whole class which was not



possible while using concrete materials in the physical setting. He focused on the fact that the virtual manipulatives are easily reproducible which was one of the most important aspects.

Aamir worked as the in-charge of mathematics so he always had some work or the other. He had his own quota of classes, class observations he needed to make and many other works related to academics. He usually had to stay up overtime to complete all his work. Managing work on time is really important in a job. When he used concrete materials to make the students do the activities, he had to devote a lot of his time in preparing materials. But when he used virtual manipulation, he found it saving a lot of his time. Occasionally, Aamir had to buy stationery, wooden materials etc. which was used for activities he made the students do. This would take up a lot of his time and also cost money. However, the use of virtual manipulatives has made the teachers' work easier as this hassle is not quite there when we search for virtual manipulatives.

In my view, when I used to prepare materials in the physical class, I used to spend a lot of time researching and executing the findings of my research to prepare fitting materials. I used to take the help of my colleagues while reproducing materials to manage my workload a little. I sought their help as I instructed them to replicate the materials I had prepared. Tentatively, it took me a little more than an hour to prepare materials for a class. But in contrast, when I started preparing virtual manipulatives, it saved me a lot of time. I was able to finish preparing materials in 10-20 minutes. I could easily surf the internet and find the perfect fitting material that covered the course. This saved a lot of my time and made my work easier. Mostly, I felt stressed in putting the effort to explain topics which seem relatively abstract to the students. Mathematics can be abstract at times, so to explain the relatively abstract topics of

mathematics is a challenge teachers have to face every day of the week. But as a blessing in disguise, virtual manipulatives come up as a tool that helps teachers explain relatively abstract topics in an easier way by the use of picture and its animations. Dahal et al., (2019) outlined that visualization softwares like Geogebra helps to change the abstractness of mathematics to concreteness and it can be helpful for visualization as well. This kind of virtual tools will help the teacher to make the students understand difficult topics of mathematics through visualization and animations that is an important trait of virtual manipulative.

In the case of Bishnu virtual manipulative has helped him explain the concepts of prime and composite number swiftly which in turn has made his work easier. Bishnu recalls the physical class where he manually uses the sieve of Eratosthenes. He instructed the students to cancel the multiples of 2, 3 and so on. After all the multiples of numbers were cancelled out, the remaining numbers other than 1 was prime numbers and all the cancelled numbers was composite. This activity took a lot of time physically, however the virtual manipulative visnos.com made Bishnu's work easier by speeding the process. Hence, virtual manipulatives made Bishnu's work easier as a teacher.

### **Virtual Manipulatives in Teacher's Professional Development**

Before the lockdown/COVID-19 pandemic, Shyam was hesitant in using the ICT tools as he thought he was not perfect in using those ICT tools and did not want to use it as he was not super confident in it. However, once COVID-19 happened, he got used to the virtual manipulatives and applied them in the class. When he was able to properly implement virtual manipulatives, he felt more empowered as a teacher. He feels that he has grown professionally and is more confident. Virtual manipulatives made Shyam realize many things that he never thought of, even after teaching maths

for a lot of years and gaining a lot of experience. When Shyam used virtual manipulatives, he had self-reflected his teaching and learning as a result of which he realized that  $y = \sin x$  and  $y = \sin^{-1}x$  are reflections of each other on the line  $y=x$ . He used to teach to graph the function  $y = \sin x$  in Grades 9 and 10 whereas he used to teach the inverse circular function in +2 where graphing  $y = \sin^{-1}x$  is of the essence. Since Shyam taught both the concepts in isolation, he never realized the relationship between the two. But one day when he was graphing both the graphs together in Desmos, Shyam came to suddenly realize this relationship between  $y = \sin x$  and  $y = \sin^{-1}x$ .

In my view, when I started my teaching career back in 2012, I did not know much about virtual manipulatives. The only computer applications I used were 'Word' and 'Excel', that too very surfacely. However, 10 years down the road, I have been able to understand and implement a lot of virtual applications which have helped me add another dimension to my teaching. After the development of such virtual skills, I have been able to give training where I teach other teachers about various virtual manipulatives.

Aamir felt a professional upswing after implementing virtual manipulatives in the class as he had learnt a variety of new things. He admitted that his computer skills were under par before the lockdown but after the teaching and learning went virtual, he was forced to learn the various computer applications. Due to this conundrum, he had to rapidly inculcate computer skills in himself or else the teaching and learning situation might reach a standstill. He recollects the initial struggle to learn various virtual applications like GeoGebra and Desmos. He constantly went to his cousin for support as he was an IT professional. But now as he falls back to all the grapples,

everything seems worthwhile. He feels like he is more empowered by the new skills that he has honed.

Aamir also talks about his vision while teaching in the physical class. He was so obsessed with finishing the course on time that he did not give much focus on conceptual understanding. He was trapped in the vortex of encircling winds of making students practice routine sums repeatedly. While doing so, he had subconsciously shifted his priority away from injecting conceptual knowledge in the students. He used to focus on problem retention, however if the students did not understand the concepts, the retention of problems was very short-term. But as he started to use the virtual manipulatives to teach, it was a stimulation for him as well. Now he believes that the first and foremost thing that a student should grasp is the concepts which can in turn help them to gain more accuracy in the problems that they solve. This is how his mission and vision has shifted after the use of virtual manipulatives in teaching mathematics and that has also empowered him as a teacher.

In Bishnu's case too, professional development was well acknowledged. Bishnu was also reluctant to use the virtual manipulatives in teaching mathematics before the pandemic. However, as the catapult needs to be held back to propel it to greater heights, the pandemic made Bishnu discover the untouched horizons of mathematics teaching which he was unaware of. This empowerment helped him a lot to gain confidence in his teaching. Even though he was a good teacher in the physical setting, he always felt that the lack of ICT skills would come back to haunt him someday. However, due to the pandemic he could strengthen his weakness and improve his overall teaching.

### **Virtual Manipulatives in Conceptual Understanding of Mathematics**

When students learn mathematics, they gain two kinds of knowledge, i.e., conceptual knowledge and procedural knowledge. Even if the student can do sums with accuracy, it does not necessarily mean that the student has understood the concept of the problem. The concept is actually the idea behind the problem and to make the students understand that, might be a challenge.

Shyam emphasizes that many virtual manipulatives have helped him make his students understand varied mathematical concepts. He gave an example of the relation between the slope of a line and the angle of inclination of the line with the  $X$ -axis. He used the slider available in Desmos to increase the angle of a line with the horizontal axis ( $X$ -axis). When he did that the students could understand that as the angle increases the slope of the line increases and vice-versa. This activity made the students understand what slope really is and made them clear that the factor that affects the slope of a line is the inclination of the line with the  $X$ -axis. Shyam also added when he taught factorization by the help of the algebraic tiles using Braining Camp, he could make the students understand that the rectangular arrangement of a quadratic equation is basically factoring. The students could link the factorization of quadratic equations with factors of any number. If we say 3 and 2 are the factors of 6, then we can keep 3 counters in the row and 2 counters in the column which makes a rectangular arrangement where a total of 6 counters will be present. The activities like these which are done through the use of virtual manipulatives aided students to gain conceptual knowledge of mathematics.

Aamir accentuates that the virtual applications have helped in triggering his students' conceptual understanding in many ways. He added the context of teaching equations by using PhET interactive simulations. While teaching equations, Aamir has felt that most teachers only focus on the process of solving equations but do not

inculcate the main concept of equations in the students. Students gradually understand the procedure to solve equations but when it comes to concepts regarding it, the students are found wanting. After the use of PhET interactive simulations to teach the concept of equations, students tried various ways to maintain the weighing balance in equilibrium position. While doing so, students understood that the weighing balance will only stay in equilibrium, if both the sides have equal weight. For some time, they struggled to keep the same weights on both sides, however when they were able to keep the same weights on both sides, they appreciated the equation as a mathematical model having equal worth on both sides. Aamir was really happy to see the students be conscious of the idea of equations via the activity.

In my proficiency, I have also found students struggle to exhibit clarity in what the equation actually is. The students have already got into the groove of solving routine problems and that is not of any difficulty to them. However, when it comes to answering conceptual questions regarding equations, there is hesitancy which is actually due to the lack of conceptual understanding.

## CHAPTER VI

### MY LEARNING AND REFLECTION

In this chapter I have put forward my learnings based on my research findings. I have also included my pain and pleasure from the identification of my problem to the formulation of my research question to the complete go through of my research process. While doing so I have included the hassles, dilemmas, reliefs and learnings of the research process. I have also inculcated the dos and don'ts during my time dedicated in this research. Finally, I have incorporated the possible implications of my research along with my future directions.

#### **Reminiscing my Problem Identification**

Since the turn of the decade, COVID-19 was the prime reason for the commencement of virtual teaching and learning in Nepal. Majority of teachers and students did not have much idea about the virtual teaching and learning scenario in Nepal. But slowly as the pandemic took its toll, the teachers had to shift to the virtual mode to resume the studies for the students as their learning had been at halt due to the unprecedented pandemic. Only a few teachers could feel the ease in the transition from the physical class to the virtual class and most of the teachers in my circle seemed to struggle in this transition as well. Conducting a virtual class for the first time amidst the mental block created by the pandemic was something everyone was struggling with. In the school that I was teaching, we were advised to stay in each other's class for technical help. I used to stay in the class of other mathematics teachers of our school and they used to stay in my class as well. I gave as well as received honest and critical feedback to/from my colleagues for the betterment of our classes. I used to help them enrol the students in the Zoom class by performing the

role of the host. However, quality teaching and learning in the initial phase of the pandemic was of the minimum priority. Everyone was thinking of regulating the classes and other things glided off the priority list. However, as time rolled on, the mathematics teachers from my school felt the need of using the virtual manipulatives in teaching mathematics to make the students understand mathematics concepts well. They started using the virtual manipulatives in teaching mathematics and started to make the class interactive. The students in the initial phase of the pandemic did not interact much in the virtual class. The students were predominantly disengaged as a result of which teachers felt the need to introducing interactive virtual manipulatives in the class which they felt would break the monotony. Some teachers felt the virtual manipulatives helped their students to get out of the bubble and make their class interactive, whereas some teachers felt really uncomfortable while using the virtual tools as they were not quite used to in using the virtual tools.

When I was studying in the second semester at KU, we had to tentatively choose the research agenda for our dissertation. Even then, our classes were conducted online due to the pandemic as a result of which I had thought of doing research that reverberates with the virtual learning. After discussion with my course facilitator, I was adamant on doing my research on the understanding of the teachers in using virtual manipulatives and uses of virtual manipulatives in teaching mathematics. My course facilitator inspired me to take up this research agenda as this was an area that was not rigorously researched upon. I along with my course facilitator felt that this was an area that needs to be researched on as virtual teaching and learning had taken huge strides in the past year or so all over the world because of the pandemic. I was ecstatic to identify a problem in mathematics teaching that I was interested to research on. I felt that I had taken a head start because it was merely the



second semester and I had a research agenda to work on. I always wanted to finish my dissertation on time as I had heard the stories of my seniors including my elder brother as they had told me how dissertations can be daunting if you do not work continuously. This is what motivated me to work perpetually in my research agenda. With subtle changes and fine tuning in the third semester, I was ready to work on the problem I identified 'Teachers' understanding and uses of virtual manipulatives in teaching mathematics'.

### **My Methodological Flashback**

It was great that I had my problem identified very quickly and my course facilitator gave me a thumbs up to go with it. However, choosing my methodology took up a good amount of time. In the initial phase, I was dialectic on how to go about researching in this agenda. Since my brother did an autoethnography in his M.Ed., I was sure that I would not want to go with the same. I pondered hard and long about going through with action research. I discussed a lot of times with my supervisor, course facilitator and head of department on how action research can be performed with this research agenda in mind. I continuously talked to them and found most things on how to implement action research. In my priority list, completing my dissertation on time was in the pole position. Since it was uncertain times due to the pandemic, I could not be sure that the schools would not be shut physically if COVID cases shot up. If the schools were shut, it would be difficult for me to execute the 15-day long action plan. After thinking about these spanners in the work, I left the action research behind and took up the narrative route. I was confident that I can take the interview of my research participants physically or even virtually if need be. After being in dilemma for 2-3 weeks, I finally confirmed that my methodological orientation is going to be narrative inquiry. After this experience, I have come to

understand that selecting the methodology is equally important to problem identification and can be daunting at times if we do not have a clear insight on what we want to do. If I have the opportunity to do another research in the future, I am going to plan out more subtly.

### **Formulation of my Research Question**

After I was crystalline about my methodology, I worked on my research question. Since I was about to do narrative inquiry and my paradigm was interpretivism, I made 3 research questions initially that probe to find the answers of my research problem. After discussion with my supervisor, we limited my research question to only one which is “How do mathematics teachers narrate their understanding and uses of virtual manipulatives in teaching mathematics?” This question was the accumulation and finer version of the 3 research questions I made earlier. Since the research question varies according to which methodology one is researching, it is important to be transparent with our methodology before we start preparing our research questions. In the second semester, we were taught about all the research philosophies and how we can formulate research questions guided by those philosophies that helped me take huge strides forward as I did not have much of a hassle to shape my research questions that suit my research philosophy.

### **Proposal Perfected Painstakingly**

After having developed research questions that fit perfectly on the shoes of the research agenda, I swiftly moved on to the next hurdle i.e., the proposal. While doing so, I had to read, read and read. Sometimes I needed to re-read my own words in order to understand the research agenda I have chosen and most importantly not to get off track. It might sound bewildering, but this was a matter of fact. I used to think that if someone delegated me a topic, I could easily research on that topic and then my

command of the English language might play its role which would help me leapfrog and probably write better than one expects. But to be afraid that was not the case. Writing on a random topic with no structure is a whole lot different from structured writing. To be able to fill in my experiences with the research agenda and to find fitting literature was a challenge. Somehow, I was able to construct my background without moving off track from my agenda. However, the real trouble came when I reviewed various literature. I had to select few themes while doing the thematic review. After the selection of themes, I had to read a lot of articles to find out the literature that best suited those themes. I also added my experiences as a teacher to support the papers I had included in the thematic review. This gobbled up a lot of time as this was the first instance when I read many articles. Finding parts of the paper that fit into my theme was a challenge of its own. This is where I got stuck for a long time. With perseverance and determination, I completed the thematic review. Since this was a learning curve for me, I did not eat up much time when it came to reviewing theories and past studies. By then, I already knew the proper technique to scan articles. I took notes of the page number of articles and collected the name of the writers during the scanning of the paper. This was an important trait that I learned during my thematic review. This trait was of the essence in the next part of the literature review i.e., theoretical and empirical review. Proper note-taking while going through various papers helped me complete my theoretical and empirical reviews in a short time. After succeeding literature review, I moved on to the methodology section. As I discussed the methodology of my research with my course facilitator before even starting to write my proposal, I was crystal clear about the research philosophy I was following and the research methods as well. Due to this clarity, I could complete the

methodology section in no time. After completing the first three chapters, I was able to submit my proposal at the end of the third semester.

### **Purposive Participant Selection**

The proposal defence was done and dusted. I was in the process of looking up for participants who are fit and willing to be a part of my research. Since my research was a narrative inquiry, I had to purposively find those teachers who have been using virtual manipulatives in teaching mathematics. I had a difficult time reaching out to the teachers and selecting them. I approached a few teachers initially and once I talked with them, I found out that they were not using virtual manipulatives in teaching mathematics. I had to filter those teachers and move ahead to find those who were using virtual manipulatives. After having scanned many teachers, I finalized two participants in the initial phase. I started to build rapport with them when I was still searching for my third participant. After a month-long hassle, I was able to get my third participant as well. The most important thing that I learnt from the process was that this selection procedure would take up a lot of time, so researcher probably needs to pan out his/her research participants from the outset.

### **The Quest for the Questionnaire**

The participant selection and the preparation of the questionnaire took place side by side. I prepared a questionnaire containing 20 questions and submitted it to my supervisor. After my supervisor had a look, he gave me feedback on a few fronts and told me to recapitulate and simplify a few terms that we as education students know but our participants might not. I used the term 'meaningful mathematics learning' in my questionnaire. My supervisor told me that meaningful is a subject to interpretation. So, if I have to get the most out of an interview, I need to make specific questions not general ones. I admired the feedback of my supervisor and made

amendments sooner rather than later. Subsequently I fine-tuned my questionnaire and ended up with 12 interview questions in my questionnaire but as my interview was a semi-structured one, I could do few tweaks in the questions as per the flow of my interview. This was an enriching learning experience for me.

### **Interview**

The questionnaire was hot to trot; timings were fixed for the interview with the participants. I met two participants physically and one participant virtually in the first instance. I linked up with the first two participants in the school they teach. I recorded the audio of the entire conversation of the first day which was instrumental. This recorded audio helped me later that night when I listened to the audio and transcribed the narratives of the participants. However, the third participant was only able to give me time virtually as he was not able to meet me before 8 in the evening. I recorded the video of our entire conversation on google meet, which is where I reverted back to, in order to transcribe his narratives. It was not easy to fix the time with the participants as they all were working mathematics teachers who worked till 7-8pm. With constant follow-up and reminders, they were able to manage time from their hectic schedule. Subsequently, I also understood that after the interview, we need to transcribe the narratives on our device as soon as possible because if we procrastinate doing that, the frame of mind of the participant and the interviewer will be different and things can mean a whole lot different.

After the first phase of interviewing and transcribing, I started to write chapter IV, where I wrote the narratives collected from the interview of the participants. While doing so, I felt the need to interview the participants one more time as clarification on a few fronts was required. I met the participants one more time to ask

them about a few things I had confusion about, which helped me shape my chapter IV well.

### **Implications**

As mathematics teachers, it is instrumental that we focus on the concepts, rather than inculcating only procedural understanding and mark scoring traits in the students. Virtual manipulatives reinforce the conceptual understanding of students in mathematics as a result of which it is very important for the teachers to know about it. This study focuses on mathematics teachers who want to teach students in an interactive fashion through the use of virtual manipulatives. It is also for those teachers who are not yet confident of implementing virtual manipulatives in the classroom but are interested to learn more about it so that they can apply them in the near future. This study serves mathematics teachers, head of departments and curriculum experts who want to improve the mathematics teaching and learning scene in Nepal through the proper use of interactive virtual manipulatives.

### **Future Directions**

I completed my B.Sc. in 2013, and since then, I have been engaged in the teaching field. I always felt that I had enough as a teacher and took pride in the communication and mathematical skills I had. I never realized the importance of higher studies until COVID-19 gave me an opportunity to self-reflect. During the pandemic, I felt the urgency of studying mathematics education as I was someone who always wanted and has been involved in the teaching and learning field. I always wanted to improve my own teaching practice as well as helped my colleagues become better professionals.

From my childhood, I always wanted to establish a school of my own, where the teaching and learning would be unprecedented. Before I gave wings to my dream,

I needed to make sure I complete my studies. As I was in the later stages of the third decade of my life, I analysed and calculated the tentative time it might take for me to complete my Ph.D. I felt the need to complete my M.Ed. when I am 31 which would help me stay on track and not get derailed in my journey of pursuing higher education. That's why I started pursuing M.Ed. in 2020 at Kathmandu University. As I resumed my studies, I was glued to the ways of teaching and learning at the university. I received the education that was enriched with interaction and open-endedness. I was impressed with the standard of education here at the Kathmandu University, which motivated me to take up this journey with enthusiasm. My supervisor Indra Mani Shrestha is another major source of my motivation as he has always inspired me to push my limits and do better.

## REFERENCES

- Arthur, Y. D., Owusu, E. K., Asiedu-Addo, S., & Arhin, A. K. (2018). Connecting mathematics to real-life problems: A Teaching quality that improves student s' mathematics interest. *IOSR Journal of Research & Method in Education (IOSR-JRME)*, 8(4), 65-71.
- Aslan Tutak, F., Bondy, E., & Adams, T. L. (2011). Critical pedagogy for critical mathematics education. *International Journal of Mathematical Education in Science and Technology*, 42(1), 65-74.
- Boggan, M., Harper, S., & Whitmire, A. (2010). Using Manipulatives to Teach Elementary Mathematics. *Journal of Instructional Pedagogies*, 3.
- Bouck, E. C., & Flanagan, S. M. (2010). Virtual manipulatives: What they are and how teachers can use them. *Intervention in School and Clinic*, 45(3), 186-191.
- Dahal, N., Shrestha, D., & Pant, B. P. (2019). Integration of GeoGebra in teaching and learning geometric transformation. *Journal of Mathematics and Statistical Science*, 5, 323-332.
- David, B. R. (2020). What is ethics in research and why is it important? *National Institute of Environmental Health Sciences*.  
<https://www.niehs.nih.gov/research/resources/bioethics/whatis/index.cfm>
- Durmus, S., & Karakirik, E. (2006). Virtual Manipulatives in Mathematics Education: A Theoretical Framework. *Turkish Online Journal of Educational Technology-TOJET*, 5(1), 117-123.
- Fosnot, C. T. (2013). *Constructivism: Theory, perspectives, and practice*. Teachers College Press.



- Furner, J. M., & Worrell, N. L. (2017). The importance of using manipulatives in teaching math today. *Transformations*, 3(1), 2.
- Guba, E. G. (1981). Criteria for assessing the trustworthiness of naturalistic inquiries. *Educational Communication and Technology Journal*, 29(2), 75- 91.  
<https://doi.org/10.1007/bf02766777>
- Kay, R., MacDonald, T., & DiGiuseppe, M. (2019). A comparison of lecture-based, active, and flipped classroom teaching approaches in higher education. *Journal of Computing in Higher Education*, 31(3), 449-471.
- Khera, S. (2018). *You Can Win: A Step-by-Step Tool for Top Achievers*. Bloomsbury Publishing.
- Ling, T. M., & Harun, J. (2014). Instructional scaffolding in online collaborative learning environment for knowledge construction among engineering students. In *2014 IEEE 6th Conference on Engineering Education (ICEED)* (pp. 40-45). IEEE.
- Moyer-Packenham, P., & Suh, J. (2012). Learning mathematics with technology: The influence of virtual manipulatives on different achievement groups. *Journal of Computers in Mathematics and Science Teaching*, 31(1), 39-59.
- Mullis, I. V. S., Martin, M. O., Fishbein, B., Foy, P., & Moncaleano, S. (2021). *Findings from the TIMSS 2019 Problem Solving and Inquiry Tasks*. Retrieved from Boston College,
- Pather, N., Blyth, P., Chapman, J. A., Dayal, M. R., Flack, N. A., Fogg, Q. A., ... & Lazarus, M. D. (2020). Forced disruption of anatomy education in Australia and New Zealand: An acute response to the Covid-19 pandemic. *Anatomical sciences education*, 13(3), 284-300.

- Pham, S. (2015). *Teachers' perceptions on the use of math manipulatives in elementary classrooms*. Sage.
- Piaget, J. (1971). The theory of stages in cognitive development. In D. R. Green, M. P. Ford, & G. B. Flamer, *Measurement and Piaget*. McGraw-Hill.
- Rakes, C. R., Valentine, J. C., McGatha, M. B., & Ronau, R. N. (2010). Methods of instructional improvement in algebra: A systematic review and meta-analysis. *Review of Educational Research*, 80(3), 372-400.
- Reimer, K., & Moyer, P. S. (2005). Third graders learn about fractions using virtual manipulatives: A classroom study. *Journal of Computers in Mathematics and Science Teaching*, 24(1), 5-25.
- Rittle-Johnson, B., & Schneider, M. (2015). Developing conceptual and procedural knowledge of mathematics. *Oxford handbook of numerical cognition*, 1118-1134.
- Rowley, J., & Slack, F. (2004). Conducting a literature review. *Management research news*.
- Samur, Y. (2012). *Measuring engagement effects of educational games and virtual manipulatives on mathematics* [Doctoral dissertation]. Virginia Tech.
- Stella, M. E., & Fleming, M. R. (2011). Clarity in Mathematics Instruction: The Impact of Teaching Number Sense and Place Value Skills on Elementary School Students. *Online Submission*.
- Taylor, P. C., & Medina, M. (2011). Educational research paradigms: From positivism to pluralism. *College Research Journal*, 1(1), 1-16.
- Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.

Wang, A. Y., & Newlin, M. H. (2001). Online lectures: Benefits for the virtual classroom. *The Journal*, 29(1), 17-18.

Young, D. (2006). *Virtual manipulatives in mathematics education*. Sage.

## APPENDIX

**Sample of Transcribed Interview**

1. Have you taught mathematics using virtual manipulatives?

Yes, I have taught mathematics using virtual manipulatives in the pandemic. In the pandemic, we did not have any other alternative to virtual learning. So, in the time of the pandemic, I used virtual manipulatives to teach various concepts of mathematics.

2. What virtual manipulatives did you use to teach various concepts of mathematics?

Since I teach Optional mathematics, graphical representations come in handy in teaching varied concepts of the subject. While teaching Trigonometry, Coordinates, Transformation, Linear Programming etc, there is extensive use of graphs. So, I have predominantly used Geogebra and Desmos in the Optional maths class. I also used Mathway app in order to find the final answer of any sort of problem. In junior classes, I have used braining camp and Mathspad while I used to go to fill in for the absentees in the substitution classes. Specifically, I have used Algebra Tiles to teach concepts of Algebra like factorization in the smaller grades.

3. What difficulties did you face in using those virtual manipulatives?

In the earlier phase, I found the implementation of virtual manipulatives difficult as it was a new genre for me. I had only taught in the physical setting and going virtual was a new taste for me. I found it difficult to learn and apply virtual manipulatives as it was new for me. But slowly, I came to pace with it. I also found many useful looking apps in the internet which was paid versions which was also another hassle I had to go through. I found Braining camp and Mathway

as apps that were not free. Mathway used to give answers of the problems even in the trial version, however, the steps of the solution could not be seen if the version isn't paid one.

4. What sort of help did you need while using virtual manipulatives? Did you find visual helps in the internet? (YouTube or other)

When I needed any sort of help regarding the virtual manipulatives, I mostly use to go to the internet. I preferred to watch YouTube videos to find out the solution of any sort of difficulties that came across. I also used to consult with friends/colleagues some times in order to clear any sort of confusion.

5. Did you get trained or any other help from the school provided?

I did not get any specific training on the use of virtual manipulatives from the school's side. I researched on my own to find out more about the virtual manipulatives. More often than not, I used to take the advice of colleagues and other mathematics teachers as well which has helped me a lot during the implementation of virtual manipulatives in the class.

6. Did you find the use of virtual manipulatives aid better engagement in the students? Justify your statement.

Comparing the physical class and the virtual class, the students are found to be more engaged in the virtual class as the students are more knowledgeable about gadgets than the teachers. So, students use the gadgets and are interested to do new things with those gadgets. In the physical class, the physical presence of the students is there. However, we cannot say with certainty that their mind is present in the class. But in the contrary, in the virtual class, students are glued to the screens of their devices. So, virtual classes aid better engagement in the students.

7. Did not the use of physical materials reach up to the level of virtual manipulatives while teaching? Please explain your point of view.

In mathematics, even though many materials are found ready-made to apply in the physical class, few materials need to be made and improvised by the teacher itself. As I had to take 6 periods in a day, it was really tiresome to prepare concrete materials on the top of it. And to make material for each class was really a mammoth job. But in the virtual setting, the materials used in one class can also be used for the next one as it is easily reproducible which is why I preferred virtual manipulatives. On the top of that, virtual manipulatives can be used to explain concepts from the figures, step by step by the help of animations which makes the task of the teacher very easy and stress free.

8. As a teacher, using which of the two or both (concrete materials or virtual materials or both) is easier?

After having understood and become swifter with technology, I have found virtual manipulatives to be easier from the perspective of a teacher. From the perspective of time management, I have found virtual manipulatives to save a lot of time. I had to take 6-7 periods a day which is why managing time was of the essence for me. So, I have found virtual manipulatives aiding me in the teaching process. While preparing concrete materials, I used to use up 1-2 hours in preparing one activity for the class which is a drawback of using physical materials.

9. Did you find the students more/less enthusiastic while using virtual manipulatives? Why do you think so?

Yes, I found the students more enthusiastic in the class while using virtual manipulatives because of the immense interest of the students in the gadgets.

10. How did you find the use of virtual manipulatives from the point of your professional development and understanding?

Undoubtedly, I have learnt new things while exploring different virtual manipulatives. The process in itself was beneficial for me as I rediscovered my way of researching for new ideas and concepts. I will share an instance. I realized that  $y = \sin x$  and  $y = \sin^{-1}x$  are reflections of each other on the line  $y=x$ . I used to teach to graph the function  $y = \sin x$  in the school level whereas I used to teach the inverse circular function in +2 where graphing  $y = \sin^{-1}x$  is of the essence. Since I taught both the concepts to different groups, I did not realize the relationship between them. But one day, when I was graphing both the graphs together in Desmos, I suddenly came to realize this relationship between  $y = \sin x$  and  $y = \sin^{-1}x$ .

11. Did the use of virtual manipulatives burden or ease your teaching process? How?

Initially, I was confused while using the virtual manipulatives which burdened my teaching process but as time went on, I got used to the virtual tools and that is why I feel the use of the virtual manipulatives has now made my work easier by lessening my burden.

12. How do virtual manipulatives contribute to meaningful (creativity, problem solving skills, contextualized learning, motivation, conceptual) mathematics teaching and learning?

In the context of Nepal, most of the teachers teach mathematics and expect the students to solve problems. Most of us are not even interested in knowing if the students have understood the concept which is a major drawback of the Nepali system. However, with the advent of time, when I started to use the virtual manipulatives, students started to see how the formulas are derived. For e.g., why

the reflection of any point  $(x, y)$  on the  $x$ -axis is  $(x, -y)$ . The visualization of ideas was instrumental in contributing to creativity in the students.

I had taught the students about the concept of linear programming by solving the inequality by using Desmos. After I taught the student to formulate a real-world problem by using linear programming. One student showed immense creativity thought about the problem meticulously and asked me how can he maximize the production of his father's ice-cream factory. Overnight, the student had already asked his father about what materials they use in making ice-cream in their ice-cream factory and listed two materials i.e., whipped cream and Choco chips as major ingredients. This is how students were motivated and creative in the mathematics class while teaching them by the use of virtual manipulatives.