# CONCEPTUAL AND PROCEDURAL KNOWLEDGE OF STUDENTS IN MATHEMATICS: A MIXED METHOD STUDY

Netra Kumar Manandhar

A Dissertation

Submitted to

School of Education

in Partial Fulfillment of the Requirements for the Degree of

Masters of Education in Mathematics Education

Kathmandu University

Dhulikhel, Nepal

July, 2018

DECLARATION

I hereby declare that this dissertation has not been submitted earlier for the candidature for any other degree.

____________________ October …, 2018

Netra Kumar Manandhar

Degree Candidate

# DEDICATION

This work is profoundly dedicated...

To my father Indra Bdr. Manandhar and mother Indra Maya Manandhar who always instilling me to make the most out of available opportunities.

To my brothers and sister, whose frequent guidance, selfless sacrifices, and continual supports made my master degree journey possible and thank you once again for reminding me what is most important in life.

To my dear students, without your generous support and continuous encouragement this study would not have been possible. Thank you very much for encouraging me to stand tall and shoot for greatness.

To all the readers!!

*Master of Education in Mathematics Education* dissertation of *Netra Kumar Manandhar* entitled *'Conceptual and Procedural Knowledge of Students in Mathematics: A Mixed Method Study'* was presented on October…, 2018.

APPROVED

__________________________ October …, 2018

Asst. Prof. Binod Prasad Pant

Dissertation Supervisor

__________________________ October …, 2018

Mr. Shiva Datta Dawadi

Dissertation Supervisor

________________________ October …, 2018

----------------------------

External Examiner

__________________________ October …, 2018

Prof. Mahesh Nath Parajuli, PhD

Dean/Chair of Research Committee

I understand and agree that my dissertation will become a part of the permanent collection of the Kathmandu University Library. My signature below authorizes the release of my dissertation to any reader upon request for scholarly purposes.

________________________________ October …, 2018

Netra Kumar Manandhar, Degree Candidate

# ABSTRACT

An abstract of the dissertation of *Netra Kumar Manandhar* for the degree of *Master of Education in Mathematics Education* presented at Kathmandu University, School of Education on October ..., 2018.

Title: *Conceptual and Procedural Knowledge of Students in Mathematics: A Mixed Method Study*

Abstract Approved

| | |
|---|---|
| Asst. Prof. Binod Prasad Pant | Mr. Shiva Datta Dawadi |
| Dissertation Supervisor | Dissertation Supervisor |

What is knowledge of mathematics? Is it a process to be skilled, or a concept to be understood deeply or both? This has been an issue among the mathematics teachers, teacher educators as well as researchers who are in/directly related to mathematics and mathematics education. This issue ultimately leads to the debate on procedural or conceptual or both type of knowledge construction in mathematics. In Nepal, we (as a teacher and a students of mathematics) have been facing this issue since very long in teaching and learning mathematics. At the same time, students develop knowledge of algebra in the middle school grades through both ways: procedural knowledge (PK) and conceptual knowledge (CK). As I experienced as a teacher of mathematics, sometimes, students acquire the knowledge of algebra by understanding the underlying concept whereas most of the time through step – by – step procedures in problem-solving. So, based on my experience, I can say that our students have both types of knowledge in this level. However, the question is how much procedural or conceptual knowledge of algebra they acquiring.

This explanatory sequential mixed method study was carried out to figure out the level of conceptual and procedural knowledge in algebra including the relationship between these two variables. Next, to explore why students had such knowledge in the domain of knowledge that was to find the possible reasons behind the situation. The major emphases in this study were students' ability to define concepts in algebra, compare two or more algebraic expression, represent the concept with figures and diagrams, and express verbal problems into mathematical sentences and critical thinking. In the quantitative setting, the survey was conducted among 360 respondents of eight grader sample participants of 9 public schools of Kathmandu Metropolitan City. The first phase interview was taken with six participants using the semi-structured questionnaires to verify the result from the survey using student's test papers. Similarly, the second phase interview was taken with the same six respondents of the first phase and a group discussion was done with four mathematics teachers who teach in the same level.

The survey data analysis revealed that there was a low level of conceptual knowledge (average-8.56 out of 20) in algebra and it was below the average. Students had higher procedural knowledge (average-14.05 out of 20) but there was an average positive correlation ($r = 0.559$, $p < 0.05$) between these two knowledge. The first phase interview analysis disclosed that our students are still weak in reasoning, critical thinking, have less ability to compare two or more algebraic quantities as well as incapable to represent knowledge with diagrams and pictures. The second phase interview and a group discussion with teachers regarding why students had such situation disclosed that there was a poor interest of students in learning algebra. The less use of effective teaching-learning approaches, lack of connecting daily life problems with contextual examples in teaching /learning, less project work, practical

and field work inside as well as outside the classroom were the phenomenal reasons which show that we all are unable to construct conceptual knowledge in comparison to procedural knowledge in algebra.

The major implication of this study is; we need to prioritize the conceptual knowledge construction in mathematics without neglecting procedural knowledge. For this, teachers need to be aware and trained to implement effective teaching and learning approaches, teaching materials, different project and practical works that enrich the conceptual knowledge. Policy makers need to be aware of making policies towards developing CK. The curriculum developers should develop effective mathematics curriculum and plans that nurtures CK and other education related buddies including parents should emphasize conceptual mathematics knowledge development.

October …, 2018

Netra Kumar Manandhar, Degree Candidate

# ACKNOWLEDGEMENTS

My first gratitude and grateful is to the Department of Mathematics Education KU, Hattiban for providing me an opportunity to do this wonderful dissertation on the topic of 'Conceptual and Procedural Knowledge of Students in Mathematics: A Mixed Method Study'.

I would also like to express my sincere gratitude to Asst. Prof. Mr. Binod Prasad Pant who greatly extended his invaluable supports and dealt to encourage me to endeavor my responsibility during the preparation of this dissertation from the beginning to the end. He has become a great source of inspiration and always cooperated in a family environment to bring me to this stage.

My genuine gratefulness and gratitude goes to my dissertation supervisor Mr. Shiva Datta Dawadi, for his insightful, invaluable, continuous directions, suggestions and support to accomplish this dissertation. Without him, this study would not have been possible.

I owe a great debt of gratitude to all the members of the Department of Mathematics, Kathmandu University; Hattiban, Lalitpur for showing keen interest and advising me to complete this research study. Similarly, I would like to extend my thankfulness to my teacher Mr. Indra Mani Shrestha, Mr. Durga Dhakal and other teachers for their remarkable suggestions and persistent help in completing this research.

I am equally indebted to all the head teachers, teachers and students of nine different schools of Kathmandu metropolitan city for their kind cooperation for providing required data for the study. Similarly, I would like to extend my appreciation to my colleagues for providing remarkable suggestions. All the words are again incomplete to my dear students for their invaluable look up, inspiring and encouraging love and support.

At last but not the least I would like to extend my thankfulness and love to my family members and relatives for their incessant support and assistance.

Netra Kumar Manandhar, Degree candidate

# ABBREVIATIONS

| | |
|---|---|
| ANOVA | Analysis of Variance |
| B. Ed. | Bachelor in Education |
| CK | Conceptual Knowledge |
| DEO | District Education Office |
| ERO | Education Review Office |
| I. SC | Intermediate in Science |
| I. Ed. | Intermediate in Education |
| KU | Kathmandu University |
| Ma VI | Madhyamik Vidhyalaya (Secondary School) |
| MOE | Ministry of Education |
| NASA | National Assessment of Students' Achievement |
| NCTM | National Council of Teachers’ of Mathematics |
| PK | Procedural Knowledge |
| PBL | Project Based Learning |
| SD | Standard Deviation |
| SPSS | Statistical Package for Social Science |
| SLC | School Leaving Certificate |
| SEE | Secondary Education Examination |
| TU | Tribhuwan University |
| +2 | 10 + 2 (Intermediate Level) |

TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF FIGURE

# CHAPTER I

# INTRODUCTION

Effective mathematics teaching and learning in all level of education has become a big issue for mathematics teachers and educators in Nepal. There are hundreds of teaching methods invented, discovered to teach/learn meaningful and fruitful mathematics. Teacher-centered to student-centered teaching/ learning strategy, behavioristic approaches to constructivist approaches have been used for the betterment of our teaching and learning system of mathematics.

Algebra is one of the major parts of mathematics in the school level curriculum of Nepal. It is one of the six parts of the school mathematics curriculum of grade eight; Sets, Arithmetic, Algebra, Geometry, Trigonometry and Statistics. The equal emphasis is given to algebra in teaching and learning of mathematics. Algebra has been characterized as the most important “gatekeeper” in mathematics (Cai et al., 2005). In addition, the underlying concepts of algebra can be used in other area/part of mathematics. For example; we use the algebraic equation to represent geometrical figures in geometry (e.g. $x + y = 0$ represents a line passing through the origin). Algebra is another emphasis in middle school level such as grade six, seven and eight. Students should learn the most fundamental concept of algebra in these grades which are applicable to learn mathematics in other higher level grades. So the knowledge construction of algebra in these grades is like a building block for the other higher classes.

In the process of teaching and learning algebra, the major focus goes on developing conceptual and procedural knowledge. Conceptual knowledge of mathematics refers to an understanding of principles, concepts, theories, models etc. and we acquire these knowledge through viewing, experiencing, reading, or thoughtful and reflective mental activities (Rittle-Johnson, & Schneider, 2015) and procedural knowledge is a series of steps, or actions, done to accomplish a goal or solve a problem (Canobi, 2009; Rittle-Johnson, Siegler, & Alibali, 2001). We have been pursuing these two types of knowledge construction together in teaching and learning the knowledge of algebra together but our emphasis will be on a particular type of knowledge without understanding the value of the other. The prioritization in knowledge construction process may be a problem in mathematics teaching-learning system in Nepal because the performance of students in mathematics is getting low day by day.

The issue of conceptual and procedural knowledge of mathematics had become famous in the 80's after the book 'Conceptual and Procedural Knowledge: In the case of mathematics' edited by Hiebert in 1986. The issue of the relationship between conceptual and procedural knowledge of mathematics have become a kind of burning issue in mathematics education. Nepali mathematics education has been influenced by the same issue that is why the work of some renowned writers and researchers of Nepal (e.g. Luitel, 2009) and others on conceptual and procedural knowledge of mathematics are quite appreciable. These educators have claimed that we largely emphasize procedural knowledge rather than conceptual knowledge of mathematics in Nepalese context.

Through this mixed method study, the researcher wants to identify the level of knowledge (PK) and (CK) in algebra with the help of eighth-grader students and tries to figure out the possible reasons behind this situations.

## Background of the Study

A debate or a math war (Klein, 2007) between procedural knowledge/skill and conceptual knowledge is not new in mathematics teaching and learning. The relationship between conceptual and procedural knowledge has been an issue of debate among mathematics researchers in education (Zuya, 2017). And the debate is still ongoing based on which one type of knowledge is more important and which one is less and the debate has become like a chicken and egg question about which type of knowledge comes first. This debate is prominent for those teachers and the educators of mathematics who believe in meaningful and fruitful mathematics teaching-learning. As a novice researcher and a teacher of mathematics, I believe in productive mathematics which helps to put one brick in the holistic development of our students. Here productive mathematics, in my opinion, means such mathematics and mathematical knowledge which is substantially important for a human to live a life. It means to say that after developing such knowledge, one should relate the knowledge to his/her daily practice and those knowledge will lead us to a meaningful life.

However, many researches such as Rittle-Johnson & Siegler (1998), Ross (2010) etc. have done to visualize the actual and representative relationship between procedural and conceptual knowledge so that we could teach better mathematics and students could learn meaningful mathematics. As a teacher of mathematics, most of the time, I heard that for a mathematics knowledge acquisition, the rich concept should be established while teaching and procedures are also important in problem-solving. But the teaching scenario changes when we come to the classroom. We have to accept that we frequently speak to students to memorize formula, steps or procedures to solve the problems in mathematics than encourage our students to be creative, ask questions, think critically, and play with the situation so that they use their highest potentiality to grab knowledge with underlying concept which is rich in connection with deep meaning, a conceptual knowledge (Star, 2005).

There is a mixed conception in knowledge building process. The perceptions of people are different in the selection of PK and CK. The perception is well applied not only in algebra but also generally in all the contents of mathematics. On one hand, a group of people believe that mathematics is a kind of subject that demands more practice and continuous procedural effort into it. On the other hand, a group of people believe in emphasis on teaching and learning conceptual part of mathematics. They suggest emphasizing on conceptual part of mathematics is a key to meaningful mathematics learning. On the other hand, people believe in constructing both types of knowledge that leads to deeper mathematics learning. People get stuck in which to develop to master mathematics knowledge. This can be one reason that we are making mathematics more difficult and abstract and consequently Nepalese students take it as a foreign subject (Luitel, 2009, p. 3).

The more complexity occurs in teaching and learning algebra particularly. Due to its abstract nature of symbols and expressions, more of teaching instructions pursue in constructing procedural knowledge by minimizing the amount of underlying concept. For example; problems solving in indices follow the development of procedural knowledge through memorizing rules, steps and formula of indices. We feel difficult in developing the underlying concept such as the pictorial representation of the basic concept of indices, comparing quantities $1/x$ and $1/(x^2)$ and so on. So, students may have different ability to construct the knowledge of algebra in grade eight. Some are capable of developing PK and some CK. As a teacher of mathematics, we should aware of the value of knowledge and knowledge construction.

## My Journey with PK and CK

### As a Student

The prominent attention of research in learning and instruction is the central role of knowledge (Egodawatte & Stoilescu, 2015). It is generally assumed that the knowledge in the human mind of a person is made up of different components. The most widely discussed components in the field of knowledge are: conceptual and procedural knowledge (Miller & Hudson, 2007).

When I was a student of lower secondary and secondary level I always became first in every grade/class. After the completion of primary level education, I had to go to another school to complete further education so that I joined one of the high schools which was about 2.5 KM far from my home. I used to go to school with my seniors as I was in grade six. The educational practice in this school I found was totally different from my previous school where I completed my primary education. During the class of mathematics, we used to be afraid of the mathematics teacher. There was always pin-drop silence in the classroom during his period. At the beginning of every lesson, he used to encourage us to memorize all the formulae of the lesson and we used to do the same.

He was a very strict teacher that I had ever found in my school level education. He used to give us many problems every day and we all had to do. We were compelled to find the solution of every problem otherwise he used to punish badly with a long stick even if there is a single mistake. It means literally like, one wrong solution to the problem equals to one stick. The condition was horrible. Because of teacher’s rule of giving punishment, we all tried very hard to get the right solution to every problem and memorize all the required formulae. This teacher would teach us in grade 7 and 8 as well. He always used to do one particular problem of a lesson and we had to do others. He often used to emphasize routine problems that were already asked in the previous examinations. He used to also focus on the continuous practice of problem-solving and he used to say, *"Practice makes a man perfect."* After doing one particular problem many times, I was able to learn the meaning of that problem. I got experienced with problem-solving skill with strict rules, formulae and too technical step by step methods without learning the underlying concept of knowledge.

I could learn mathematics at the lower secondary level. According to my experience, I could feel comfortable while solving arithmetic computations like solving the problems of time and work in grade 8, problems of profit and loss, unitary methods, geometry, problems of finding HCF and LCM in grade 8, 9 and 10, problems regarding statistics (finding median, quartiles, standard deviation, mean deviation etc.). The knowledge that I developed in this stage is considered as 'procedural knowledge'. In the words of Rittle –Johnson et al. (2001), "Procedural knowledge is the ability to execute action sequences to solve problems. This type of knowledge is tied to specific problem types and therefore is not widely generalizable (p.1)." This explanation helps us to grab the meaning of procedural knowledge in our own context. To solve too arithmetic computations, algebraic problems (solving equation) etc. we particularly like to use previously learned step-by-step solution procedures of methods to solve the problems (Briars & Suegkerm 1984; Hiebert & Wearne 1996, as cited in Rittle-Johnson et al., 2001, p. 1).

On the other hand, the experience that I grabbed when I was in grade 9 was typically different from my experience as a student of the lower secondary level. This learning helped me to develop the deeper understanding of most of the knowledge of mathematics and made me a critical thinker in problem-solving, searching the reasons and creative aspect of mathematical knowledge. Similarly, it taught me to search the meaning of mathematics in my community practice and my day to day living life. It is because the teacher who taught at secondary level was different from the previous teachers. His teaching style was somewhat different. At the beginning of every lesson he used to connect the core idea of any lesson with our day to day practice such as; *'Have you ever got profit/loss in your daily transaction?*', '*What is the practice of interest in your society? Is it simple interest or compound interest?', 'What are the geometrical objects that you have used in your daily practice?', 'Do you have any knowledge about this and that?', 'When do you use this thing in your daily practice?', 'Do you know the actual meaning of this or that?', 'Tell me about your experience based on this mathematical knowledge'*, he often used to ask such questions and there was a natural and critical discussion between teacher and us. He often used to teach most of the lessons with the help of concrete teaching materials which really helped us to learn the concept of mathematical knowledge.

Similarly, he used to inspire and encourage us to learn the things internally with the help of maximum amount of effective teaching-learning material without rote memorization and used to say a Chinese proverb: If I see, I forget, if I do, I learn and if I practice, I excel. But this is not all the time. When the parts of mathematics and optional mathematics like arithmetic, statistics, algebraic problem like HCF & LCM, solving the equations, geometry; he often used to focus on process as well as conceptual part also. In this way, I learned mathematics at the secondary level. What I feel is; the learning process in this stage was pretty much easy. It is because we got the chances and opportunities to learn a deep understanding of concept at the beginning. I could relate mathematics to my day to day practice immediately, I could learn the core idea of the knowledge immediately, and I could learn those mathematical ideas by imparting them with my previous learning. The knowledge that I developed in this stage is more often considered as conceptual knowledge. Conceptual knowledge is considered an essential component in doing mathematics with understanding (Egodawwatte & Stoilescu, 2015). With this, we previously learn the core idea or concept of a particular problem.

The conceptual knowledge can be acquired by connecting ideas with whatever we have learned before. It is possible because we have a long range of schemas of knowledge, ideas or things. So while generating a new knowledge these schemas are helpful to generate the concept part of knowledge. Because of this component of knowledge, we learn 'why' a particular thing happens in a particular way rather than 'knowing how and what' (Hiebert & LeFevre, 1986).

In addition, let us talk about a teacher's instruction. I had encountered another mathematics teacher in grade 10 (note: this teacher was same in grade 9 to teach optional maths and compulsory maths in grade 10) who taught me to think in a procedural manner.  It was the winter season of January 2008. Our school was resumed after the completion of the mid-terminal examination. As we know that winter is considered as the coldest among the four seasons. People usually wore the warmest clothes; maybe jackets, jeans, and other warm clothes. We try to hide from the cold zone. This was an important time was of 10th standard as we all were about to take a part in the SLC examination. At that time SLC was taken as an 'iron gate' in the entire educational journey of students living in Nepal and also quite similar in these days too. During this particular time of appearing in the SLC examination, students were given enough time by their family members. But it is quite difficult in the rural area of Nepal because the students in the rural area have other own daily responsibility to fulfil on an everyday basis. Well, our condition was also the same.

After the mid-terminal examination, it was our first day to attend the class. I went to the school long holidays, so the first class was really boring. Our mathematics subject was in the fourth period before the break. Our mathematics teacher was so strict who used to give us the maximum number of problems to practice and each individual had to do all if not we would be punished by him. He was cruel and violent too so we had kept his name as ‘Hitler’.

As he used to teach both compulsory and optional mathematics, most of us did not like him. I was the first student in the class, so there was a soft corner for me and used to focus on me without listening to other’s voice. That’s why most of my friends used to ignore him. They used to feel injustice all the time. I still remember one incident which happened when I was in grade nine. In grade nine, we had one more mathematics subject as an optional subject. The same teacher Mr. Rambabu used to teach optional mathematics. Since it was an optional subject, we were 25 students among 150 students who took optional mathematics in grade nine, in the beginning. After the first two or three classes of it, our teacher started to frighten us. He told, *“Optional mathematics is a very complicated subject. You have to memorize around 500 formulae. And one more thing, do not talk about trigonometry. It consists of 300 formulae that you have to memorize.”*

After telling this, he wrote the topic name *‘Trigonometry’* on the board. Then he started writing the formulae of trigonometry below the topic. Like; formulae related to opposite relation to trigonometric ratios and so on. In between 15 minutes he wrote around 40 formulae and told us to copy all. After a while, he told us, *“You have to memorize all these formulae by tomorrow. I want you to memorize these all and tell me tomorrow when I ask you. Okay!”* There was pin –dropped silence in the classroom. We were just looking at each other’s innocent face and did not try to spell a single word. When teacher went out after his the period, our discussion started about whether to study optional mathematics or not. In the very next day, there were just eight students in the class of optional mathematics and observing this condition the same teacher told, *"Without having a capacity to memorize the formulae and rules, how dare are you taking optional mathematics? Nonsense! Fellows."* We too sat quietly without saying anything. Our condition was somehow complicated and facing so many challenges we could finish grade nine. Since the emphasis of his teaching was on procedural knowledge, we had to practice one problem maximum number of times and we had to repeat it many times. In this way practicing lots of time and with the help of rote learning, we could learn mathematical ideas and core ideas of it. But the learning condition was devastating. We had to invest a considerable time in memorization. So this is what the scenario of our mathematics teacher.

Once I was in grade ten and it was our 4th period of the day. Our mathematics teacher Mr. Rambabu entered the classroom and we greeted him by saying, *"Good afternoon! Sir."* And he too did the same. We got permission to stay back to our own seats. He then started saying, *"Before the mid-terminal examination, we had completed all the lessons besides geometry. So today we are going to study geometry."* He then wrote 'Geometry' at the top of the board. Immediately I stood up and said, *"Sir we do not know more about geometry. It is because of less time, we did not study the geometry lesson last year."* And, *"Well, do not worry my boy you will learn this year. We have two types of proof of the theorem, one is experimental proof and another is theoretical proof. Today we are going to learn how to prove the theorem by theoretically. Are you ready?"* He asked. *'Yes sir'*, we replied. Under the topic named 'Geometry', he wrote 'steps of proving the theoretical proof of theorem'.

At the same time, he was suggesting us to copy everything whatever he had written on the board. Then he wrote the following things and told us to copy them all:

Step 1: Given: - write all the information of the figure of the theorem like parallelogram ABCD or ∆ABC and other things according to figure.

Step 2: To prove: - Write a mathematical sentence which we are going to prove. Such as: Parm. ABPQ = 2 ∆ABC

Step 3: Construction: - This is optional case because it is not necessary to write in all the proof of the theorems. In this step, draw some valid geometrical shapes/line which helps us prove the theorem.

Step 4: Proof: - This is the final step of proving the theorem. In this part, we draw a table which has two columns. In the first column we write the statements and in another column, we write the reasons for those statements. In this way, we can prove any theoretical proof of the theorem. Whatever he wrote on the board, we copied. After a while, he did theorem – 1: 'The area of parallelograms are equal standing on the same base and between the same parallel lines' by using the aforementioned steps. We extracted this too. He forced us to memorize the steps and repeat the same process of proving the theorems again and again then only we can memorize. *"If you repeat the same proof of the theorem, it will perpetually be in your mind otherwise you will forget it he added".* One of our friends Ratan stood up and asked a question to sir, *"Sir, why do we memorize these all? Don't we have another method to learn the theorems?"* Sir then replied, *"Yes, we have another method too. But this method is fruitful to you. I have been teaching geometry for 10 years with the help of this method. You do not have to think about understanding. Questions will be asked and you have to write accordingly whatever I have just written. Otherwise, you will not get full marks. So the steps are necessary. You have to write as it is. Did you* *understand?"* In which sense could we say 'yes'? But unknowingly we said, *"Yes sir."* After we finish copying, sir told us to prove the same theorem -1 by changing the name of the figure. At first, some of us feel difficult, but, we write the same steps and words but we replace the name of the figures instead. We almost did the same. The bell was about to ring. Our teacher said to us, *"So, today you learned how to prove theorems theoretically. Well, tomorrow you all memorize the steps and do the theorem -1, four more times by changing the name. Okay?"* And *"Yes sir*", we replied. In this way, I practice many times and repeat the same process. At first I feel somehow difficult to learn theorem by rote memorization, using pre-determined steps, without having the concept but later on, I could do and learn the theorems. Maybe we are emphasizing procedural types of mathematical knowledge because of teacher's instruction.

These experiences explain how I develop conceptual and procedural knowledge of mathematics at the secondary level. Similarly, the experiences that I had at my intermediate and bachelor level were similar. After completing my SLC, I went to headquarter to complete my intermediate level of education. Since then, my keen interest was there in learning mathematics. I could not join I. SC. due to my poor family financial condition so joined I. Ed. by taking mathematics as a major subject. This campus/college is the central college to all the students. Most of the students who wanted to get a higher education used to come to this college. The condition of students who studied mathematics education was quite disastrous. Among around 400 enrollments in that college, we were only 10 students who studied mathematics education and the condition was still the same. In addition, the teaching and learning process was miserable. The teacher who used to teach us mathematics could complete only four lessons out of 12 lessons in grade 11. Most of the time, he used to be out of the college or town. During college time he used to teach another private school too and that is why he used to come to college very lately. Generally, he used to solve one or more problem of a particular lesson and force us to do all. He used to say, *"Now, teacher is just a helper at this stage. You have to learn in your own way. Buy the guesses and guides of mathematics and practice them frequently. First you copy as it is from the guide and then do it five or more times, then you learn how to do it. So, do not be dependent on me."*

He then used to mark the questions that are already been asked in the previous examination and say*, "These are the important questions for your examination. What you all have to do is, find the solutions to these questions by going through the guide."* In this scenario, I used to look up the guide of mathematics all the time and try to copy as it is. After doing the same problems so many times, I could somehow learn mathematics. In this way, I completed my intermediate level and came to Kathmandu to complete my bachelor level in mathematics. I came here to fulfill my desire to complete my B. Ed. in mathematics education that is why I joined one of the renowned education colleges in Kathmandu, one of the education campuses which was a leading campus under TU in Kathmandu. In this level, altogether we had to study six pure mathematics courses. All the courses were taught by different teachers of mathematics but the way of their instructions was somehow similar. In mathematics subjects like algebra, analysis, calculus, number theory etc. My teachers used to teach through too technical and procedural structure. There was no place for students' voice. Whatever they did was correct and we were forced to copy them. We used to copy all the matter whatever they wrote on the board and we used to memorize them all to pass the examinations. There were no genuine and deep understanding of concepts.

When I completed my bachelor level, I thought about my knowledge of mathematics. Was it a meaningful study? Or was it a qualitative study? I was good at the theoretical part and happy to have so much quantity of mathematical knowledge but what is about quality mathematics then? I thought many times and decided to change the university that is why I came to Kathmandu University (KU) to complete my graduation in the same stream of mathematics education. After joining this university, I realize what our mathematics should be like. In the first semester, I had the ability to teach mathematics through various approaches. I got so many opportunities to learn how to make mathematics meaningful and fruitful using various teaching and learning strategies like ICT in mathematics education, collaborative, co-operative, problem-solving strategies and approaches in mathematics. Different types of learning theories and their practical applications in teaching and learning made mathematics classes more interesting. I got the opportunity to make our own curriculum of mathematics. In the third semester of this university, the first chapter conceptual and procedural knowledge of the mathematics of ‘Recent Paradigm in teaching and learning mathematics’ encouraged me to carry out this study based on procedural and conceptual knowledge of mathematics.

**As a Teacher of Mathematics**

It could be the day of April 2010, as a student of mathematics education at bachelor level, we had to attend one and half months teaching practice. Teaching Practice for B. Ed. level students is compulsory with the expectation to learn practical teaching skills to become a teacher. In this scenario, I got an opportunity to teach the students of class 10 in a government school of Kathmandu valley and the name of the school was, Gautam Ma Vi. Even though I was not a practitioner teacher, I always had the enthusiasm to use different approaches to make mathematics colorful, learnable and fruitful to the students. It was my first day of teaching practice. After talking with my subject teacher and with school administration's permission I was heading to class 10 with some teaching material. Students of class 10 were eager for my arrival. As I entered the classroom, all the students greeted me by saying *'good morning!' sir*. We exchanged our greeting and let them to sit in a comfortable way. We introduced to each other and finally begun my very first class by saying, *"Dear students, which topic would you like to learn today?"* With some pause, one student Raj stood up and said, *"Sir our subject teacher told us to request you to teach 'co-ordinate geometry' because we just started this lesson yesterday….."* Without the completion of Raj's speech another girl named Prapti stood up and said, *"But sir I could not understand the matter properly."* Then I replied to her, *"It is okay, you will learn anyway."* Even though that was my first class, many thoughts were running inside my mind like how do they like optional maths? Is it really difficult as most of the students think? So, without wasting my time I asked, *"What do you think about optional mathematics? Do you really feel easy or do you feel complicated? Do you enjoy mathematics?"* The class became so much noisy because of my question. I requested them not to make noise and immediately one student said with monotonous voice, *"Sir! It is too much complicated and difficult. We were 15 students in grade 9 but now we are just seven."* Then I said, *"How did it happen? And what is the reason behind it?"* The first boy of the class stood up and said, *"Sir, because we do not understand most of topic and its concepts. It is more complicated because we need to memorize many formulae around 200 and 300. Our teacher always emphasizes on memorizing the formulae and steps to solve the problems."* Another student stood up and said, *"Sir this is so disastrous. I am now frustrated because of this subject. What is the meaning of such subject which does not have meaning in our daily life? I did* *not see the practical meaning of it. Why should we memorize so many formulas? Why is there only one method to accomplish the solution to the problem, why not others?"*

I was surprised by their threatening responses. It did not take much time to understand that it is the practical and common problem which is faced by students since a long time and made mathematics so much complicated. The condition was so much distressing but I tried to reply with soft voice, *"Well, we do have problems. And to solve it we need considerable time and determination too. In this one and half month teaching practice I try best to teach in such a way that you all fill mathematics meaningful and useful in your daily life. I assure you that till the end of our session you all will be able to learn many things."* I could see some happiness on their face. The problem was about creating conceptual knowledge of mathematics. In this session, I determined to create favorable environment so that student can learn meaningful mathematics and apply it to their day to day life. From the very first day, I started putting a brick to build a realistic house of conceptual knowledge of mathematics.

Then I said to them, *"Well students, today, we are going to learn about how to calculate the point of intersection of two linear equations?"* Then they replied, *"Okay sir!"* Then I started asking them questions like what is the linear equation? The first student of the class stood up and said, *"Sir, the first degree algebraic equation is called linear equation."* Then I asked, *"What does it represent?"* They got stuck. They were unanswered by seeing each other's face. I was amazed and without any delay I replied, *"It represents a straight line."* Then I constructed one straight line on the board to show it and asked them to find out any example of straight line which is just constructed on the board. Again Raj stood up and said, *"Sir it can be 2x + 3y = 0, is not it sir?"* Then I immediately replied to him, *"Yes! Raj you are right. Straight* *lines always are in the standard for ax + by = c, where a, b and c are constant. It means, the name of straight line that I constructed on the board might be 2x + 3y = 0, x = 0, y = 0, 4x – 10y =-5 or others as well."* Then I asked them, *"Did you understand my dear students?"* They replied *'yes sir'* with loud voice.

Next, I asked a question about what they have understood by the term point of intersection by drawing two intersected straight lines and asked, *"Do you have any knowledge about the point of intersection of two straight lines?"* Another girl stood up and said, *"Sir, is it a point intersected by two lines? But I do not say anything about it and just tried to explain the meaning of the word 'point of intersection."* I replied, *"You are great Ritu, because at least you are able to find out the meaning of the words."* All students were laughing at Ritu. I told them to calm down and said, *"Yes! Ritu is correct in a sense point of intersection means a particular point which is common to both intersecting straight lines. Well now I give you two equations of straight lines: x - y = 4 and x + y = 6. Now, solve these and find the value of x."* One student was shouting to others and said *'it is easy for us'*. Within 5 minutes, they were done and showed their answer. And I asked them, *"Dear students, which method do you apply to calculate the value of x and y*?" Rajib stood up and said, *"Sir, I applied elimination method (hataune bidhi* in his own words*) to find the values."* I added, *"Well! You have done a great job. Now I must tell you that whatever the value of x and y you found that is the point of intersection of the straight lines that I had drawn on the board."* All the students were in amazement with a similar kind of voice *'Huh!!'*. Immediately the first student started telling us, *"Sir! If the co-ordinate with x –value and y –value is the point of intersection, then we know the various method of solving the two linear equations."* I replied, *"Yes! Mahesh you are right, the co-ordinate with x –value and y –value always represents the point of intersection of the* *two given straight lines. Now you guys tell me, which are the methods that they have already learnt to find the value of x and y from the two given linear equations?"* Mahesh replied, *"Sir! By using graphical method, substitution method we can also solve the equations."* I replied, *"Yeah you are right!"* Hrithik who used to speak very less in the class replied with soft voice, *"Sir, we learned to find the x –value and y – value through matrix method, can we use this method to find out the point of intersection?"* Immediately I replied, *"Yes! Definitely you can Hritik. You can even use matrix method. And there is one more method that you have already learned and that is cross multiplication method."* Arati replied, *"It means that we have various methods to solve two linear equations. Which one is most appropriate sir?"* I liked her appropriate question because most of the students want to apply suitable method and replied to her, *"You are correct, we have various methods and you can apply or use any method that you find appropriate. So, did you understand all? There are still some methods which you will learn in the further classes."* Students replied with loud voice, *"Yes sir."* The bell was about to ring, the first student stood up and said, *"Sir! Honestly, I was unaware of such thing. I knew about such methods but did not know how it can be applied to solve other problems of mathematics. So, today's class is very fruitful to us. And thank you very much sir."* Other students agreed with the first student. I was happy by seeing this and my first effort to build a particular concept was really good indeed. Eventually, I gave them 4 or 5 sets of problems regarding to find out the point of intersection of two straight lines and headed towards the office. The experience that I had represented is about conceptual knowledge of mathematics. If a learner constructs a conceptual knowledge of mathematics, he/she can understand about it and he/she has a multiple ways to learn it and solve it. We have multiple methods to do a task or solve a problem.

## Problem Statement

According to my experiences, as a teacher and a student, the process of knowledge construction in mathematics, was more of procedural knowledge in comparison to conceptual knowledge. It may happen because of the way I was taught, the environment I was grown up and the instruction or strategies of my teachers of mathematics which were more emphasizing procedural knowledge by minimizing the amount of conceptual knowledge. Theoretically, it is said that both knowledge are important in mathematics. Researches such as Ross (2010) and Ghazali & Zakaria (2011) etc. have shown the positive relationship between these two knowledge. Further, these researchers emphasize on the construction of both knowledge together rather independently. However, the importance has been given to the construction of a greater amount of conceptual knowledge in comparison to procedural knowledge (Rittle-Johnson et al., 2001). Similarly, in Nepal, we have been talking about constructing both PK and CK in mathematics.

Let us assess the past results of SLC/SEE and the achievement in mathematics subject of the students at the secondary level as well as the status of the students who got success in the past few years, the achievement level has been decreased gradually. According to the status result of MOE published in 2012, the percentage or the regular passed students in mathematics were 52.71% and 47.229% were failed. Similarly, in 2013 the percentage of passed students decreased by approximately 6% and it was 47.1%; that means 52.90% students failed in mathematics that year. In 2014, the percentage of failed students in mathematics increased and it became 65%. Similarly, in 2015, about70% of students failed, the percentage of failed students increased by approximately 5% in comparison to the previous year (MOE, 2012, 2013, 2014, & 2015). SEE result of 2016 and 2017 had been published in the GPA system. In these two consecutive years, the performance in mathematics of students does not seem good. The data provided by the Office of Controller of Examination (years 2013/14/15) has revealed that the achievement of students of grade 10 in mathematics is decreasing. In the year 2013, around 39% of students had succeeded in mathematics and in 2014, it was 36.5% and 35.31% all over Nepal (MOE, 2013, 2014, & 2015). If we look at our previous history of achievement of students in SLC and SEE particularly of mathematics subject, we can visualize our status of mathematics education system in the school level and consequently it has still been devastating.

Next, the middle grades such as six, seven and eight are taken as a backbone of higher grades in school level. On the other hand, the concepts in algebra are useful in constructing knowledge of other aspects of mathematics (Welder, 2006) such as geometry, arithmetic, trigonometric etc. I am confident that students who are weak in algebra definitely be weak in other aspects of mathematics. This area includes mathematical knowledge through abstract symbols and expressions. However, one of the largest assessments done by NASA in 2013 under education review office (ERO), MOE Nepal among eighth-grader students claimed that our students are weak in reasoning, critical thinking and they are weak in making figures and shapes. Among the achievement ranges from 28 to 38 percentage in the test of mathematics, students scored only 28% in algebra which was significantly lower than the national mean 35% (NASA, 2015). Similarly, students had scored 34% in geometry, 37% in sets, 38% in arithmetic and statistics. This scenario clarifies that our students are weak in knowledge of algebra in comparison to other parts of mathematics (NASA, 2015). At this stage, what is the genuine problem? Is it a problem of emphasizing more on procedures to solve the problem without learning the underlying concept in algebra? Why our children are not enjoying learning concept in algebra properly?

On the other hand, NASA assessment in 2013 found that students of grade 8 in mathematics have low ability to solve complex problems (Higher ability) and that is only 25% of the maximum score in higher ability related questions (NASA, 2015). Students' performance found better in lower cognitive skills but poor in higher cognitive skills such as analysis, evaluation, applying and creating the gained knowledge and skills in a new situation. Students were much better in recalling types of questions, numerically 46% (NASA, 2015). These phenomena clearly show that our students are forced to generate knowledge through memorization, they are evoked to recall and remember the procedures and algorithms in algebra without understanding the representative concept.

There has always been a question regarding instruction of teacher in algebra. As a practitioner-researcher, a students and a teacher of mathematics, most of the time I used to start teaching my students how to factorize the expression without teaching what is factorization, how we can use in your contextual setting; most of us teach indices by writing different rules of indices such as multiplication rule of indices, division rule of indices, and so on. Have we ever used the pictorial form or other manipulative to teach these concepts? Have we ever presented the example like the use of $5^x$ in their daily life? Next, the problem is with the comparison of two algebraic expressions. We should admit the fact that generally, students get stuck in comparing ½ and ¼. It is even complex for them to compare 1/x and $1/(x^2)$. Let us try to ask this question with our students. I assure you that your students probably are unable to compare. Have we ever taught our students to compare two or more algebraic expressions? Another fact is; we teach our students to solve a linear equation like 1/(x

$-5) = 1$ where $x \neq 5$. As a student in grades eight, nine and ten. I frequently used to ask the question about why there is $x \neq 5$ in the question. But my all teachers had the same reaction and said that it is not important to learn. More important is to learn how to find the value of x to get marks. They thought that I was raising the stupid question. Have you ever faced such situations in your academic journey as a teacher or as a student of mathematics? I have faced many times. If there is no reason to learn those concepts, then why do we learn?

If we observe the situation of our students, there is mathematical anxiety in students while learning mathematics. Mathematical anxiety, in the words of Richardson and Suinn (1972), means, “Feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations” (p. 551). Now, coming to algebra, students have much more anxiety while learning. As a teacher of mathematics, have you ever used project based approach of teaching? Have you ever used projects, field work and practical work kind of things in your teaching and learning algebra? There is always questions from students' side such as “what is the use of such mathematical knowledge, sir?” “What will be the condition if I am not good at mathematics right now?”, "If there is no proper connection between the ideas we derive of mathematics and our life then what is the actual use of studying it?”, “We have done so many problems based on profit and loss, simple and compound interest, time and work, algebra, trigonometry and so on mathematical knowledge every day but we could not do the real-world problems related to these concepts?”, “Should I be mastered in computational fluency or conceptual part of mathematical knowledge?”, “Teacher always insisted that mathematics is an inseparable part of human life and it is everywhere, if it is so, why should we be taught so called sophisticated mathematics which has no relation to our real life?" There are other various questions which can be raised by students every single day in learning mathematics. After all, having no meaning of it, they tend to decrease their interest of learning mathematics; that is why there is mathematical anxiety within students.

There are several factors which embody the mathematical anxiety. The major one might be teacher's instruction which is more of strategic. When teacher's instruction is only on developing the knowledge on the sophisticated routine problem by neglecting the core ideas of it, students may feel isolation of not being able to connect mathematics with day to day practice.

Most of the students claim that mathematics is a subject which can be learned only by those students who have a big memory container of mathematics (Luitel, 2013). This conception gives rise to a culturally de-decontextualized mathematics. Luitel (2013) has further discussed mathematics as a pure body of knowledge which can be strengthen through mathematical tasks, muted symbols and mechanical procedures. By feeling such things students get tensed and stressed while studying mathematics so that they always be the victim of mathematical anxiety.

While summing up, the major problems found were the growing disinterest of students in learning mathematics, students' decreasing level of performance in SLC/SEE, Students' lower cognitive level in algebra in comparison to other aspects of mathematics, mathematical anxiety in students. The knowledge construction process through PK and CK in algebra is taken as influential factors in these situations. The researcher has a strong belief in these phenomena. Therefore, this research study is carried out to measure the level of conceptual and procedural knowledge of students of grade eight. Next, this study reveals with the major quality findings about what are the tentative reasons of being such situations in algebra teaching of grade eight.

## Purpose of the Study

Based on my experience, in general, we are persuaded to develop both procedural and conceptual knowledge in grade 8 and particularly in algebra. Thus, this study was done on the basis of the following purposes:

1. To identify the level of students' procedural and conceptual knowledge in algebra.
2. To explore why students develop conceptual knowledge (CK) or/and procedural knowledge (PK).

## Research Questions

To address the aforementioned purposes of this study, I have constructed two research questions and they are as follows:

1. What is the level of eight grader students' conceptual and procedural knowledge of algebra?
2. Why do students develop conceptual or/and procedural knowledge?

### Statistical Hypotheses

The following eight hypotheses have been formulated:

Hypothesis 1: There is no relation between procedural and conceptual knowledge of students in algebra.

Hypothesis 2: There is no significant difference between the mean marks of students in procedural knowledge in algebra based on their gender.

Hypothesis 3: There is no significant difference between the mean marks of students in Conceptual knowledge in algebra based on their gender.

Hypothesis 4: There is no significant difference among the mean marks of students in procedural knowledge based on their father's education.

Hypothesis 5: There is no significant difference among the mean marks of students in conceptual knowledge based on their father's education.

Hypothesis 6: There is no significant difference among the mean marks of students in procedural knowledge based on their mother's education.

Hypothesis 7: There is no significant difference among the mean marks of students in conceptual knowledge based on their mother's education.

Hypothesis 8: There is no significant difference among the mean marks of students in PK and CK based on their parents' occupation.

## Significance of the Study

Researches or studies are very much meaningful and significant for everyone. In a broad manner, research or study postures a present phenomenon/situation of any subject area. Researches are useful and valuable in an academic sector also. It helps people to understand the present condition/issues/problems in education and provides authentic solutions to sort out those issues. Policymakers can take benefit from research and it helps them make different realistic plans such as; plans regarding curriculum development, teacher development, de/contextualization of the content or matter etc.

This study is a kind of comparative study on conceptual and procedural knowledge of mathematics which discloses the true condition of these knowledge of algebra in grade eight. These two types of knowledge are the issues in teaching and learning mathematics of Nepal. A group of people believe and assert that procedural knowledge has to be constructed rather than conceptual part of it whereas the next group of people are exactly against the first view. And there are some others who believe in constructing both type of knowledge.

The findings and exploration may be helpful to resolve the problems of prioritizing knowledge construction of mathematics in school level. Findings may be useful for both private and community schools in teaching and learning mathematics even though the research was done only in the public schools. It may be helpful for policymakers to create and implement a reliable, viable and valuable plan to promote real and in-depth understanding of the knowledge of algebra. Similarly, it may be useful for curriculum developers to make a dynamic curriculum to get the highest outcomes. This study may assist school mathematics teachers to change/improve the way they teach and perceive mathematics. On the other hand, this study may be valuable for those people, teachers, educators who want to see the significant changes in mathematics and mathematics education of Nepal.

## Chapter Summary

This is the beginning chapter of this study. This chapter had entangled the introduction of this research study, the background of the research study, as a researcher; my journey with PK and CK – how I encountered with these ideas, problem statement of the study, purposes and research questions, the hypotheses for the study and significance of the study. I have found the main difficulties in problem statement which were; increasing disinterest of students towards learning mathematics, decreasing performance in SLC/SEE, below the average level of performance in algebra and mathematical anxiety. On the other hand, there were two purposes to conduct this study; one was to identify the level of PK and CK of students and the other was to figure out the possible reason behind the situation. Similarly, two research questions and eight hypothesis were formulated. The main significance of this study was to resolve the belief system of people's prioritization of knowledge construction.

# CHAPTER II

# LITERATURE REVIEW

In this section, the researcher tries to conceptualize the significant part of this study and related terms with the help of related literatures. Similarly, researcher tries to present the core ideas and concept of this research study, findings of different available researches, journals regarding procedural and conceptual knowledge of mathematics. Moreover, researcher tries to envisage about how these two types of knowledge are developed through different available resources. This section has been divided as thematic, theoretical and empirical review.

## Thematic Review

This section constitutes the major theme/themes of this research study. The very first research question of this study wants to scrutinize the level of mathematics knowledge based on procedural and conceptual knowledge of the students of grade eight in algebra. On the other hand, the second research question asks us to explore about how and why such knowledge are/is generated in algebra. Consequently the major themes of this study are procedural and conceptual knowledge of mathematics. Let us first understand what conceptual and procedural knowledge of mathematics are all about.

### Conceptual Knowledge of Mathematics

The word 'concept' according to oxford dictionary (2005), "An idea or a principle that is connected with something (p. 313)." Concept is an idea which is generated through connections. To learn a concept of a particular idea we need to have ability to connect with other ideas or something. For instance, to learn a concept of volume of a concrete solid object (maybe cylinder, prism, cone etc.) one should connect this idea with weight, mass, cubic measure etc. The knowledge based on different connections or concepts is often called conceptual knowledge. In the words of Hiebert & Lefevre (1986), “A knowledge that is rich in relationship. It can be thought as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information which linked to some network (p. 3-4).” Creating concept of idea is to connect that indication with other information. For instance, to learn what ‘interest’ is in mathematics, we connect this information with other information such as its definition, its practical uses in our day to day life, and we try to search the relationship of interest with principle, amount, interest rate, time etc.

Therefore, there is some kind of network of knowledge while establishing it. Conceptual knowledge of mathematics is generated through pre-requisite concepts, ideas and information. In the process of learning, we behold a range of pre-requisite information and ideas. When we look into this world after our birth we gradually develop mental pictures or schemas of something by interacting with environment, with different people, and so on. These schemas are very much powerful to create other concepts. Establishing conceptual knowledge of mathematic means to learn about ‘why’ it happens in a particular way (Hiebert & LeFevre, 1986). It gives multiple perspectives to think about a particular problem. It means a learner has different methods to tackle with the problem of mathematics.

Once we develop conceptual knowledge of something, then certainly we try to figure out its meaning, why it is important in my life, its pros and cons, and solely its generalization to other fields as well. Having conceptual knowledge means not only having known about definition, formulae and procedures but also are being able to justify it (Zoya, 2017). So having conceptual knowledge about something means one come up with reasonable answer to it.

**Procedural Knowledge of Mathematics**

According to oxford dictionary (2005), procedure means ‘a way of doing something, especially the usual and correct way’. Procedure or process is a systematic way through which something has to be done. If we see our day to day practice, naturally, we are trying to pursue a systematic way to live our life so that we can say human life is a kind of routine system, I guess. From the beginning to the end, when we go to bed, in general, most of the people try to make a schedule and follow accordingly. We need everything in a proper order. Mostly what we think is, to get success or to accomplish a goal we need to pursue a particular way which should be planned in advance. If I talk about acquiring or establishing knowledge, knowledge which is generated through a systematic way or procedure is called ‘procedural knowledge’ that is knowledge through process (Hiebert & LeFevre, 1986).

In the process of establishing knowledge in our mind, we follow a strict and a particular predetermined rule, process to get predetermined answer is known as procedural knowledge. If one tries to grab or behold any idea of mathematics through rote memorization, by using step by step method to get the answer or let us say by using too algorithmic problem-solving method to solve the problems of mathematics then he/she tries to develop procedural knowledge. Such knowledge of mathematics sounds like a tool-box that includes facts, skills, procedures, algorithms or method (Barr, 2003). In fact, we try to pursue too mechanical process, steps to get the right answer. We are bounded to follow the process to accomplish the solution to a particular problem rather than getting into its conceptual part. The whole process is done through rote memorization. If one important step is done wrong, then the right solution to the problem might be impossible. So, to get the right solution to the problem, every step should be memorized and done properly and systematically. There is a group of people who believe in procedural knowledge of mathematics. They think that mathematics knowledge can be acquired through mechanical algorithmic way too if we try it for maximum times where repetition is must. If we do the same problem repeatedly at times, we will reach in a certain condition where core idea of that problem can be acquired.

Procedural knowledge is ‘knowing how or knowing what’, or the knowledge of the steps required to attain the various goal. In the words of Byrens and Wasik (1991), “Procedures have been characterized using such construct as skills, strategies, productions, and interiorized actions" (p. 777, as cited in Rittle-Johnson & Schneider, n.d., p. 7). If we look into this view, we should have a particular skill to solve a particular mathematical problem. That means, the procedural knowledge cannot be widely generalized (Rittle-Johnson, Siegler, & Alibali, 2001). Rittle- Johnson and Schneider (2015) have emphasized the following key point regarding procedural knowledge; (1) algorithms – a predetermined sequence of actions that lead to the correct answer when executed correctly, (2) possible action sequences that must be sequenced appropriately to get the solution to a given problem. So, procedural knowledge in mathematics is a knowledge that can be constructed through stated procedures to get a right answer. If one step is executed wrongly, the answer will not be possible.

**Relationship between PK and CK**

There are numerous research studies to figure out the relationship between these two types of knowledge. As I stated before, the issue of giving emphasis on which one type of knowledge development is not new for us. Probably, as a teacher of mathematics, we have been coming across this issue all the time. If we see the existent context of Nepali mathematics education and its teaching and learning system, perhaps we teachers are somewhat compelled to focus on a single procedural or conceptual knowledge. It is due to our national objectives that focus more on getting better final outcomes rather than neglecting what students or learners learn in the process. We have more content based mathematics curriculums at every level of school education that force us to learn or at least know about more and more contents of mathematics.

Another important factor might be people's perception towards mathematics and mathematics education because most of us believe in a quantity of mathematical knowledge and skills. We have a lot of experiences of getting the concept of mathematics through procedures. It is because, as we know that, our teaching instruction is widely focused on procedural knowledge. Most of the teachers of mathematics have been pursuing one size fit all or pipe instruction (Luitel, n. d.) approach as a core instruction of mathematics. In the name of the requirement for the future and future generation we try to cover all the concept of mathematics in a span of time. As I started off with the discussion about the relationship between these two fundamental mathematical knowledge PK and CK, there are some available recent literature that help us behold how these two knowledge have been used.

## Empirical Review

This section constitutes the empirical part of this research study through available literatures. Moreover, researcher tries to figure out the research gap in this section. As I previously mentioned, the debate between PK and CK is not new for us. It is considered that this debate or issue became famous probably since 1986 after the book published by Hiebert (1986) on 'Procedural and Conceptual knowledge: In the

Case of Mathematics'. However, Scheffler (1965) and others identified these two kinds of knowledge PK and CK. Similarly, the equivalent terms relational and instrumental understanding were used by Skemp in 1976 (Long, 2005). After this numerous research studies have been done regarding these two fundamental knowledge. There are some research studies that claim in the development of one type of knowledge particularly and in contrary, there are other research studies have been done that assert in the construction of both type of knowledge iteratively. Different researchers and theories have different prediction on the inter-relationship between conceptual knowledge and procedural knowledge. One major difference is described by Rittle Johnson, Siegler and Alibali (2001) who have differentiated among the theories of knowledge construction based on the concept – first view, procedure –first view, inactive view and iterative process.

According to *concept-first view*, children will initially acquire conceptual knowledge, for example by listening to verbal explanation and practicing as well as derive procedural knowledge from it (Schneider & Stern, n.d, p. 195). It means children first learn concept of the knowledge either through parents or by natural setting and then procedural knowledge is achieved through constant practice. According to *process- first view*, it asserts that children first learn procedural knowledge through own interest and then later build conceptual knowledge of it (Karmiloff-Smith, 1992, as cited in Zoya, 2017, p. 3). It means, process-first theory, on the contrary of concept-first, asserts that children first acquire procedural knowledge in a specific domain of knowledge, for instance by trial and error process, and then gradually develop abstract conceptual knowledge from it by reflection. And the other theory of generating or establishing knowledge is iterative process. Similarly, according to inactive view, child develops conceptual and procedural knowledge independently (Haapasalo & Kadijevich, 2000 as cited in Rittle –Johnson & Schneider, n.d.).

According to iterative perspective of knowledge, the relationship between conceptual and procedural knowledge is bidirectional (Rittle-Johnson, Siegler, & Alibali, 2001). According to this view, children build one kind of knowledge with the continuous help of the other. That means the positive change in one kind of knowledge supports the similar change in another kind of knowledge. So, they both go hand in hand that is why the process is bidirectional.

A research study done in 2001 by Rittle-Johnson, Siegler and Alibali on developing conceptual understanding and procedural skill in mathematics through two experiment conducted with fifth and sixth grade students about learning decimal fraction has concluded that the construction of knowledge should be based on iterative process. That means one type of knowledge leads to a positive development of the other. After the accomplishment of this study they have presented a model of iterative theory of knowledge construction which is given below:

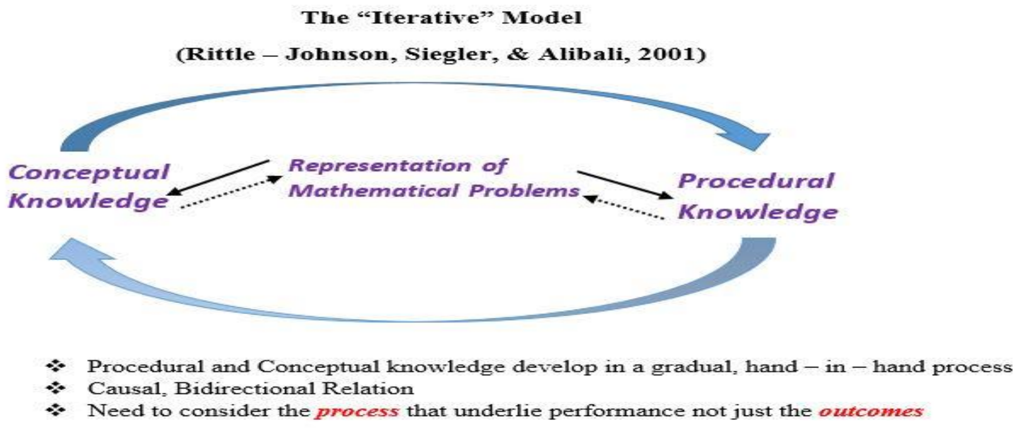



Fig. *1*: Iterative Model of PK and CK

As a practitioner-researcher and a teacher as well as student of mathematics, I do believe in this theory of knowledge construction. Based on my experience, a learner who can learn mathematics using both processes is more intelligent in mathematics than a learner who learns mathematics through an individual process. It is because Conceptual and procedural knowledge are interrelated (Baki & Kartal, 2004; Hiebert & Lefevre, 1986).

Zaini (2005) conducted a study to investigate conceptual knowledge on the topic of fraction among trainee teachers in colleges. The result showed that the level of conceptual knowledge of trainee teachers is moderate (Zakaria, Yaakob, Maat, & adnan, 2010).

Abd Rahman (2006) revealed that the students’ conceptual knowledge of algebra is extremely low. Chappell and Killpatrick (2003) stated that teaching based on the concept has a better effect in improving understanding of students without sacrificing procedural skills.

Another research study done by Ross (2010) in his doctorate dissertation among the $8^{th}$ graders students to measure their conceptual and procedural knowledge in algebra found that there is strong positive correlation between these knowledge.

A research done by Ghazali & Zakaria (2011) that was carried out among 132 secondary level school students to compare students' procedural and conceptual understanding in algebra found the high level of procedural understanding but a low level of conceptual understanding in algebra. The survey research study shows the comparative judgement about students' procedural and conceptual knowledge in algebra. Moreover, as in this study, students have a low level of conceptual knowledge comparing procedural knowledge in algebra. This research also revealed that the relationship between students' procedural and conceptual knowledge in algebra has the average positive correlation ($r = 0.512$, $p < 0.05$) between students' procedural and conceptual knowledge in algebra.

## Theoretical Review

This section consists of theoretical perspectives regarding this study. Researcher tries to present particular kind of theoretical perception based on procedural and conceptual knowledge of mathematics with the help of related literatures. There are different theories arise which help develop procedural and conceptual knowledge of mathematics.

Theory/theories will help us to reach our research agenda and understand in-depth of it. It shows the clear way to walk on the specific way. We can see our problem with our theoretical eyes and so-called philosophical eye. As far as I know, Knowledge of mathematics can be seen and evaluated theoretically. Since this study is going to put emphasis on exploring the students' level of understanding based on two components of knowledge: conceptual as well as procedural and to explore the meaning that why it happens particularly to the students of the grade 8 in algebra, so the constructivism and cognitive theories can be useful to carry out this mixed-method research study.

Some learning theorist believe in learning can be possible through continuous mental processing (cognitivist), some believe in stimulus and response that is learning through stimulation and inspirations (behaviorist), and some believe in learner's own effort into it and his/her past experience (constructivist). This research study is going to figure out the practice of conceptual knowledge (CK) and procedural knowledge (PK) in the algebra of grade eight students. For this, two types of learning theories; cognitive and constructivism learning theories are applicable to justify my study and findings. Let us take each in turn.

**Cognitive Learning Theory**

Cognitive learning theory believes that a learner constructs or develops the knowledge of mathematics through his/her active mental process (Yilmaz, 2011). In this process, the previously learned conceptions remain the same in the brain and learner use prior knowledge as a schema to form a new knowledge. In this view, construction of procedural knowledge of mathematics is a kind of mental process so that this theory is applicable to justify the meaning and value of procedural knowledge of mathematics in the process of data collection as well as analyzing both forms of data. This theory helped the researcher to examine and explore the meaning of participant's responses towards procedural knowledge of mathematics.

**Constructivism Learning Theory**

This theory always keeps the learner at the center of learning. It means, learner plays the key role to the knowledge development. Constructivists believe in learner's own effort to construct knowledge individually or through social interaction. Constructivists believe that people construct the understanding or knowledge of the world through experiencing things and reflecting on those experiences (Von Glasersfeld, 1995). And the experiences are very rich in developing new knowledge. Conceptual type of knowledge development needs the learner's own experiences. To evaluate students' conceptual knowledge of mathematics and to justify it theoretically, constructivism theory is a necessary condition for this study. The exploration of the conceptual kind of mathematical knowledge can be examined through the constructivist eye. Researcher uses this theory to provide more information about the conceptual type of knowledge, to examine the relationship between conceptual and procedural knowledge of mathematics and to interpret the findings.

## Chapter Summary

In this chapter, the researcher has tried to relate the study with the help of related literature. This was done through thematic, theoretical and empirical review sections. This chapter has explained how PK and CK in mathematics teaching and learning are the major emphases since very long time. The relationship between PK and CK have been shown in the chapter with the help of related literature. Similarly, the researcher wants to use cognitive and constructivism theory of learning to carry out this study.

# CHAPTER III

# RESEARCH METHODOLOGY

Methodology gives a complete framework for any research study. It shows the way of how we complete a particular study. In this scenario, the main purpose of this chapter is to build up a clear-cut framework for this research study. This chapter incorporates different approaches to this research such as methodology, research design, nature and sources of data collection, selection of the study area, population and sample for this study, selection of the sample schools, sample students and teachers for the interview, data collection and data analysis procedures, data collection and generation tools, validity, reliability, and trustworthiness of this study,. Similarly, ethical considerations and summery of this research have been included.

## Methodology

According to my research purposes and questions, the sequential explanatory mixed method research design was applicable to carry out this study meaningfully. And I see this method was significant to address my issue. “Mixed method research is a procedure for collecting, analyzing and ‘mixing’ both quantitative and qualitative data in a single study or a series of studies to understand a research problem (Creswell & Plano Clark 2011, as cited in Creswell, 2015, p. 535). The basic assumption of this method is the uses of both quantitative and qualitative methods, in combination, provide a better understanding of the research problem and questions than either method by itself. For this design, we need to understand both quantitative and qualitative research methods. This research design consists of merging, integrating, linking, or embedding the two ‘strands’ or both types of data are mixed (Creswell, 2015). Among the different forms of mixed-method design, to address the issue or the research purposes and questions of this study, sequential explanatory mixed method design was appropriate. In this mixed method design, the researcher first collected the quantitative data and analyzed it and the process was followed by qualitative data collection and result interpretation in the second phase. To address the first research question, the researcher collected quantitative data through a survey. The survey was conducted among the randomly selected students of grade eight of Kathmandu metropolitan city. In addition to this, to explain the quantitative finding from the first phase and elaborate the possible reasons behind the quantitative findings; interviews were done with purposively selected teachers and students.

**Ontology**

Ontology is the theory and philosophy of reality. It deals with what is the reality out there. This is a philosophical study of nature of being, existence or reality as well as relations of reality (Saunders, Lewis, & Thornhill, 2009). It is a study of what truth is and what is false. Generally, we have two kinds of realities all in all. The very first one is singular reality and the second one is multiple realities. The purpose of this study is to compare eight grader students' level of procedural and conceptual knowledge in algebra and explore about it. This study meets both post-positivist and constructivist approaches. In this scenario, singular and multiple realities can be used. In the first phase, this study compared the level CK and PK through a survey. The survey found the singular reality of whether procedural knowledge or conceptual knowledge of students is strong. On the other hand, in the second phase of this study, it figured out why one or both kinds of knowledge students develop in this level through the interview with some students and teachers. Interview found the different reasons as multiple realities why students develop such knowledge.

**Epistemology**

This is another worldview of philosophy which deals with how we develop knowledge. That means, it is a theory of knowledge (Sounders et al., 2009). Epistemology is a study of how an individual acquires and develop knowledge. This is an area of philosophy concerned with the nature of the justification of human knowledge (Hofer & Pintrich, 1997). While developing mathematical procedural and conceptual knowledge students are guided by both subjective and objective knowledge. Mostly, post-positivism refers to the objective kind of knowledge whereas constructivism refers to the subjective knowledge. In the study, I believe that knowledge can develop not only by the subjective way but also by the objective way. That means both forms of knowledge are used in the study.

**Axiology**

This worldview of philosophy deals with the theory of value. While using the concept of criticism (why, how, why not?), "belief about the meaning of ethics or moral behavior" is mainly focused on interaction which "respect" and "beneficence" for cultural norms and social justice respectively (Mertens, 2007). The value can be different and distinct from person to person. In this study, I value the perception of every participant. Similarly, my subjective and objective value have been included.

## Nature and Source of Data/Information

The data in this study is mixed in nature. It means the researcher collected both quantitative and qualitative forms of data. The first phase survey comprised quantitative data as numbers and statistical information whereas the second phase 'interview' involved qualitative data such as students' and teachers' perception, their experience, pictures, texts, their feelings, emotions etc. towards PK and CK in algebra. The crucial sources of both kinds of data were the responses of students of grade eight as well as mathematics teachers who teach in grade eight.

## Selection of Study Area

Selection of the study area, I think, is one of the challenging tasks for the researchers. To address constructed research questions and fulfill the purpose is very challenging in nature. And also, the researcher should be aware of the target population. In this study, the researcher selected students of grade eight of public schools in Kathmandu metropolitan city of Kathmandu district as well as some mathematics teachers who were teaching in the same level to get all the required data and information.

## Population and Sample of the Study

All the students of grade eight of public/government schools of Kathmandu metropolitan city are the target population of this study. The probability sampling technique is appropriate for this study. Among the various probability sampling, simple random sampling is used to select a sample for this research study. Taherdoost (2016) asserts, “The simple random sample means that every case of the population has an equal probability of inclusion in the sample (p. 21).” In this sampling procedure, each member of the population has an equal chance of being selected in the sample.

Kathmandu metropolitan city was selected for the study and all the students of grade eight of public schools from metropolitan city were considered as the population for this study. According to the statistical data of DEO of Kathmandu district, there 61 schools were running in the year of 2074 and 2075 (See appendix – A). Among these schools, N = 4458 students were studying currently in grade eight. Therefore, all the 4458 students were the population of this study. Out of these students, the sample for this study was selected by using the following formula and presented below,

$$Sample\ size = \frac{X^2 N\, p(1-p)}{d^2(N-1) + X^2 p(1-p)}$$

Where,

$X^2 = Z^2$ (tabulated value of Z from normal distribution)

*P* = the population proportion (assumed to be 0.50 since this would provide the maximum sample size).

*d* = degree of accuracy expressed as a portion 0.05.

Now,

$$n = \frac{(1.96)^2\ \times 4458 \times 0.50 \times 0.50}{(0.05)^2 \times 4457 + (1.96)^2\ \times 0.50 \times 0.50} = 354$$

Therefore, n = 354 was the sample size for this study. However, for some convenience 360 sample students were selected. So, the sample for this survey was, n = 360

**Selection of Sample Schools**

While inspecting the nature of students of 61 schools (see Appendix- A), this metropolitan city, the nature of students' distribution based on schools was heterogeneous. Among 61 schools, about 15% i.e. approximately nine schools were selected. While selecting sample students,

No. of sample students from each school = $\frac{360}{9} = 40$

Now, for this sample in each school, researcher separated schools that has students greater than 40. Next, among these nine schools, students were selected randomly for the survey study.

## Sample Students and Teachers for the Interview

For the qualitative data collection, face to face interview was conducted among six students from two schools (see Appendix – C). Three students were selected from each school who already appeared in the survey with the help of the purposive sampling method. The interview was done in two phases. The very first interview was conducted to understand whether students in algebra have actual procedural and conceptual knowledge with the help of their test papers. This was to verify the finding in the first phase (survey) of the data collection. The performances of students in algebra test were categorized into three groups; high mark, moderate mark and low mark achieving group. So, students were selected according to their ability of performance. Three girls and three boys students were selected in which two (one male and one female) of each group. On the other hand, these six students were interviewed again to address the second research question in the second phase. Moreover, four mathematics teachers (see Appendix – D) from four different schools were selected for the open discussion.

## Data/Information Collection and Analysis Procedures

Since this study follows explanatory mixed method design, both numeric and narrative forms of data were collected from the survey, in-depth interview with students and an open discussion with teacher. Dividing the data collection procedures into two phases: the first quantitative through survey and the second qualitative through in-depth interview with students and open group discussion with teachers, the researcher collected both forms of data sequentially. Numeric data was collected through an achievement test paper and narrative data was collected through semi-structured questionnaires.

After collecting the numeric form of data in the first phase, the researcher used the 23rd version IBM Statistical Package for Social Science (SPSS) software to draw out descriptive and inferential analysis. To address the first research question and purpose numerical comparison was needed to compare eighth-grade students' procedural and conceptual knowledge in algebra. For this, statistical analysis was used to draw the conclusion. The descriptive statistics mean, standard deviation, correlation as well as regression were calculated. Similarly, for the inferential statistics; t-test, one-way ANOVA (F – test) were used for hypotheses testing.

Whereas in the second phase, to verify the findings from the first phase (survey) themes were generated through the narrative analysis. It means, from the collected qualitative information, themes were generated. In the case of narrative analysis, the researcher collected stories of the selected participants, their experiences, field notes, as well as their perception towards the research questions. Similarly, the researcher recorded the interview and the entire scenario to transcribe them in a thorough manner. At last, the researcher analyzed these data to get the result.

## Data Collection/Generation Tools

To conduct the survey, an achievement test paper was developed. Based on the suggestions of different mathematics educators, teachers and researchers; test questions were developed to measure both knowledge of students in algebra, however, the major focus was in basic concept of algebra, factorization and the linear equations (see appendix – F). Similarly, the test paper was translated into the Nepali language with the help of language experts and mathematics teacher educators (see Appendix – G). Four questions were developed to measure the procedural knowledge by giving five marks to each question. Among these four questions, 2 of them were of factorization and remaining two were of equation solving. These questions were developed on the basis of the prescribed curriculum for grade eight of mathematics curriculum of Nepal. Similarly, 20 objective questions were developed to measure the conceptual knowledge in algebra. Every question had 4 distractors. The first four questions were from factorization (One was of the form $ax^2 + bx + c$ and another using the formula $a^2 – b^2$) and equation solving (one linear and another quadratic). To measure the procedural knowledge, the task is almost always solving problems, and the outcome measure is usually accuracy of the answers or procedures (Canobi, Reeve & Pattison, 1998; LeFevre et al., 2006; Schneider & Stern, 2010; as cited in Rittle – Johnson & Schneider, 2015). The first four questions were produced to measure the algorithmic procedure of students to get the correct answer.

On the other hand, the multiple choice questions were developed to measure students' conceptual knowledge in factorization (as well as their prior knowledge in algebra and factorization) and equation solving. The very first question is about defining factorization in words. Having conceptual knowledge means one has to have ability to explain what it is, for example, defining the equal sign (Knuth, Stephens, McNeil & Alibali, 2006; Rittle – Johnson & Alibali, 1999; Rittle – Johnson & Schneider, 2015). Questions no. 4 and 9 were developed to measure student's representational knowledge in geometrical figures. This measure has been categorized into implicit measure of conceptual knowledge. The translation of mathematics abstracts into figure and representational system (Hecht, 1998; as cited in Rittle – Johnson & Schneider, 2015). For instance, placing symbolic number on number lines (Siegler & Booth, 2004; Thompson, & Schneider, 2011; as cited in Rittle – Johnson & Schneider, 2015). Next, questions like no. 2, 3, 6, 17 and 18 were developed to measure the prior knowledge of students in factorization and equation solving. These were used to measure their conceptual knowledge as it is directly related to linking core concept of mathematical ideas. These questions consist of core terms like factors in the factorization, degree of any algebraic expression, terms of any algebraic expression; similarly, recognizing linear equation among different equation, number of solution of linear equations etc. These key facts help to understand how much our students are able to grab the core ideas of concept. Questions such as 5 and 7 were developed to measure the students' ability to compare the quantities. The first question is about finding the expression equivalent to the given algebraic expression. The very next question no. 7 is to determine the ascending order of the given three expression $1/x$, $1/(x^2)$ and $1/(x^3)$. These also belong to the implicit measure of conceptual knowledge as stated by Rittle – Johnson and Schneider in their book chapter. For example; indicating which symbolic integer or fraction is larger (or smaller) (Hecht, 1998; Laski & Siegler, 2007; Rittle – Johnson & Schneider; 2015). Other questions such as questions no. 8, 10, 11, 12, 13, 14, 15, 19 and 20 were developed to measure students' critical thinking, representation of verbal knowledge into the mathematical structure. These questions are related majorly to our day to day life. Things such as age, numbers as well as different quantities we are surrounding by. So, the knowledge is directly linked with our daily life. Further, question no. 16 was developed to measure students' explanation in procedures. So, students should explain where the process is wrong or right. For e.g. students should evaluate unfamiliar procedures (Rittle – Johnson & Schneider, 2015).

At first, a set of semi-structured questions was developed to verify the result of the first phase survey study (See appendix – H). To address the second research question, second phase interview with the same students of the first phase was done. For this, second interview checklist for the students was developed (see appendix – I). Similarly, an open group discussion was done with four teachers from four different schools was done to capture their perspective to find the possible answers to the second research questions. For this also, a set of semi-structured questionnaires was developed (see appendix – J).

**Validity, Reliability and Trustworthiness**

"Validity refers to the development of sound evidence to demonstrate that the test interpretation (of scores about the concept or construct that the test is assumed to measure) matches its proposed use" (AERA, APA, NCME, 1990, as cited in Creswell, 2015, p. 159). It means validity includes the degree in which our test or other measuring device is truly measuring what we intended it to measure. To establish the validity; content, criterion and construct are used in quantitative study.

In this study, to maintain the validity, the researcher had developed the test items in such a way that they represent the content area (algebra) to be measured. The researcher had developed the test only to measure the CK and PK of students in algebra. The items were developed with the help and suggestions of mathematics subject experts, subject teachers, researchers as well as the curriculum of mathematics of grade eight. Also, the dissertation supervisors had helped to develop the test items. Moreover, the item analysis was done. It is a process of examining the responses to individual test or set of questionnaires so as to assess the quality of those items and test as a whole. This is the process of finding out the difficulty index, discrimination index and the effectiveness of distractors of items while being reused or reproduced (Gajjar, Sharma, Kumar, & Rana, 2014). 26 multiple choice questions (see Appendix-E) were selected for the pilot texting. On the other hand, 25 students were selected for this test from one of the schools besides the sample. Using item analysis, difficulty index, discrimination index and the power of distractor were found. In this analysis, some questions were found to be eliminated. Questions having difficulty index ranging from 0.30 to 0.69 were selected and some other questions were revised. Next, questions with discrimination index value greater than 0.4 were kept and having index from 0.2 to 0.39 were revised and selected. Out of 26 questions, only 20 questions were selected for the survey (see Appendix – F).

Similarly, reliability means that scores from an instrument are stable and consistent. That means the scores of the tool should be nearly the same when it is used by different researcher. Through the response of students in the pilot survey, the researcher calculated the Chronbach Alpha to measure the internal consistency of the questionnaires. The reliability of this test by Chronbach alpha value was found to be $\alpha = 0.80$; this means that the questions have high internal consistency.

For the second part of this study, the researcher had taken trustworthiness as a quality standard. Trustworthiness, in a qualitative study, is about establishing the four things credibility, transferability, dependability as well as confirmability (Shenton, 2004). To maintain the credibility of the qualitative data researcher spent the prolonged time to collect data, qualitative data was examined through member checking method. Similarly, to maintain transferability researcher used purposive sampling procedure and thick description of the data and findings. The researcher believes that the finding and the data can be used in other similar circumstances also. The concept of procedural and conceptual knowledge in mathematics is a key issue to all the level of teaching and learning. So I believe that this study is helpful for other level and context as well. And to maintain conformability researcher used participant’s response and there was no potential biasness. Researcher assured that the researcher’s bias does not skew the interpretation of what the research participants said. And finally to maintain dependability researcher tried to maintain the consistency of the data and findings through audit trail method.

## Ethical Consideration

The matter of ethical consideration occurs at every point in the research process (Creswell, 2015). In this regard, research must follow the provision to maintain ethical issues in his/her study. Ethical issues are ongoing matters in the research process. Ethical consideration constitutes sampling procedures, respect participants, informed consent, maintaining the privacy of every participant, use of language etc. In this regard, the researcher considered the ethical issues and respected the participants during the study. In this study, all the participants were informed before the procedures of data collection in both forms of data collection. Similarly, they were informed to be free to respond, it means there were no forces and boundary to respond. In case, if respondents did not want to involve during the data collection, they could leave anytime and anywhere. Participant’s right to respond was fully respected by the researcher. Researcher strictly persuaded the provisions and rules while collecting both forms of data. The details of respondents were kept confidential and to maintain it researcher used the pseudo-name instead of their original name in this study. During this research, no participants was hurt physically and emotionally. The researcher was respecting their cultural, community and social practice from the depth of the heart. While collecting qualitative data people from different background were treated equally. The emerging issues of privacy, anonymity and confidentiality were maintained precisely.

## Chapter Summary

All in all, the sequential explanatory mixed method was used in this research study to address both research purposes. Qualitative and quantitative forms of data are integrated to get a better result. Sample students for the study were selected randomly from the nine schools of Kathmandu metropolitan city by assuming the all the schools in Kathmandu metropolitan city as a target population. Six students and four teachers were selected randomly to conduct second phase interview. Pilot testing was done beside the sample schools and got reliability alpha value 0.08 and sample questionnaires for the survey were selected by analyzing questions' discrimination level as well as difficulty level with the item analysis approach. Ethical things were taken seriously by the researcher according to the norm and value of research study.

# CHAPTER IV

# DATA ANALYSIS AND INTERPRETATION

In this chapter, the researcher tries to analyze and interpret both types of data; first quantitative and second qualitative. Similarly, the researcher tries to explain the research findings and its generalization to the population.

## Quantitative Data Analysis and Interpretation

The survey was conducted among the 360 students of grade eight from nine different public schools of Kathmandu metropolitan city. The survey among these students was conducted to measure the procedural and conceptual knowledge in algebra. In addition, students' gender, their parents' occupation and education have been taken as influential factors in the development of procedural and conceptual knowledge in algebra. Among 360 respondents, 175 were boys and 185 were girls. Similarly, in parents' occupations, the researcher has taken 5 different professions such as teaching, agriculture, government job, business and other professions. In parents' education, the researcher has taken six different level of education such as literate, SLC, +2, bachelor, master and other. Among 360 respondents, there were124 respondents whose fathers were just literate, 106 were SLC graduate, 35 were 10 + 2 graduate, 20 were bachelor graduate, 15 had completed their master's degree and remaining 60 were others. Similarly, academic status of respondents' mother also found different as 161 were just literate, 90 were SLC graduate, 25 of them 10 + 2 graduate, 12 of them had passed bachelor, 7 of them were holding the master degree and 65 were others. There were 12 respondents' parents who were from the teaching profession, 42 of them from government job holder whereas 85 of them from business and 62 of them were from agriculture occupation and remaining 159 were in other professions.

On the other hand, the following statistical hypotheses were formulated;

- Hypothesis 1: There is no relation between procedural and conceptual knowledge of students in algebra.
- Hypothesis 2: There is no significant difference between the mean marks of students in procedural knowledge in algebra based on their gender.
- Hypothesis 3: There is no significant difference between the mean marks of students in Conceptual knowledge in algebra based on their gender.
- Hypothesis 4: There is no significant difference among the mean marks of students in procedural knowledge based on their father's education.
- Hypothesis 5: There is no significant difference among the mean marks of students in conceptual knowledge based on their father's education.
- Hypothesis 6: There is no significant difference among the mean marks of students in procedural knowledge based on their mother's education.
- Hypothesis 7: There is no significant difference among the mean marks of students in conceptual knowledge based on their mother's education.
- Hypothesis 8: There is no significant difference among the mean marks of students in PK and CK based on their parents' occupation.

**Comparison of PK and CK of Students in Algebra**

In this part, the researcher has tried to give the answer to the first research question. For this, the researcher has tried to figure out the level of respondents' procedural and conceptual knowledge of algebra, the relation between them, how much they are correlated and independent of each other with the help of statistical value.

The following table shows the condition of respondents' procedural and conceptual knowledge in algebra.

Table 1

*Procedural and Conceptual Knowledge of Students in Algebra*

| Indicator | N | Minimum | Maximum | Mean | SD |
|---|---|---|---|---|---|
| Procedural Knowledge | 360 | 0 | 20 | 14.05 | 6.344 |
| Conceptual Knowledge | 360 | 1 | 20 | 8.56 | 3.912 |

The above table shows that among 360 respondents, the mean mark of respondents in procedural knowledge is 14.05 with standard deviation 6.344 whereas the mean mark of respondents in conceptual knowledge is 8.56 with standard deviation 3.912. If we compare the mean marks of both of the knowledge of respondents, there is a huge gap between them. The mean marks show that students are good at the procedural knowledge of algebra in grade eight. Comparatively, respondents are weak in conceptual knowledge of algebra in grade eight. The standard deviation values show that the marks of students in procedural knowledge are more deviated from the mean mark. This evidently means the marks in conceptual knowledge are more consistent than the marks in the procedural knowledge of respondents in algebra. In addition, there is greater variability in procedural knowledge in comparison to the conceptual knowledge of respondents.

A research done by Ghazali & Zakaria (2011) that was carried out among 132 secondary level school students to compare students' procedural and conceptual understanding in algebra found the high level of procedural understanding but a low level of conceptual understanding in algebra. The survey research study shows the comparative judgement about students' procedural and conceptual knowledge in algebra. Moreover, as in this study, students have low level of conceptual knowledge comparing procedural knowledge in algebra.

**Correlation Between PK and CK of Respondents**

Several researches have been done to analyze the relationship between procedural and conceptual knowledge. One of the researches which was conducted by Rittle – Johnson and Alibali (1999) from Carnegie Mellon University to examine the relations between children's conceptual understanding of mathematical equivalence and their procedures for solving equivalence problems (e.g., 3 + 4 + 5 = 3 + …..). They had done this research study in $4^{th}$ and $5^{th}$ grade students and found that there is strong relationship between these two knowledge. They stated, "These two types of knowledge do not develop independently rather positive development of one knowledge leads to development of the other" (p. 175). In reference to this study and other related study, the researcher has strong belief in the positive relationship between these two types of knowledge. Regarding the relation between these two knowledge of respondents in this study, the following table shows the actual relation by Karl Pearson's correlation coefficient.

Table 2

*Correlation Coefficient between Procedural and Conceptual Knowledge*

| | | PK | CK |
|---|---|---|---|
| PK | Pearson Correlation | 1 | .559** |
| | Sig. (2-tailed) | | .000 |
| | N | 360 | 360 |
| CK | Pearson Correlation | .559** | 1 |
| | Sig. (2-tailed) | .000 | |
| | N | 360 | 360 |

**. Correlation is significant at the 0.01 level (2-tailed).

The above table shows that the correlation coefficient between procedural and conceptual knowledge of respondents is $r = 0.559$. This evidently shows that there is a moderate positive correlation between these two knowledge of respondents in algebra. This clearly means that the development of one kind of knowledge helps to develop another type of knowledge. For example, if students are familiar with the process of factorization of an algebraic expression; he or she is also developing the related concept such as defining factorization, relating factors in a diagram or pictures, comparing the expression and understanding the key terms like factors, degree as well as terms etc.

There is similar conclusion in a research study done to measure the relationship between students' procedural and conceptual knowledge in algebra which concluded that there is average positive correlation ($r = 0.512$, $p < 0.05$) between students' procedural and conceptual knowledge in algebra (Ghazali & Zakaria, 2011).

Another research study done by Ross (2010) in his doctorate dissertation among the 8th graders students to measure their conceptual and procedural knowledge in algebra found that there is strong positive correlation between these knowledge (Ross, 2010).

Further, we can test against the research hypothesis there is no significant relationship between these two types of knowledge. This also helps us understand whether there is relation between these two knowledge or not. The above table shows that the significant correlation r value 0.00 (0% approx.) is less than that of alpha value 0.001 (1%). In this condition, we are able to accept the research hypothesis. So, we conclude there is relationship between the two types of knowledge.

**Linear Regression between PK and CK of Respondents**

Here, the researcher has analyzed the effect of procedural knowledge (independent variable) in conceptual knowledge (dependent variable) to understand how much change occurs in the increase or decrease in the procedural knowledge of respondents. For, this here the researcher has used a regression model to analyze the statistical data of effect on conceptual knowledge with respect to procedural knowledge.

Table 3

*Regression Model of Conceptual and Procedural Knowledge*

| Model | | Unstandardized Coefficients | | Standardized Coefficients | t | Sig. |
|---|---|---|---|---|---|---|
| | | B | Std. Error | Beta | | |
| 1 | (Constant) | 3.716 | .410 | | 9.062 | .000 |
| | Procedural Knowledge | .345 | .027 | .559 | 12.961 | .000 |

a. Dependent Variable: Student's marks in conceptual knowledge.

In the above table, the statistical p – value is 0.000 which is less than alpha value 0.05 which is statistically significant that the regression model can predict the outcome variable (CK) with respect to the change in independent variable (PK). This tells us that respondent's procedural knowledge affects their conceptual knowledge in algebra. We have R-square value and it is 0.313. This value means that CK can be explained about 31% by PK which is moderate value. Additionally, there is 31% total variation in the dependent variable i.e. in CK because of PK.

The linear regression equation is;

$$CK = 3.716 + 0.345\ (PK).$$

Next, the B coefficient represents how much units CK increases for a single unit increase in the predictor that is PK. It means 1 point increase on the PK corresponds to 0.35 increases on the CK. So we got the conclusion that, if we increase PK by 1 unit, there will be 0.35 unit change in CK.

**Comparison of PK of Students Based on Their Gender**

It is widely accepted that gender is one of the factors in the development of mathematics knowledge. A study by Mathema and Bista (2006) found a strong positive correlation between gender and students' performance in SLC (Belbase & Panthi, 2017). In this research study, the researcher is going to test whether the gender of the students influences the development of procedural knowledge of mathematics in grade eight. The following table shows the phenomenon of gender influence in the procedural knowledge of students in algebra.

Table 4

*Procedural Knowledge of Students Based on Gender*

| Indicator | Gender | N | Mean | Std. Deviation | t - value | Sig. (t - tailed) |
|---|---|---|---|---|---|---|
| Procedural knowledge | Boy | 175 | 14.82 | 6.266 | 2.225 | 0.027* |
| | Girl | 185 | 13.36 | 6.350 | | |

*t -value significant at *p > 0.05*

From the above table, the statistical significant t-value 0.027 (2.7% approx.) < alpha value 0.05 (5%) which is statistically significant. In this condition, we can conclude that there is significant difference between the mean marks in procedural knowledge of respondents according to their gender. If we talk about mean value, we can see a slight difference between the mean values of boy and girl students. The mean mark of the boy is 14.82 whereas 13.36 is of girl. This significantly shows that the group of boys is quite better than the group of girls in procedural knowledge of algebra in grade eight.

**Comparison of CK of Students in Algebra Based On Their Gender**

The following table shows whether gender matters or not in the development of conceptual knowledge of students in algebra.

Table 5

*Conceptual Knowledge of Students Based on Gender*

| Indicator | Gender | N | Mean | SD | t-value | Sig. (2 - tailed) |
|---|---|---|---|---|---|---|
| Conceptual knowledge | Boy | 175 | 8.63 | 4.087 | 0.316 | 0.752* |
| | Girl | 185 | 8.50 | 3.759 | | |

*t -value significant at *p > 0.05*

The above table shows that the significant t-value of the statistic is 0.752 (75.2%) and it is greater than alpha value 0.05 (5%). In this position, we should accept our research hypothesis. In conclusion, there is no significant difference between the average performance of boys and girls in conceptual knowledge. This means the conceptual knowledge of students is not affected by the gender of the students. If we talk about mean marks, the mean marks of both groups of students are about the same 8.63 and 8.50. This shows that conceptual knowledge of boy and girl students is below the average.

**Comparison of PK of Students Based on Their Father's Education**

Here, the researcher assumes that parents' education is another significant influential factor in developing procedural and conceptual knowledge of students in algebra. The researcher has tried to figure out whether there is an influence of parents' education in the development of knowledge through One Way ANOVA test. The following table shows the condition of students in procedural knowledge of algebra according to their father's education.

Table 6

*Procedural Knowledge of Students Based on Their Father's Education*

| | Sum of Squares | Df | Mean$^2$ | F | Sig. |
|---|---|---|---|---|---|
| Between Groups | 154.463 | 5 | 30.893 | .765 | .575 |
| Within Groups | 14737.758 | 354 | 40.377 | | |
| Total | 14892.221 | 359 | | | |

*F-value significant at *p > 0.05*

The above table shows that the significant f – value is 0.575 (i.e. 57.5% approx.) and this is greater than alpha value 0.05 (i.e. 5%). Hence, we conclude that there is no significant difference among the means of students' procedural knowledge according to their fathers' education. This clearly means that the father's education does not influence their children's procedural knowledge of algebra.

**Comparison of CK on the basis of Respondents Father's Education**

The following table shows the condition of student's conceptual knowledge in algebra based on their father's education.

Table 7

*Conceptual Knowledge of Students Based on Their Father's Education*

| | df | Mean Square | F | Sig. |
|---|---|---|---|---|
| Between Groups | 5 | 30.905 | 2.048 | .071 |
| Within Groups | 354 | 15.093 | | |
| Total | 359 | | | |

*F-value significant at *p > 0.05*

The above table shows that the significant f –value 0.071 (7.1%) is greater than alpha value 0.05 (5%). Now in this condition, we should accept the fact that there is no significant difference among the average marks of students in conceptual knowledge of algebra according to their father's education. This evidently means that father's education does not influence the conceptual knowledge development of the respondents.

**Comparison of PK of Students Based on Their Mother's Education**

The researcher has believed that a mother's education influences their children in knowledge development. In the same way, mother's education can be a significant factor in procedural and conceptual knowledge development of students in algebra.

The following table shows whether the mother's education level matters in the development of procedural knowledge.

Table 8

*Procedural Knowledge of Students Based on Their Mother's Education*

| | df | Mean Square | F | Sig. |
|---|---|---|---|---|
| Between Groups | 5 | 39.151 | .972 | .435 |
| Within Groups | 354 | 40.264 | | |
| Total | 359 | | | |

*F-value significant at *p > 0.05*

The above table shows that the significant f –value 0.435 (43.5%) is greater than alpha value 0.05 (5%). In this condition, we should accept our research hypothesis that there is no influence of mother's education in the development of procedural knowledge of students in algebra. This evidently means that whatever the level of children's mother education is, they have about the same level of procedural fluency in algebra. In this study, the respondents have the same level of procedural knowledge in algebra on the basis of their mother's education.

**Comparison of CK of Students Based on Their Mother’s Education**

Mother education can be a factor in CK development of algebra. The question arise as is there any significant difference among the mean marks of respondent's conceptual knowledge in algebra with respect to their mother’s education? The following table shows whether a student's conceptual knowledge in algebra is dependent or not on their mother's academic achievement.

Table 9

*Conceptual Knowledge of Students Based on Their Mother's Education*

| | df | Mean Square | F | Sig. |
|---|---|---|---|---|
| Between Groups | 5 | 20.698 | 1.359 | .239 |
| Within Groups | 354 | 15.233 | | |
| Total | 359 | | | |

*F-value significant at *p > 0.05*

The above table shows that the significant f –value 0.239 (23.9%) is greater than the alpha value 0.05 (5%). In this position, we should conclude that there is no significant difference among the students' mean marks of conceptual knowledge in algebra. Mother's academic achievement is an independent factor for the development of respondents' conceptual knowledge. Whatever the education level of student's mother, all the students has all most about the same level of conceptual understanding of algebra as other students whose mother has the different education level.

**Comparison of PK and CK of Students Based on Their Parent's Occupation**

It is widely accepted that parent's occupation is also one of the factors in learning mathematical knowledge. In this study too, the researcher has taken it as a factor influencing in student's procedural and conceptual knowledge development in algebra. The following table shows that whether there is significant difference among the mean marks of procedural knowledge of respondents on the basis of their parent's occupation.

Table 10

*Procedural Knowledge of Students Based on Their Parent's Occupation*

| Indicator | Df | Mean Square | F | Sig. |
|---|---|---|---|---|
| Between Groups | 4 | 60.793 | 1.519 | .196 |
| Within Groups | 355 | 40.025 | | |
| Total | 359 | | | |

*F-value significant at *p > 0.05*

The table 10 tells us that the significant f – value 0.196 (19.6%) is greater than that of alpha value 0.05 (5%). In this situation, we say that there is no significant difference among the mean marks of respondents according to their parents' occupation. This evidence shows that parent's occupation does not play the role in the development of procedural knowledge of respondents in algebra.

In addition, the following table and explanation show whether there is effect of parent's occupation on their children's procedural knowledge development in algebra.

Table 11

*Conceptual Knowledge of Students Based on Their Parent's Occupation*

| | df | Mean Square | F | Sig. |
|---|---|---|---|---|
| Between Groups | 4 | 14.062 | .918 | .454 |
| Within Groups | 355 | 15.320 | | |
| Total | 359 | | | |

*F-value significant at *p > 0.05*

Here, we obtained the significant f – value 0.454 (45.4% approx.) greater than alpha value 0.05 (5%). These statistical values force us to accept the research hypothesis that there is no significant difference among the mean marks of conceptual knowledge of students in algebra which proves that the occupation of parents is an independent factor in developing conceptual knowledge of algebra in grade eight.

Summing up, the above analysis and interpretation evoke us to conclude some of the major highlights here. The level of conceptual knowledge is low in comparison to procedural knowledge in algebra. Students, in grade eight, have a high level of procedural knowledge that is the knowledge of skills, action sequences and step-by-step procedures without explicit reference to mathematical ideas (Marchionda, 2006). The result is similar to the finding of Ross (2010) and Ghazali & Zakaria (2011). Students are below the average in procedural knowledge. Still, there is an average positive correlation between students' procedural and conceptual knowledge. This means that the knowledge development process in algebra should be iterative (Rittle – Johnson et. al., 2001). If we talk about gender dependency in these two knowledge, data suggests that gender does not matter in conceptual knowledge but it matters in procedural knowledge. Similarly, parent's education and their occupation are the independent factors in students' procedural and conceptual knowledge in algebra.

## First Phase Qualitative Data Analysis and Interpretation

According to the first phase data collection and analysis, the first research question was addressed by the result of quantitative findings. The researcher found that there is a gap between conceptual and procedural knowledge of eighth-grader students in algebra. Also, the researcher found that students have greater procedural knowledge of algebra in comparison to conceptual knowledge. Similarly, we saw that male students are somewhat better than female students in conceptual knowledge. However, the quantitative finding has shown that gender does not matter in the development of procedural knowledge of algebra.

Next, the researcher wants to explain and verify whether students have lower conceptual knowledge in algebra through the interview. This time, the researcher wants to figure out the reason why did such result occur in the first phase? How did it happen? Was the result true? The secondary qualitative data collection was done to help explain and elaborate on the quantitative finding and results of the first phase (Creswell, 2015). Therefore, the second phase qualitative data collection was done through the interview to support and verify the result of the first phase.

First, 20 multiple choice questions were asked in the survey where each question has 1 mark for conceptual knowledge related questions. Therefore, the researcher assessed the conceptual knowledge of respondent by assuming 20 as full marks. Next, to conduct this interview, the researcher separated the answer sheets and categorized them into three parts: lower mark achieving group (who achieved ≤5), moderate mark achieving group (who achieved 9 – 11) and higher mark achieving group (who achieved 18 – 20). Among these group, the researcher selected one boy and one girl students from each group for the interview. It clearly means that there are six individuals from three groups who had been selected for the interview. The researcher used their answer sheets to take the interview and to ask the questions like how and why they selected those options for the questions in the survey; to measure their actual conceptual knowledge in algebra. The interview was done with these six individuals separately and it was done in two different schools, three students from each school (see Appendix – C).

While measuring the conceptual knowledge, I have used the measuring task developed by Rittle – Johnson and Schneider (2015). In their words, “Measures of conceptual knowledge vary in whether tasks require implicit or explicit knowledge of the concepts (p.3).” With the help of interviewee’s answer sheet, I have used different implicit and explicit measuring tasks. For the implicit measuring tasks I have tried to address their ability to provide examples for the related concept, their representational knowledge with diagrams or pictures (linking with pictures Q. No. 4 and 9 were asked again and tried to get whether they have the concept of pictorial representation), comparison of quantities. Similarly, other measures like if they have multiple methods to solve a problem as well as how much he/she knows about such knowledge. On the other hand, in the explicit measure, I have asked the questions to evaluate their ability to define the concept such as what the factorization is, what linear equation is. Students were asked to explain why the procedures work and why does not in a single problem (Q. No. 16). In addition, one critical question (Q. No. 11) was asked to measure their ability to solve the verbal problems in mathematics. (For the entire information such as questions and the checklist of the interview, see the appendix - H).

To take the interview, I had also recorded the entire interview so as to transcribe them later.

## Student's Conceptual Knowledge in Algebra

With the starting of every interview with every individual, I asked them some personal questions such as the meaning of their name, their actual hometown, their interest so that they may be open to give the answer and provide genuine information and also for the closeness to the participants. At the very beginning, I appreciate their involvement in the survey as well as in the interview.

### Students' Ability to Provide Examples

The very first question I had asked them was; *"can you give me any other example of algebraic expression?"* I asked this question to measure their ability to provide example for the related concept of algebraic expression. Some of the discussion went in the following way:

(Note: I: Interviewer and R: Respondent)

I: *What do you know about algebraic expression?*

R: *Sir, definition? I don't read definitions?*

And with a little pause *….umm I think one of the examples can be 2x – 6.*

Another respondent from the higher mark achieving group was asked to give the example of algebraic expression? And she replied with soft voice,

R: *Sir, I think,* $x^2 + 6x + 3$.

Similarly, another respondent from moderate achieving group replied:

R: *Umm…I actually don't know the definition and example kind of things*.

Other three interviewees replied the same thing that they don't know the proper example of algebraic expression. One of them said, *"I am not good in mathematics and I did not understand what my teacher taught but I learn to solve problems with the help of example. So, if there is simple problem given, I can do. However, I do not know the example of algebraic expression."*

If we analyze the above conversation, we can probably assure that our students in the middle grade (as well as other grades) do not have much ability to provide the example of the related concept. As a researcher, I accept this statement. When I was in middle grades such as six, seven and eight; I was very good in mathematics. I could solve probably all the questions of the exercise of the textbook. There was an incident in math class when I was in grade eight. It was a day after winter vacation, one of the teachers from other school came to visit our school. He entered our classroom and started saying, "I am here to ask some questions to assess whether you are multiple intelligent or not. We said, "Okay! Sir." With a loud voice. He said that he was going to ask some question from the textbook and also request us to say if we know. As I was the first in the class, my friends pointed my name by saying "genius of mathematics" or" hero in mathematics" and said, "Netra is ready to answer, sir." So, I raised my hand and teacher asked me to give an example of a linear equation. I was overwhelmed and panicked by the question because it was from algebra but also being best in algebra, I replied with a very soft voice, "*Sir, is that* $x + y = 5$*? But I am not sure"* Then sir replied, *"Can you create one more?"* I then replied, *"Sir, is* $x^2 - y^2 = 0$*?" "No!"* sir replied. He continued, *"How did you create these two different examples?"* I then said, *"Sir, I think linear equation means having the same degree of x and y. There is degree one, one of x and y in the first example and 2 in the second example."* All the students' eyes were on me. Then sir said, *"Netra! You have done a great job. However, the first equation you gave was the equation of a line but not the second one."* I then spoke very surprisingly, *"Why sir?"* Sir replied, *"Do you know what linear equation is?"* I then replied with a very soft voice, *"Sir! It is the equation of a line. Nevertheless, I am not familiar with the equation of the line and I don't know the proper example of it. I can solve the problems given by teachers and problems given in the exercise book."* At last, teacher said, *"linear equation means, an equation whose degree is one. Now, you understand?"* I then replied, *"Oh! I see. So, the equation like* $y = 2x + 5$ *is also the equation of a straight line. Right sir?"* Then that teacher replied, *"Yes!"*

The ability to consider and evaluate examples for the related concept shows one level of conceptual knowledge of that concept (Rittle-Johnson & Alibali, 1999). However, the interview with these six individuals presents the fact that our students are somewhat weak in considering examples of the concept such as algebraic expression, linear equation etc.

**Students' Representational Knowledge in Algebra**

Another key term to measure conceptual knowledge of the learner is to see whether he/she has the ability to represent mathematical knowledge with pictures such as representing the symbolic number with pictures (Hecht, 1998). Here, in this study, two questions Q. No. 4 and 9 were asked to measure students' pictorial representational knowledge in factorization and linear equation. Interviewees were asked how and why they choose the correct or the wrong option. In addition, they were asked to represent some other concepts in the diagram.

Question number 4 asked to represent the factorization of $x^2 + 7x + 12 = (x + 3). (x + 4)$ in the diagram or figure. Respondents had to choose the figure from the option. The correct option was area of rectangle expressed into length × breadth. For this, among six respondents in the interview, 3 of them chose the option (a) picture of rectangle with $A = x^2 + 7x + 12$ where length = x + 4 and breadth = x + 3.

Among these interviewees, the conversation of one of them and the interviewer as follows;

I: *Why did you choose this option?*

R: *Umm! I think there is (x + 3) and (x + 4) and they are length and breadth. Also, the area of rectangle is A = l × b. So, I chose this option.*

I: *Why did you not choose other option? What about other options?*

R: *Sir, I think they are not related to length × breadth*, so.

I: *Have you ever used pictures to represent factorization of any expression?*

R: *Sir, I think we have used paper folding method to show* $(a + b)^2 = (a + b) \times (a + b)$. (Showing in the diagram)

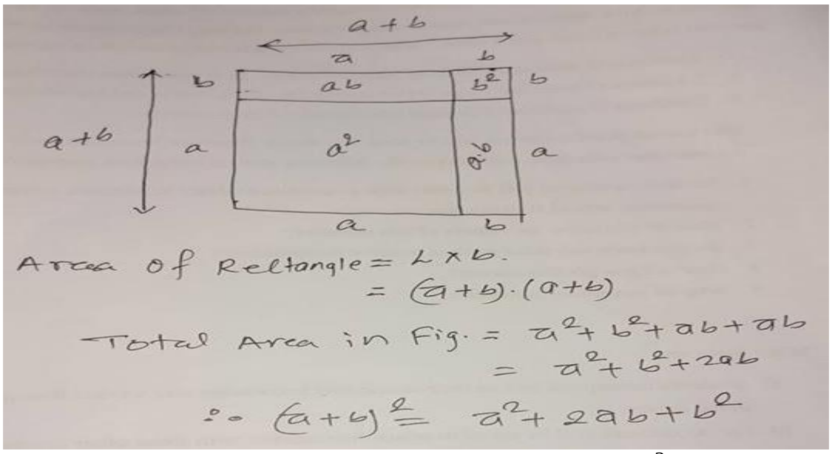



Fig. *2*: Model of (a + b)$^2$

*However, in other cases, I don't know.*

The other respondent replied;

R: *Sir, I thought that we have used this diagram may be grade six or seven while proving* $(a – b)^2 = a^2 - 2ab + b^2$*. So, I quickly remember the thing and I chose this option. But I do not have ability to represent all these expression in pictures or diagram.*

Another respondent replied:

R: *Sir, I chose this option. But I do not know why. I do not know how to draw the figure of questions (factorization).*

If we compare the response of these three respondents, the first respondent used the formula to find the area of the rectangle but he/she does not have the concept to represent all the factorization problems into the diagram. On the other hand, the second respondent used his prior knowledge and already established knowledge to choose the correct option. In this, he somewhat had the conceptual knowledge as Hiebert & Lefevre (1986) asserted that CK is a knowledge which is rich in relationship and connections. Here, prior knowledge can be taken as a concept. However, he/she also do not have the proper concept of factorization of the second-degree polynomial that can be represented pictorially as the area of the rectangle is equal to length time's breadth.

Another respondent chose the figure of a cuboid and I asked him.

I: *Why did you choose this figure to represent the factorization of the given expression?*

R: *Umm! Sir, it is difficult to say. I do not know how to draw the figure of it.*

Another question was Q. No. 9 which was; what is the diagram of $x – y = 0$? Among the interviewees (who chose the correct option), one of them replied in the following way:

I: *why did you choose this option for the diagram of* $x – y = 0$*?*

R: *(Being confident!) Sir, because, umm! When we put* $x = 0$ *and y will be 0. When we put* $x = 1$*, then y will be 1 and so on. I mean, the values of x and y are equal to each other. So, if we put these in the graph, the graph is definitely like this* (pointing to the picture).

I: *Why did not others be?*

R: *I don't know actually. But I think other diagram does not follow the rule as I said earlier.*

Other respondents expressed that they were not familiar with these concept.

One of the weird answers from a respondent (but wrong) was; the selection of option (d); line parallel to y –axis. And the conversation was like this:

I: *why did you choose this option?*

R: *Umm! Sir, I think* $x – y = 0$ *means* $x = y$*. That means x and y are equal. In the same manner, I thought that, line is vertical when* $x = y$*. Is not it sir? So, I thought it is right.*

I: okay, then *what about horizontal line?*

R: *Umm! I do not know much more about it.*

It seems that students do not have proper knowledge about liner equation and its pictorial representation.

As a teacher of mathematics and a practitioner researcher, for the better understanding of an underlying concept, one should have sound ability to represent mathematical knowledge with pictures and diagrams. Students should understand how that knowledge be used in their context as well as their physical world. The human mind can do this when there are visual things like pictures, diagrams, graphs or table etc. (NCTM, 2014). However, in this survey with the reference to the interview, our students do not have proper knowledge about the pictorial representation of the concepts in algebra.

**Students' Knowledge in Comparison of the Expressions in Algebra**

It is widely thought and accepted that the concept of comparison of two or more quantity is generally difficult for the learners as well as others. In the case of mathematics, most of us feel difficult when we compare two or more different quantities. We can take an issue that most of the students in upper primary and middle school grade feel difficult to compare two distinct fractions. Many research studies have been done to claim that students are very poor in comparison of fractions such as determining the bigger among ½ and ¼ (Philippou & Pantziara, 2012); also in comparison of decimal expression such as determining the large one among 0.25 and 0.5 (here is misconception of students that $25>5$ implies that $0.25 > 0.5$). And coming to algebra, it is more difficult as it constitutes abstract symbols and representations (variables and constants).

In the part of comparison of two algebraic expressions, I have asked two questions;

The very first one is Q. No. 5 which was to find the expression equivalent to $x^2 + 7x + 12$. We can see the scenario that most of the students get stuck to find the algebraic expression equal to a given expression. They see the expressions $x^2 – 5x + 2$ is different from $2 + x^2 – 5x$. They think that the value of expression changes if we change the position or place of its term or terms. In the same way, among the respondents who chose the correct option, one of them responded as follows;

I: *Why did you choose this option? What is the reason of choosing this option?*

R: *Sir, I thought that, equivalent means having equal expression, same sign? So, in 12 – (- 7x) – (-$x^2$), if we multiply – 7x by -, - $x^2$ by – given in front of it, then the expression is 12 + 7x + $x^2$.*

I: (stopping him). Taking little pause….*Well, are 12 + 7x + $x^2$ and $x^2$ + 7x + 12 equal? What about their order?*

R: *Sir, does order matter? I think in the expression, we can keep 12, 7x and $x^2$ anywhere we want without changing their sign. But if there is equal to 0, then sign will be changed.*

Another respondent who chose a correct option replied that,

R: *Sir, I thought that; if I do like this: 12 – (- 7x) – (-$x^2$) = 0, then take all the terms to right hand side, then I get $x^2$ + 7x + 12. That is why they are equal.*

According to these two respondents, they have different methods to explain why they did it right. It seems that they have little bit ability of comparison of the expressions. Nevertheless, respondents who chose the wrong answer replied that they properly do not know what equivalent algebraic expressions are. As I explained earlier, they expressed that the value of the expression changes when the place/s of the term/s is/are changed.

The next question was Q. No. 8 and it was; which one is big 1/x and $1/(x^2)$? Here, I was astonished by their responses because no one of these six respondents was able to compare these two quantities. It is a kind of headache for mathematics teachers and students to grab the concept of comparing given two or more quantity. For example; in fraction, perhaps, students feel difficult to compare fraction like 1/5 and 1/25. They probably say that (1/25) is greater than (1/5). It is because, they think that 25 is greater than that of 5. I faced the same situation here. All the responses were same. Let me take you through another conversations here.

I: *Well, which one is big for a positive value x; x or $x^2$?*

R: (With little pause). *Umm! Sir, $x^2 > x$ because $x^2$ has more power than x.*

I: *Well, that is okay. Then tell me which one is big 1/x or $(1/x^2)$ for a positive value x?*

R: *Umm! Pause….Sir, I think $(1/x^2) > 1/x$.*

I: *How is it true? Can you explain it?*

R: *Sir, because the expression having more power is always greater. Here, $(1/x^2) = (1/x)^2$ and $(1/x) = (1/x)^1$. Here, power of $(1/x^2)$ is 2 and 1/x is only 1. So, $(1/x^2) > 1/x$.*

I: (Now, I asked a numerical problem). *Which one is big ½ or 1/8?*

R: (A small pause). *Sir, in 1/8 the power of 2 is 3, right?* (He asked me and I replied, "*yes*"). *So, definitely 1/8 is greater than ½.*

I asked the same questions to other respondent but I got similar responses from them. The expression having more power is always larger than expression having less power. Here, we can say that students do require knowledge of comparison of the expression. Even, students are not able to compare numerical quantities which are given into fraction. In this case, they have poor conceptual knowledge of fraction as well as fraction in algebraic expressions.

**Students' Explicit Conceptual Knowledge in Algebra**

Now, in explicit measure of conceptual knowledge, I have evaluate respondents' ability to define concept or terms, for instance defining the equal sign (Knuth, Stephens, MCNeil & Alibali, 2006), ability to explain why the procedures work, for instance, an ability to explain why is it ok to borrow when subtracting (Fuson & Kown, 1992) and some of the major focus areas on the ability of critical thinking. First, I asked the questions about defining the factorization and then linear equation. Most of them replied that they do not emphasize on reading definitions, do not spend time in conceptualizing the definition of core ideas, terms etc. Two of the participants replied that factorization is the process of conveying the expression into the product of its two or more factor. They created this definition with the help of example done in the procedural knowledge questions. On the other hand, students do not know about what linear equation is. They understand what the figure of a line is but they do not know about the equation of a line. This clearly implies that they are not more familiar with the definition and core concept of linear equations. So, they have somewhat weak conceptual knowledge in the linear equation.

I had asked another question to evaluate their ability about why a procedure work. For this, I had used a sample question from the survey question that was Q. No. 16. In this question, students were asked to determine a false step and step 2 is false in the solution. The response was common to all as in the following conversation.

I: *Well, the step that you chose is false? Why did you choose this one?*

R: (Making a little pause). *Umm! Sir, in the solution, the first step is right. But in the second step, +x is taken from the right side to the left and added it to 2x to make 3x which is completely false.*

I: *Why?*

R: *It is because sir, when we change the side of any term (kura in nepali), we should change the sign. Like in the step two, while taking +x to left side, then there would be –x and we ultimately would get 2x – x which is x only. Because of this reason, the step 2 is false eventually.*

This means that students are quite familiar with steps and they somewhat know how to perform these action sequences while solving a problem. This quite seems that the ability to action sequences to solve problems comes under procedural knowledge (Rittle – Johnson & Alibali, 1999). However, most of the students are quite familiar with how to do them and their explanations were quite clear.

The other respondents who chose a wrong option, they also have a common opinion that they do not know how to do it. It means they are not properly capable to explain whether the given step is wrong or right.

At last, I had asked a verbal problem to evaluate respondents' ability to think critically and change verbal problem into mathematical sentences. They were asked Q. No. 12 and it was; Jiya is exactly five years older than Rina. Let J stands for Jiya's age and R stands For Rina's age. Which of the following is an equation to compare Jiya's and Rina's age? Respondents who chose the correct answer had the same opinion as:

I: *Why did you choose the option J = R + 5?*

R: *Umm! Sir, the question said that J is five years older than R. That means, umm! We should add the number 5 to R. I think older means more so we need to add the number and younger means we need to subtract.*

In this situation also, students felt difficult when they were asked other verbal problem to express them in a mathematical sentence. Another group of students who chose the wrong option expressed that older means multiply and younger means divide. I requested them to give one example from their daily life but they were unable to create a verbal problem and felt difficult to relate to their living life. One of them asked, *"Sir, can we relate these mathematics concepts to our day to day life?"* I then replied, *"Yes, we can. For instance; your dad is 25 years older than you. That is, suppose your dad is 39 (asking with her) now and your age is 14, then tell me how can we do with 25 to your age to reach to your dad's age?"* She certainly replied, *"Sir, we should add up 25 years to reach my dad's age." "Did you get that how we can relate mathematics to our living life?"* I asked. She then replied, *"Yes, sir a little bit."*

Another weird answer is; *“Sir, I have used less and greater while solving the problems in profit and loss where I multiply when there is profit and divide when there is a loss. So, I did the same here.”* This was quite surprising. I did not know why she relates this concept to profit and loss but here, the concept was not right. The condition seems that students of grade eight in algebra are weak in critical thinking and verbal problem-solving.

All of these explanations indicate that students of grade eight are weak to some extent in the conceptual knowledge of algebra. They are weak in defining the core ideas, giving the definition of concept to some extent. They are weak in generating examples. They are weak in expressing abstract knowledge through pictures and diagrams to most of the extent. They are mostly weak in comparing the quantities of algebraic expressions. This thing forces us to generalize the fact that students have a very lower level of understanding of core ideas in comparison to the procedural knowledge of algebra. Similarly, our middle schools' students are weak in critical thinking and meaning-making. These things encourage us to declare that our middle-grade students' conceptual knowledge is lower than procedural knowledge in algebra.

## Second Phase Qualitative Data Analysis and Interpretation

After analyzing the data obtained from the first phase and interpreting them we came to know that eighth-grader students have lower conceptual knowledge in algebra in comparison to procedural knowledge. The above analysis and interpretation leads us to accept the fact that student have lower knowledge level such as they have lower ability to provide examples of the related concept, compare two distinct quantities, define and interpret the relevant concept, represent knowledge into figures and diagrams and to think critically. Perhaps, students have good skills in explaining procedures in problem-solving and procedural fluency when questions, like factorize the expression, solve for the value of x, are directly asked. So, the first phase qualitative finding helps us understand the realistic scenario about the respondents having lower conceptual knowledge in comparison to procedural knowledge in algebra.

In this sequential explanatory mixed method study, first, as a researcher, I have drawn the quantitative finding with the evidence by collecting some qualitative data through the interview. Now, it is the time to respond to my second research question which emphasizes to figure out the possible reasons why students have such knowledge in their domain of knowledge. For this, I conducted second phase interview with the same participation of the first phase interview and with four mathematics teachers of four different schools to understand the possible reasons of being lower conceptual knowledge of students in algebra. I developed two different sets of the semi-structured question, one set for students and another for teachers (see Appendix – I and J).

**Voice of Students**

I have categorized these six students into two groups – one group is high mark achiever group and another is low mark achiever group. Respondents in high mark achiever group were counted in talented students and low mark achievers were counted in below the average students. So, from high mark achiever group, I have taken here representative respondent (Rashmi) and a representative from low mark achiever group (Raja) to transcribe their voice regarding my questions. Response from High Mark Achiever Group

Rashmi: *Sir! Good morning*.

Researcher: (greeting) *good morning. Well, what is your name?*

Rashmi: *Sir, its Rashmi.*

Researcher: *Okay! Rashmi. I am sorry to disturb your class of mathematics*.

Rashmi: *It's okay sir.*

Researcher: *How are you then?*

Rashmi: *I am fine, thank you sir! (It seems that you are fine too. Hahaha!!)*

Researcher: *Well! You have got me right. Now I will ask few questions for the purpose of my research study. It is about your learning interest and other things. Are you ready?*

Rashmi: *Yes! Sir.*

Researcher: *Okay. Tell me, what is your emphasis while solving problems in algebra? Do you focus more on learning steps or concepts or both?*

Rashmi: *Sir, I am good in mathematics. I like mathematics most but I do like other subject too. But regarding your questions, I like to learn mathematics in both ways. However, if it is about algebra, it is difficult for me to say sir.*

Researcher: *Could you make it clear please?*

Rashmi: *Sir, most of the contents in algebra are abstract in nature such as long division, indices, linear equations as well as verbal problems. So, I do learn these concept with the help of formula or rules. But I know how to factorize a given algebraic expression, solve linear equation. Do I make my concept clear sir?*

Researcher: *I will tell you later. Okay then how much do you guys have conversation and critical discussion inside the classroom while learning mathematics concept in algebra?*

Rashmi: *Sir, it is almost impossible to discuss all the students inside the classroom. I do discuss with my talented friends about problem solving of algebra. If there is any confusion, I ask with teachers. But there are other students they do not discuss about problems and solution inside the classroom.*

Researcher: *How much do you enjoy learning algebra?*

Rashmi: (little pause) *Umm! Sir, I do enjoy learning algebra when I am able to do problems otherwise sometimes it would be a headache for me too. I solve problems with the help of guidebook and examples done in the book.*

Researcher: *Well. You have good habit of solving problem. I appreciate it. Now, I ask some questions about your teacher's instruction and support in learning algebra. Are you ready?*

Rashmi: *Okay sir. I am ready.*

Researcher: *What is your teacher's focus on teaching mathematics? Do he emphasize a particular problems by saying it is important for your examination?*

Rashmi: *Sir, our mathematics teacher teaches us in a good manner because he gives the answer of all our questions. If we are near examination, he then starts emphasizing questions which had already asked in the examination.*

Researcher: *Do your teacher put emphasis on a particular students like you?*

Rashmi: *Yes, he does sir. There are so many students in the class. So, how can he give chance to all students?*

Researcher: *Oh! That's right. Umm. Now, do you guys had project work, field work and practical work ever in mathematics or particularly in algebra?*

Rashmi: *No! I don't think so sir. I think we had not get such things. However, Oh! I remembered that sometimes our teacher tells us to bring chart paper by writing formula on it.*

Researcher: *Okay. Rashmi, thank you so much for your valuable time.*

Rashmi: *Thank you so much sir!*

It is a representative example of talented students, how he/she involves in learning, what support he/she has with teachers and the reflection towards teachers' teaching method.

Low Mark Achiever's Perception (Raja)

Here is the conversation with low mark achiever group representative who is below the average in mathematics.

Raja: (Greeting) *Namaste sir!*

Researcher: *Namaste. Its Raja, am I right?*

Raja: *Yes sir!*

Researcher: *Well, how are you up to?*

Raja: *I am good, sir. And you?*

Researcher: *I am good too. Well, Raja I am going to ask some questions regarding your learning journey of algebra. Are you ready? Please be open to respond, okay?*

Raja: *Okay, sir I am ready.*

Researcher: *Raja, what is your emphasis when you solve problems in algebra?*

Raja: (with soft voice and being nervous). *I am not good in mathematics. Last year, I hardly passed the exam but failed in mathematics. However, regarding your questions, I would like to learn mathematics by using step by step approach more and concept very little. I try to learn solutions. I do one or two steps and I forget again. I forget to do the steps all the time. The teacher said that I needed to memorize the formula and steps to make it error free. But what to do I don't enjoy memorizing things. So, I spent my time playing with friends or sometimes I read other books.*

Researcher: *Oh! That sound little strange. So, do you get equal support from your teacher while learning mathematics?*

Raja: *No, it is definitely not. They moreover emphasize their so-called talent students. We have 49 in the class and how is it possible to give equal chance to all. So-called talented students will ask the questions frequently with teachers and they do not give us chance to ask. On the other hand, we too do not show daring in asking questions. It is because it is sure that our teacher scolds in the name of failure. So, disgusting nah sir?*

Researcher: *Yes, it is a bad habit. What do you think about your teacher' instruction in algebra? Does he/she use contextual examples? What about teaching materials? Have you ever got project and field work as a home assignment?*

Raja: *Sir, most of the time my teacher just write the solution and formula on the board and we extract as it is. Another thing is; sometimes he uses contextual examples (I think once in the verbal problem solution unit about father and son's age). Regarding* *project work and other practical work, we never had such things in algebra learning, sir.*

Researcher: *Okay! Thank you so much Raja.*

Raja: *Thank you sir.*

Interview with six participants were a great and unforgettable moment and one of the achievements for me. I was wondered and astounded by some of their responses in the interview that was quite matched with my personal experience. When I was a student in the middle grades of my schooling I was encountered with two or more mathematics teachers. I had different mathematics learning experience with them as each individual has a distinct version of the style. So, I was filled with a different style of learning mathematics. Out of these teachers, some of them used to emphasize more on learning steps and procedures so that we could execute them correctly. For example, in the solution of finding the value of x in $2x - 5 = 7$.

Step 1: Keep the variable terms to the left hand side means add (+5) to both sides to get 2x on the left.

$2x - 5 + 5 = 7 + 5$

Step 2: Now, cancel (-5) and (+5) to get $2x = 12$

Step 3: Divide both side by 2 to get only x on the left.

i.e. $(2x)/2 = 12/2$

Here, I sometimes got confused doing directly that if $2x = 12$ then what to write is it $x = 12/2$ or $x = 12 - 2$ (laughing! Huh!!)

Step 4: Now, at last we get $x = 6$ which is the correct answer.

I think this is too mechanical steps to perform to get a problem solved. However, some of the teachers taught me/us to learn mathematics knowledge through manipulatives and concrete object so that we could learn them with deep understanding. Here, I have developed two types of approach of problem-solving but not distinctly, both processes were going together, hand to hand. In the same way, the respondents in the interview expressed the same thing that they would like to learn mathematics in both ways. It shows that students' interest is to go with both skills and understanding. Nevertheless, there is a question; do all the students learn mathematics in the same way? The answer is; yes. Since these both knowledge are not separated, everyone develops both type of knowledge (Rittle-Johnson & Alibali, 1999). However, it depends upon how one is emphasizing the knowledge development process. As a teacher of mathematics, there exist some students whose major focus is on only one part of knowledge construction and very minor focus on another. For example, in this interview, one student had expressed that he studied mathematics with the help of solutions done in the guidebook or the example done in the 'solved problems' in the exercise book. He then totally extracts the steps one or many times and learns how to solve similar types of problems. On the other side, if there is given different problems in the exercise, he stopped and could not go further as there would not be any rules to do it. He also has the conceptual knowledge in this process to some extent.

Another problem is that there is no proper space for the discussion inside the classroom. Students are busy with their personal stuff these days. I came to know from the interview that students talk with friends while teachers are teaching in the classroom about activities in social media such as Facebook, YouTube, tweeter and other means. They spend time with friends when there is no class by chattering about different movies and serials. It is seen that, students in the front row or who are called talented only discuss the problems of mathematics. Some of the respondents expressed that there are discussions about mathematics problems when there is an examination near them. Therefore, it concludes that students do not spend time in critical discussion and group discussion.

Next thing is about mathematics teachers' instruction approaches inside the classroom and support to build the knowledge. They had the similar expression to this issue as well. Their teacher emphasizes both processes of knowledge development, through the process as well as concepts. However, their major focus is always on memorization of formula, steps and skills. Their teacher sometimes uses the teaching materials, pictures and diagrams while teaching. They responded that their teachers do not majorly focus on providing and creating examples related to their own daily life as well as they very rarely encourage students to create examples. On the other hand, respondents expressed that they do not have mathematics workshop, fieldwork and projects work inside and outside the classroom.

**Teachers' Reflection in Developing Students' PK & CK in Algebra**

Now, this section deals with teachers' reflection about the situation of PK and CK in algebra. The interview in a group was done with four teachers of different government schools of Kathmandu metropolitan city. These four teachers were Paudel, Shimkhada, Kedar and Nidhi. All the four teachers were invited at a particular place to meet and for the discussion. They were well informed previously about the issues and topic of discussion. Among these four individuals, Paudel sir was a coordinator of mathematics in his school and had 15 years of teaching experience in teaching government schools. Shimkhada sir had 10 years, Kedar sir had 7 and a half years and Nidhi mam had 5 years of teaching in government schools. They shared their opinion about what is happening inside/outside the classroom and to teaching the meaningful concept of algebra in middle-grade students. The group discussion was done and audio was recorded so as to transcribe their voice later. Here the conversation begins:

I: *Namaste to you all.*

All of them greeted me by saying ‘Namaste’

Researcher: *Well, first of all, thank you so much for managing your time and being here.*

All of them: *You are welcome sir. And thank you so much for providing this opportunity to us.*

Researcher: *I told you all about the purpose of this study. Let us have some meaningful discussion on the issues that I am going to raise. First of all, I would like to share the result of survey. Sir, my analysis found that our students' have a lower level of conceptual knowledge comparing procedural skills in algebra.*

They were also surprised but not so distinctly because they are familiar with the situation of students in learning algebra. On the basis of this discussion, the following themes are generated regarding PK and CK constructions in algebra.

**Teacher Wants Their Students to build PK as well as CK**

Researcher: *Well, what is your major emphasis while teaching and learning the concept of algebra?*

Kedar: (gathering confidence) *I put my emphasis on learning of both concepts and procedures to grab the proper understanding of algebra. What do you other think?*

Other teachers agree with the view of Kedar sir.

Researcher: *Well, Paudel sir, do you want to add something to it?*

Paudel: *Of course, sir. It is true that knowledge in algebra are most often considered as abstract in nature. At times, it is too vague to clear the concept because of abstract symbols and expression. Even teachers feel difficult to understand the core concept.*

*Many times, I felt challenged to clear the concept of indices and solving polynomial equations. So, in these lessons I prefer, to teach rules (in indices different rules using chart paper), formula and steps so that they do not do wrongs while solving the problems.*

Researcher: *I agree with you, Paudel sir. It is because as a teacher in the middle graders' students, I feel tough to build the concept of long divisions into students so that I encourage them to memorize steps such as first factoring the expressions of numerator or denominator, and cancelling the like terms etc. Moreover, we give emphasis students to build both knowledge.*

Nidhi mam: *As you said Netra sir (*I told them that our students are weak in comparing the quantities in algebra such as 1/x and $\frac{1}{x^2}$ most of the students are below the average to compare these types of things). *Some extra talented students who are above the average create their own method to understand them but it is not similar to all. Also, there is a problem of representation in pictorial form. Here, we can present the concept of fraction and decimal with the help of number line but what about algebraic expression. I think, we do not find materials to teach such concept.*

Others agree with her perception. I can understand her issues.

The iterative learning process model can be appropriate to understand the core concept and to solve the problems in algebra. For example, the core concept of factorization should be constructed with the help of pictures and diagrams such as to represent the factorization of quadratic expression, we can use the area of rectangular shape with length and breadth. At the time of concept construction of this matter, we can introduce the rules to factorize the expression. The iterative model is applicable in better understanding of mathematics concepts (Rittle-Johnson et al., 2001).

**Teacher's Instruction can be a key Factor to Develop CK**

The approach that mathematics teacher use to teach mathematical content matters in construction of knowledge. The following discussion clarifies the importance of teacher's instructional strategies in knowledge development.

Researcher: *Well, let us talk about our instructional strategies inside the classroom. What do you think about teachers' instruction in promoting conceptual knowledge of algebra?*

Shimkhada sir was waiting his turn and he said,

Shimkhada: *This can be a reason. Teaching approaches are also the influential factors in building the conceptual and procedural knowledge of mathematics.*

Kedar: *I think, we have similar types of teaching approaches inside the classroom that is most of the time the approach is lecture including inquiry with students, and sometimes teacher – students' discussion.*

Nidhi mam: *There is a problem in the discussion, collaborative teaching and demonstration inside the classroom because of time constraint to cover a wide range of contents and number of students. In collaborative and co-operative approaches to teaching every student gets chance to learn the concept with the formation of the group. But what to do?*

Shimkhada: *You are right Nidhi mam. In these teaching strategies, students get the opportunity to perform a task or accomplish a task until he/she understand and there is continuous facilitation and support of teacher. Although teachers were familiar with the concept of these teaching approaches, they are unable to use them in their own context.*

Paudel: (Being confident). *Well, Netra sir we are somehow familiar with these approach but what to do? Because of time constraint and big number of students, we are unable to use these approaches.*

Others agree with his view.

Teacher's instruction inside the classroom matters. We can see visualize the scenario that almost every time students are dependent upon the teacher's instruction of teaching the concept. In algebra lesson also, students need the continuous facilitation and encouragement from teachers to create the concept. Similarly, the discussion inside and outside the classroom can be a meaningful strategy to develop conceptual knowledge in algebra. Based on my experience as a mathematics teacher, most of the time, I encourage my students to take an active participation in the discussion. This strategy in teaching reduces the distance between teacher and student which is more applicable to understand student's problems and ability in learning mathematics. I let them put their understanding about the matter without hesitation and encourage them to reach their highest potentiality. For this, teaching and learning approaches such as collaborative, co-operative, inquiry based, project based etc. which encourage us for critical discussion can be implemented.

Similarly, teacher should spend more time with students to develop better understanding in not only algebra but other contents of mathematics. A research done by Rittle – Johnson, Fyfe and Loehr with 180 second grade children in the United States by using a randomized experiment in which children received a classroom lesson on mathematics equivalence. This research has concluded that within a single lesson, spending more time on conceptual instruction may be more beneficial than spent teaching a procedure (Rittle – Johnson, Fyfe & Loehr, 2016).

**Real life Examples can Enhance Deeper Understanding, Challenging Though**

As a teacher and a student of mathematics, I am inquisitive to find the real world meaning of mathematical concepts. I try to relate the concepts like profit and loss, simple interest, linear equations and so forth to my day to day life. This takes a lot of time and effort but once we are able to relate the concept then the next aha! (Eureka) moment could not be expressible. The following discussion presents the importance of real life examples in mathematics teaching and how much it is difficult for teachers to construct.

Researcher: *Sir, I think, the other part or condition can be uses of contextual examples and daily life-related problems with every concept of algebra in developing students' conceptual ability, is not it?*

Teachers: *Yes, it is.*

Researcher: *If it is so, could you put your opinion on this?*

Nidhi Mam: *Sir, we have faced students from various and diverse community and cultural practices. Considering and creating an example with respect to each practice is almost impossible for a teacher.*

Shimkhada: (Stopping Nidhi mam). *Sir, I want to add to this, the creation of a representative example is also a challenging job. Perhaps, I felt that teachers are trying to use concrete and real-life examples in algebra lesson.*

Paudel: *I try to use real-life examples when I teach some basic concept like variables and constant such as the amount of consumption of water in daily life (a variable) and time in hours (constant). Students naturally try to construct and relate them to their practice. So, we need to do it regularly and make a habit to teach meaningful mathematics.*

Kedar: *Nevertheless, what about creating real-life examples in factorization, solving the polynomial equation of degree two as well as in indices. I think it is challenging.*

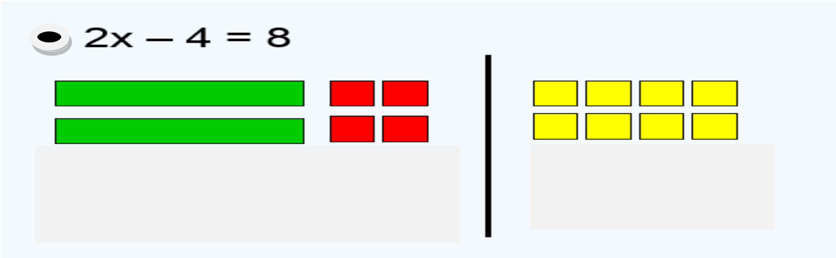
Researcher: *I agree with you, sir. As a mathematics teacher, I am facing the same difficulties because of an abstract concept in algebra, teachers are unable to create daily life-related problems.*

**Use of Teaching Materials can Enhance Conceptual Knowledge**

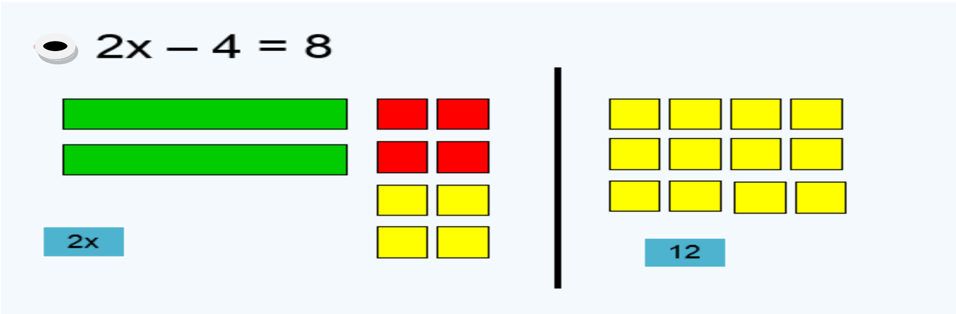
It is widely accepted that the importance of teaching-learning materials in mathematics teaching could not be expressible in a sentence. The effective and concrete manipulatives help construct better understanding of concept. In my personal view, the use of materials is essential to enrich CK in mathematics. Let us have the following discussion.

Researcher: *Well, let us talk about another fact when we use manipulative, teaching and learning materials that directly influence in building conceptual and procedural knowledge of algebra. In my career of teaching in middle grades, I thought that it is the building block for the grades 9 and 10 and definitely for SLC/SEE and other higher grades. So, I thought students should build the clear-cut concept of every mathematical knowledge for meaningful mathematics learning. For this, concrete teaching aids help build the real understanding of an abstract concept. They promote in effective learning of the concept. When I was a teacher in middle grade, I used to take help of teaching materials to build the concepts such as* $a^2 - b^2$, $(a + b)^2$, $a^3 - b^3$ *and so on. I used to use algebra tiles to build the concept of addition, subtraction, division, multiplication of algebraic expressions. Similarly, to clear the concept of factorization, solving the linear and quadratic equations, I used to use algebra tiles. Here is one example I would like to share with you; in algebra tiles, there are two colours. Let us denote red color as negative (-) and yellow color as positive (+). The* *area of square having length x is $x^2$ (yellow) and $-x^2$ (red), rectangle having lengths x and 1 is x (yellow) and –x (red), and unit square 1 (yellow) and -1 (red).*

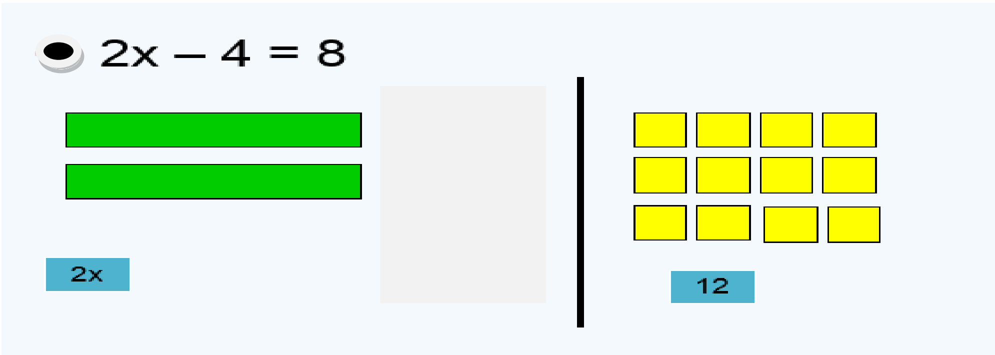
- Fist set the algebra tiles model as shown in the figure.




- Next, to make 2x alone add 4 positive unit tiles to both sides



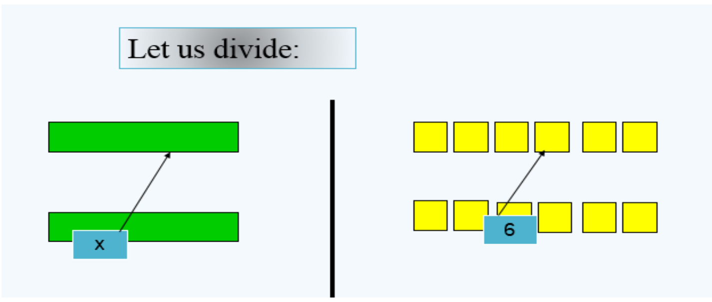

- Now, take away 4 negative and positive tiles from the left side. Then, we get




- Let us divide, at last we get

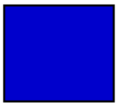
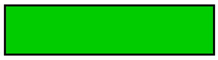
6 equal units for 2 same x.

In the diagram, the equal each x has equal 6 units in the right hand side.



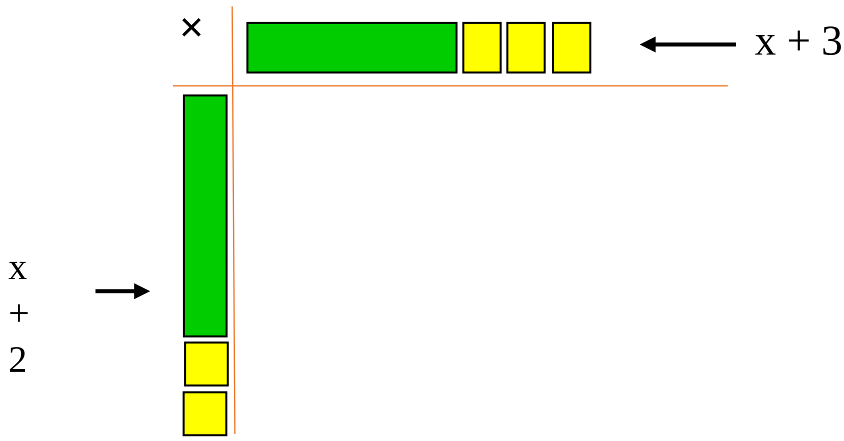

Hence, x = 6.

Another example is for multiplying (x + 3). (x + 2)

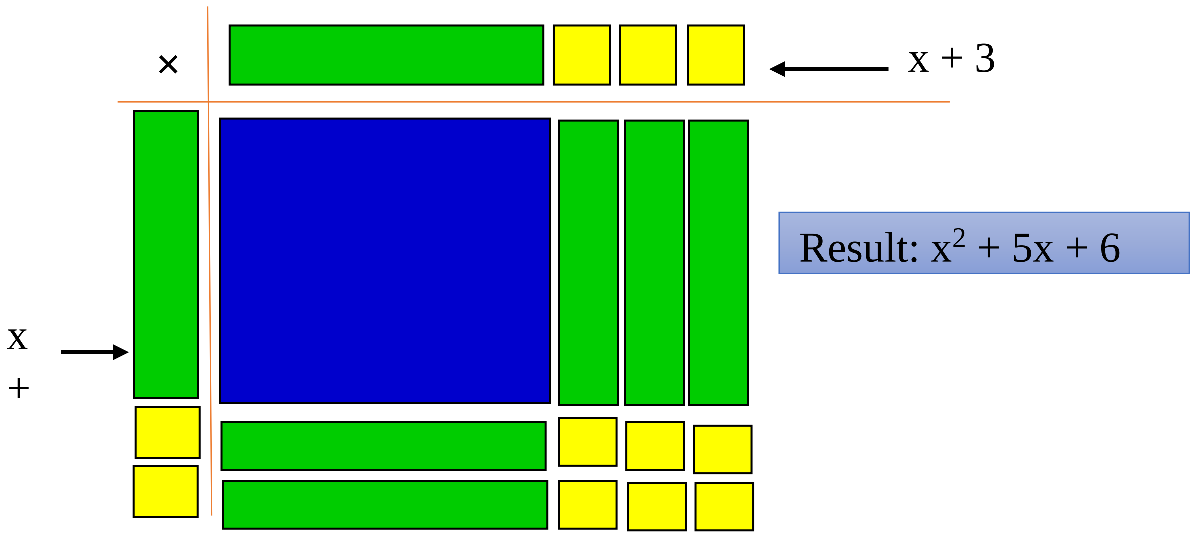
Requirement:- Square tile:  Rectangular tiles: 

Unit: 

First, set the two expression in the product mat as follows.




- Now, multiply each tile of column and row and input the tiles in the result section. For example, x.x = $x^2$, x.1 = x, 1.1 = 1and etc. as in the following diagram.




∴ (x + 3). (x + 2) = $x^2$ + 5x + 6

In this way, we can teach some of the major knowledge of algebra through algebra tiles. Have you ever used this?

All appraised me. And they said, *"Yes, sometimes. But we have not used this material yet in the middle grades."*

Paudel: *It's really a great thing sir. I think we need to learn something from you. However, using teaching materials is not everyone's cup of tea. There are many emerging problems.*

Shimkhada: *Exactly sir. First of all, we are not trained to use all the materials so that we sometimes use teaching materials in algebra lesson.*

Nidhi: *I mostly use teaching materials like the model of* $a^2 - b^2$, $(a + b)^2$, $a^3 - b^3$ *etc. and number line chart.*

Kedar: (with surprised voice)! *What are you saying? Teaching materials in algebra teaching? I do not think so because we can use teaching materials except the models of* $a^2 - b^2$, $(a + b)^2$, $a^3 - b^3$ *etc. Only these, how can we maximize students' conceptual knowledge?*

I added, "*The condition of using teaching materials in algebra lessons is almost pathetic in the government schools of Nepal because there are always the issues of funding, material management, well-trained teachers to use and teacher professional training. Because of these reasons people are making and taking algebra as more complex, complicated and abstract concept."* They all agree with me.

The use of teaching materials in algebra teaching is essential to construct the understanding the concept of mathematics. Similarly, effective and concrete teaching learning materials should be used to develop knowledge of algebra. Among the teaching materials, ICT tools in mathematics teaching is considered widely as one of the most innovative and creative approaches. This approach is essential to develop necessary knowledge and skills as well as to re-shape the viewpoints of mathematics (Koparan, 2017). ICT tools such as Geo-gebra, Google-Sketchup and other tools are useful to construct better understanding of mathematics. These are useful to create knowledge through audio-visual medium so these are applicable for both teachers and students.

**Project work, Practical work and Field are the Other Major Issues in CK**

I believe that project, practical and field works are the phenomenal things to be implemented in mathematics teaching-learning that encourage pupil to develop conceptual knowledge. The following discussion presents the importance and the situation of the use of these things in our mathematics teaching and learning.

Researcher: *Let me talk to you about another key emerging issue in building conceptual knowledge in algebra. It is the frequent implementation of project work, field work as well as practical work activities. Could you please put your ideas on it?*

Paudel: *It is the most important issue to promote conceptual knowledge in algebra.*

Shimkhada: *Sir, I have no experience of practicing these kinds of stuff inside/outside the classroom.*

Nidhi mam*: It is true sir. First, we need proper teacher training on this matter. Project and practical kind of things are vigorous but rigorous to conduct and assess.*

Kedar: *We do not have a concrete plan to do it. School-related bodies did not show their interest to implement and government also does not have the strong plan.*

Paudel: *These things are true and stop doing practical work. However, making project-based learning in algebra is too much difficult. Another thing is no proper space for project work kind of things given by curriculum.*

Others agree with his perception.

The proper and continuous use of these techniques and ideas support students' holistic development not only educational development. In my opinion, these things are essential to make the abstract concept of algebra more meaningful and fruitful. It is viable that every student gets benefited by these approaches. Project-based learning is an approach that puts students in a position to use the knowledge that they get, effective in helping students understand, and retain information. It includes building skills such as reasoning, critical thinking, communication and collaboration as well as cooperation (Stivers, 2010). In this sense, PBL is a dynamic approach to teach students to explore real-world problems and challenges, simultaneously developing 21st-century skills while working in small collaborative and cooperative groups. I have experienced that schools are rarely using projects and practical work as there are many obstacles like a time-consuming in creating, thinking, planning as well as assessing the project is really a challenging job. My research shows that many schools deny the concept of project or research base assignments in mathematics as they think that they need trained or skillful teachers, good financial condition and many more. It is more rigorous when it comes to algebra. At last, I thanked them for their valuable time.

The above discussions, analysis and interpretation clarify why our students in algebra have lower conceptual knowledge in algebra. They hate mathematics as they think that it is not useful in daily life and very boring in comparisons to other subject matter. They feel algebra is more challenging most of all as it has not any sense practicality, no real-life meaning and application, less interesting like a poem, story and other things. They feel detachment of mathematics and their real world. Similarly, the teachers' emphasis of developing skills of memorizing steps, formula; their teaching strategies and support to students inside and outside the classroom, poor use of teaching manipulative concrete materials, audio-visual aids and contextual related problems and examples; almost no use of practical work, project-based works, mathematics related activities are the reasons that our students are below the average in conceptual knowledge.

## Chapter Summary

This chapter has incorporated the entire analysis and interpretation of both forms of data; quantitative and qualitative. The survey data analysis and interpretation were done with the help of SPSS and results were able to address the first research question. This means students had a lower level of CK in comparison to PK. Students are better in PK than CK in algebra. However, the average positive relation was found between PK and CK which points out the fact that one leads to the positive development of other. On the other hand, the two-phase interview with students and group discussion with teachers were presented which were able to address the second research question about why our students have lower CK in algebra. The first phase interview revealed that our students are weak in both implicit and explicit conceptual knowledge. The second phase interview and a group discussion with teacher analysis revealed that the major reasons behind the lower CK are; students' disinterest towards learning, teachers' major emphasis on PK, teachers' one way instruction approaches (almost no room for discussion inside/outside the classroom), minimum or no use of real life problems and examples while teaching and learning, very less use of manipulatives and effective teaching materials and very less or no use of project, practical as well as field works in teaching and learning algebra.

# CHAPTER V

# FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

In this explanatory sequential mixed method study, at first quantitative study was done with the help of survey and next qualitative study was done to help explain, elaborate the quantitative finding as well to figure out the possible reasons behind the result of the first phase. The main purposes of this mixed method study were to measure the level of procedural and conceptual knowledge of grade eight students' in algebra as well as to explore why students develop such (procedural or conceptual) knowledge in the domain of knowledge. For the survey study, 360 eighth grade students were selected randomly from nine different schools. Four procedural knowledge-based test questions and twenty conceptual knowledge based multiple choice questions were developed with the help of subject experts, teachers, mathematics teacher educators, curriculum experts and other experts to use them in the survey.

Similarly, in the second phase six students were selected from two different schools to understand and verify their response in the survey; to understand the actual reason of their understanding in the survey. And in the third phase, two different interviews were conducted with six students form two schools and 4 mathematics teachers to understand the possible reasons regarding the finding of the quantitative analysis. Now, this chapter establishes the discussion on quantitative findings, qualitative findings and integration of both findings. Similarly, the ultimate conclusion from this entire study is discussed. Moreover, this chapter incorporates the recommendations at the end.

## Quantitative Findings

After obtaining the required data and responses from 360 sample respondents from nine different schools, data analysis and interpretation were done with the help of 23rd version of IBM SPSS software. The major purpose of quantitative data collection and analysis was to identify the level of students' conceptual and procedural knowledge in algebra. Similarly, the aim was also to figure out the relationship between these two knowledge, level of student's procedural and conceptual knowledge with full respect to their gender, their father's and mother's education as well as their parent's occupation. Therefore, according to quantitative data analysis and interpretation, we came to the following major findings:

- Conceptual knowledge of students' is comparatively low and below the average but procedural knowledge of students' in algebra is high. The mean mark of students in procedural knowledge was 14.05 with standard deviation 6.344 whereas the mean mark in conceptual knowledge was 8.56 with the standard deviation of 3.912. Therefore, students are good in procedural skills in algebra and they are weak in the conceptual part.
- There is an average positive ($r = 0.559$, $p < 0.05$) relation between procedural and conceptual knowledge of students in algebra which is moderately positive. This correlation shows that one type of knowledge development help support the positive development of the other and vice-versa.
- Regarding gender influence in students' conceptual and procedural knowledge in algebra, there was a significant difference between the mean marks of students in procedural knowledge. Boy students were better than girl students in the procedural skills of algebra. It means boys were better in applying step – by – step procedures to solve problems. In contrast to this, there was no gender influence in

the development of conceptual knowledge in algebra. It enlightened us that level of conceptual does not differ by the gender of students.

- This research clearly clarifies that father's and mother's education influence in students' conceptual and procedural in algebra, there was no effect of parents' level of education in the level of procedural and conceptual knowledge of algebra. In this study, eighth grader students' procedural and conceptual knowledge in algebra were independent of six different level of their parents' education – literate, SLC, +2, Bachelor, Master and Other.
- At last, parents' occupations did not influence the level of procedural and conceptual knowledge of students in algebra. It concluded that whatever students' parent's occupations are, they have about the same level of procedural and conceptual knowledge in algebra.

Therefore, the quantitative finding shows that this research study is able to address the first research question.

**Qualitative Findings**

The first phase interview with six sample responded of two schools selected through purposive sampling method that verified students’ true level of conceptual and procedural knowledge in algebra. The analysis was done in chapter IV. It clarified that students have the low level of CK in comparison to PK in algebra. They are comparatively weak in providing different examples of a concept, defining the concept, comparing two or more quantities in algebra, the representational ability of a concept with figures as well as critical thinking. It disclosed the fact that students have lower cognitive ability. On the other hand, in the second phase interview, after obtaining the data from six students who participated in the first phase interview and group discussion with 4 mathematics teachers, the analysis and interpretation were done. This interpretation highlighted the following qualitative findings:

- Students were influenced and encouraged to generate procedural skills, memorize formulae and steps to solve the problems. Most of them felt that mathematics is more difficult in comparison to other subjects. They felt that they are below the average in learning mathematics. They did not see the practical use of mathematics in their living life. So, most of them study mathematics with the wishes of just to pass the examination.
- They spend more time on learning procedures rather than understanding the underlying concept of algebra. They did not have the critical discussion inside and outside the classroom about mathematics and particularly about algebra.
- Students raised their voice on teachers' instruction and support while learning algebra. Most of them did not get the equal opportunity of learning mathematics inside the classroom. Psychologically, average and below the average students feel inferior when their teachers emphasize talented students inside the classroom.
- Most of the time, there is a rush in students to just pass the examination. They all have a kind of anxiety or fear of being failed. In this situation, most of them are trying to learn and memorize steps so that they leave the conceptual part of learning.

On the other side, group discussion with teachers highlighted the following key points that why students are having lower conceptual knowledge in algebra:

- Almost all teachers wanted their students to learn both concepts and procedures of mathematics learning. So, the emphasis is on both type of knowledge construction.

- Regarding algebra, because of its abstract nature, they are automatically forced to teach to construct PK with the minimum focus on the underlying concept.
- Teaching methods/approaches are the prevalent factors in the construction of conceptual knowledge in algebra. However, still, our teachers in government schools are forced to implement lecture and sometimes inquiry methods in teaching mathematics. Because of lack of well-trained teachers, poor management of teacher training and professional development programs, a heavy amount of contents, large number of students as well as school-related issues affect the implementation of collaborative and co-operative approaches and other child-friendly approaches. In this genuine situation, teachers are in a rush to finish the course in the allotted time.
- Another reason that stops teacher constructing a wide range of conceptual knowledge in algebra is; students from the different community practices. Students can learn concepts of mathematics when they play with problems related to their day to day life using more contextual examples. However, in the context of Nepal, it is very challenging to create problems and examples representing each community practice of students. So, creating representative examples in algebra lesson to represent each student's practice is almost impossible for all the teachers.
- The other factor affecting and stopping the creation of conceptual knowledge of algebra into students is the use of teaching materials and manipulatives. Teaching the concept of algebra needs a greater amount of concrete materials. Effective use of ICT technology can help build the conceptual knowledge in algebra. But, there is the problem around it. Teachers take it as a difficult task to construct manipulatives for each concept and use it inside the classroom. Therefore, teachers are forced to encourage students to learn steps and procedures to solve

problems in algebra rather than using and constructing teaching materials to teach the conceptual part.

- Project-based learning, fieldwork and practical works are considered the weapons/mediums of constructing a real and authentic understanding of mathematics. During the time of research, very fewer mathematics teachers were founded using teaching material. The rare and poor use of these things can be a reason why our students are weak in the conceptual part of knowledge.

## Conclusions of the Study

Procedural and conceptual knowledge are the major focus in teaching and learning mathematics throughout the world. This is a debate among the people from the early 80's to till now about which is important, which we should give more emphasis as well as the debate of the relationship between these two types knowledge. In the context of Nepal, this has been an issue of discussion in the construction of knowledge of mathematics. Algebra is a part of our mathematics curriculum from the lower primary grades to university level. In the teaching and learning field, Nepal also gives the major emphasis on learning algebra in all grades. However, the abstract nature of algebra, makes students feel more difficult matter in every grade. On the other hand, teachers of mathematics take algebra as a difficult matter to teach in comparison to other matters. Students are failed to conceptualize the different concept of algebra.

Middle grades in schooling are considered as the backbone of knowledge construction in Nepal. Students get chance to reshape their knowledge of mathematics in these grades. Coming to algebra, it is important that students should learn the core and deep concept of algebra because after the accomplishment of these grades they do not have enough space to construct conceptual part of the knowledge of algebra in higher grades because of the recent content nature of mathematics.

Knowledge construction process differs from a person to person. There are people who can learn mathematics with the help of algorithmic step-by-step procedures and there are the people who can generate mathematical knowledge with deep understanding of the concept and here is a major thing, everyone has both the conceptual and procedural knowledge. However, the amount of each type of knowledge depends upon how an individual is constructing knowledge, his/her past experiences as prior knowledge as well as the environment of learning mathematics. People who are deviated to construct PK they almost have greater PK of mathematics in comparison to CK. Similarly, people who are deviated to construct the underlying concept of every mathematical knowledge have the greater CK.

According to my experience, we should generate both types of knowledge but the major emphasis should be given to constructing conceptual knowledge. More conceptual knowledge of any concept of mathematics help people to build the procedural skill of solving problems. So, the researchers, cognitive psychologist as well as teachers suggest constructing deeper and realistic understanding of concept so that learning becomes meaningful in mathematics to understand the knowledge and develop procedural fluency.

As a teacher and a student of mathematics, I have faced so many things in teaching and learning scenario of mathematics. As a student of mathematics, I was good in mathematics with the greater amount of PK and minimum amount of CK. Similarly, as a teacher of mathematics, I have been repeating the same thing as my teacher used to do with us. There are real problems in teaching and learning mathematics. Coming to algebra, because of its use of abstract symbols, variables as well as expressions; there are challenges to build a proper understanding of each underlying concept; concepts such as factorization of any expression, indices, basic mathematics operations in algebra in middle grades, solving linear and quadratic equations etc. Behind these challenges, there are the reasons. The very first reason is heavy content nature in algebra. Another thing is our teaching instructional strategies which emphasize more on examinations rather than the understanding. Similarly, less use of teaching materials, trained to use only duster, marker, pen and delivering lectures to solve the mathematical makes it more complicated and terrific. We just force students to memorize formula, steps and problem-solving methods. If they are not able to do so, we put ours blames towards them in the name of unintelligent fellow. Another big problem I have faced is the problem of practicality in mathematics teaching. Around 95% of public schools do not use project, practical and field work kinds of stuff in teaching and learning mathematics.

This explanatory mixed-method research study was able to address the aroused issues about conceptual and procedural knowledge of students in algebra. Similarly, the study was able to provide possible reasons to improve our teaching and learning process as well as the prioritization in knowledge development.

## Recommendations

According to all the data analysis, finding and interpretation; I, as a researcher, in this stage has some significant recommendations to teachers, policy makers and curriculum developers and including further research possibilities.

### Recommendations for the Teachers

The finding from the quantitative data analysis and interpretations shows that the level of conceptual knowledge of our students in middle grades is low and the level of procedural knowledge is high in algebra. This clarifies that our students are provoking to use algorithmic stepwise procedures and formulas to construct mathematical knowledge in algebra. Students are weak in constructing the meaning of core ideas, understating the value of knowledge, creating an innovative approach to learn algebra and they are weak in critical thinking. Students have a poor representational knowledge and comparison ability in algebra. Thus, in this phenomena, we as a teacher and facilitator of mathematics should understand the scenario and understand that how much conceptual knowledge is important in building the realistic and fallible understanding.

Another thing is the way of teaching. It is called instructional approaches that we have been frequently implementing to construct the knowledge of algebra inside and outside the classroom. Different research studies have claimed that effective implementation of collaborative approaches, inquiry-based approaches, and co-operative learning approaches are the phenomenal factors to create real understanding into students. We should accept that these are the magnificent tools in constructing conceptual knowledge in algebra.

Similarly, the proper use of instructional materials helps students understand the concept in algebra. As we know that algebra is a kind of abstract in nature and there are always challenges to create teaching materials and manipulatives to all the concept of algebra. But we can try to be innovative and creative to construct related teaching materials. We can make algebra teaching meaningful with the proper and effective use of ICT. Similarly, we should support every student in learning algebra by giving equal chances to present their thoughts and encourage them to be more creative learner. This can be done by introducing contextual and daily life-related problems in every algebra class.

Another thing is to implement the project, fieldwork and practical work in our teaching. Through these approaches, students can understand the real-life application of algebra. Then, they themselves will be ready to take challenges in learning. Obviously, these things make abstract knowledge of algebra practical and reduce the negativity of students in learning algebra.

**Recommendation to Policy Makers and Curriculum Developers**

As we can see, our students in government schools are below the average in conceptual knowledge of algebra than procedural knowledge. Both are equally important in problem-solving, however, the major focus should be given to constructing conceptual knowledge in algebra. But there is a similar situation in other parts of the mathematical concept too. Here the policymakers should be aware of making new policies and implement them in schools primarily in teaching and learning context. They need to think now for making creative and applicable policies to make mathematics teaching more meaningful and prioritize the conceptual knowledge part of every mathematical knowledge.

Similarly, they should be aware of making teaching and learning policies which effectively support the conceptual part of knowledge creation and development without neglecting procedural part also. There should be implementation of ICT policies, creative and innovative teaching methods, use of effective teaching aids, project-based, practical based as well as fieldwork based teaching policies to get the desired result.

On the other hand, curriculum developers of mathematics need to think on the matter while making the curriculum of mathematics. They should be aware of including representational knowledge (figures, diagrams, graphs, pictures etc.) in every concept of algebra. They should include the example or problems which are directly related to our day to day life and contextual examples in every concept of algebra. People are making algebra more and more complex and theoretical. So in such case, curriculum developers need to think about the strategy of making algebra more fallible, viable, fruitful and meaningful. In the same way, proper directions for teachers with appropriate procedures should be included. Similarly, the curriculum should include at least one project work in each of the concepts such as in algebraic expressions, factorizations, indices and others. At last, they need to think of a concrete plan to construct and promote conceptual knowledge together with procedural knowledge in learning mathematics.

**Recommendation for the Further Research**

As a researcher, we cannot include all the things in a study. We cannot address each and every perspective in a single study. In the same way, this research study is unable to cover all the emerging things and issues around conceptual and procedural knowledge of algebra. I have some recommendation for the further study:

1) This study was completed to measure the level of eighth grader students' procedural and conceptual knowledge in algebra. Therefore, the future researcher may choose other broad areas such as whole mathematics in grade eight, may choose the whole lower secondary level or other school levels. Similarly, he/she can scrutinize the other area not only to measure the level of knowledge.
2) The issue of conceptual and procedural knowledge in mathematics is a very wide and broad area to cover. However, this study could only cover students' level of this two knowledge through this mixed method study. So, the future researcher can do the same research using considerable time.
3) Because of time and financial constraints, this research study could incorporate the sample respondents of public schools of Kathmandu metropolitan city, Kathmandu. In

this situation, there are some circumstances to generalize the results and findings to the broader population may be to the whole country or the whole world. So, the future researcher can take a large sample so that the result could have been generalized to a wide range of population.

4) This study was able to find the positive relation between PK and CK but not able to find the possible reasons regarding this relation. The future researcher can carry out a study to figure out this.
5) As a researcher, in Nepal, I felt that probably there is no separate research study had been done in this area. I have put my enthusiasm and great effort to accomplish this study. Therefore, for the future researcher, the issue of procedural and conceptual knowledge of mathematics can be a place to show your endeavor.

## Chapter Summary

This is the ultimate chapter in this research study. It has included the discussion of major finding from both firms of data collections and interpretation, the major conclusion of the study with researcher's experience as well as the recommendations for the teachers, policymakers and curriculum developers. In addition, the future possibility for the researcher in this area has been included.

# REFERENCES

Abd Rahman, Z. (2006). *Mastery of algebraic concepts and attitudes towards algebra among secondary school students* (Unpublished Master of Education Project Report). Universiti Kebangsaan Malaysia.

Baki, A., & Kartal, T. (2004). Kavramsal ve işlemsel bilgi bağlamında lise öğrencilerinin cebir bilgilerinin karakterizasyonu. *Türk Eğitim Bilimleri Dergisi*, *2*(1), 27–46.

Belbase, S., Panthi, R. (2017). Teaching and Leaning Issues in Mathematics in the Context of Nepal. *European Journal of Educational and social Science*, 27

Byrnes, J. P., & Wasik, B. A. (1991). Role of conceptual knowledge in mathematical procedural learning. *Developmental psychology*, *27*(5), 777.

Cai, J., Lew, H. C., Morris, A., Moyer, J. C., Ng, S. F., & Schmittau, J. (2005). The development of students' algebraic thinking in earlier grades. *Zentralblatt für Didaktik der Mathematik*, *37*(1), 5-15.

Canobi, K. H. (2009). Concept-procedure interactions in children's addition and subtraction. *Journal of Experimental Child Psychology*, *102*, 131–149. doi: 10.1016/j.jecp.2008.07.008.

Chappell, K. K.,& Killpatrick, K. (2003). Effects of concept-based instruction on students' conceptual understanding and procedural knowledge of Calculus. *PRIMUS. 13*(1), 17-37.

Creswell, J. W. (2015). *Research design: Qualitative, Quantitative, and Mixed Method Approaches* (5th ed.). India: Pearson.

De Jong, F. (1987). Differences in the self-regulation processes between successful and less successful students and the prediction of learning performances in case of comprehension and learning of text. In P. R. J. Simons & G. Beukhof(Eds.), *Regulation of learning* (pp. 33–45). Den Haag: SVO-Selecta.

Egodawwatte, G. & Stoilescu, D. (2015). Grade 11 students' interconnected use of conceptual knowledge, procedural skills, and strategic competence in algebra: a mixed method study of error analysis. *European Journal of Science and Mathematics Education, 3*(3), 289-305.

Fuson, K. C. & Kwon, Y. (1992). Korean children's understanding of multi-digit addition and subtraction. *Child Development*, *63*, 491 – 506. Doi: 10.1111/j.1467-8624.1992.tb01642.x.

Gajjar, S., Sharma, R., Kumar, P., & Rana, M. (2014). Item and test analysis to identify quality multiple choice questions (MCQs) from an assessment of medical students of Ahmedabad, Gujarat. *Indian Journal of Community Medicine: Official Publication of Indian Association of Preventive & Social Medicine, 39*(1), 17.

Ghazali, N.H., & Zakaria, E. (2011). Students' Procedural and Conceptual Understanding of Mathematics. *Australian Journal of Basic and Applied Science, 5(7)*: 684-691, 2011. ISSN: 1991 – 8178 .

Habila, E. Z. (2017). "Prospective Teachers' Conceptual and Procedural Knowledge in Mathematics: The Case of Algebra." *American Journal of Educational Research*, *5*(3), 310-315. doi: 10.12691/education-5-3-12.

Hecht, S. A. (1998). Toward an information-processing account of individual differences in fraction skills. *Journal of Educational Psychology, 90*, 545–559. doi: 10.1037/0022- 0663.90.3.545.

Hiebert, J., & LeFevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and* *procedural knowledge: The case of mathematics* (pp. 1–27). Hillsdale: Lawrence Erlbaum Associates, Inc.

Hofer, B., & Pintrich, P. (1997). The Development of Epistemological Theories: Beliefs about Knowledge and Knowing and Their Relation to Learning. *Review of Educational Research*, *67*(1), 88-140. Retrieved from http://www.jstor.org/stable/1170620

Klein, D. (2007). A quarter century of US ‘math wars’ and political partisanship. *Journal of the British society for the history of mathematics*, *22*(1), 22–33.

Knuth, E. J., Stephens, A. C., MCNeil, N. M., & Aliali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for research in Mathematics Education, 37*, 297 – 312

Koparan, T. (2017). Analysis of Teaching Materials Developed by Prospective Mathematics Teachers and Their Views on Material Development. *Malaysian Online Journal of Educational Technology, 5*(4), 8-28.

Long, C. (2005). Maths Concept in Teaching: Procedural and Conceptual Knowledge. *Journal for research in Mathematics Education, 7*, 97 – 112.

Luitel, B. C. (2013). Mathematics as an im/pure knowledge system: Symbiosis, (W) Holism and synergy in mathematics education. *International Journal of Science and Mathematics Education, 10*(6), doi:10.1007/s10763-012-9366-8

Luitel, B.C. (2009). *Culture, Worldview and Transformative Philosophy of Mathematics Education in Nepal: A Cultural-Philosophical Inquiry.* Curtin University of Technology.

Marchionda, H. (2006). *Preservice Teachers' Procedural and Conceptual Understanding of Fractions and the Effects of Inquiry-Based Learning on this*

*Understanding.* (Unpublished Master Dissertations). Retrieved from https://tigerprints.clemson.edu/all_dissertations/37

Mertens, D. M. (2007). Transformative paradigm: Mixed methods and social justice. *Journal of Mixed Methods Research,1*(3).

Miller, S. P. & Hudson, P. J. (2007). Using evidence-based practices to build mathematics competence related to conceptual, procedural, and declarative knowledge. *Learning Disabilities Research & Practice, 22*(1), 47–57.

Miller, S. P., & Hudson, P. J. (2007). Using evidence-based practices to build mathematics competence related to conceptual, procedural, and declarative knowledge. *Learning Disabilities Research & Practice, 22*(1), 47-57.

Ministry of Education (MOE). (2015). *Report on national assessment of student achievement (NASA) 2013*. Bhaktapur, Nepal: The Author.

Ministry of Education (MOE). (2015). *Status report -2015*. Retrieved from: http://www.doe.gov.np/assets/uploads/files/5d346f4a67463dc1687d1a5568b6f100.pdf

NCTM. (2014). *Principles to Actions: Ensuring mathematical success for all.* Reston, VA: NCTM, National Council of Teachers of Mathematics.

Oxford Dictionary. (2005). *Advance oxford English Dictionary* (7th ed.). UK: Oxford University Press.

Pantziara, M., & Philippou, G. (2012). Levels of students' "conception" of fractions. *Educational Studies in Mathematics*, *79*(1), 61-83.

Preston, R. (2008). Mathematics anxiety: Research and implications for middle school students and teachers. *Conference Proceedings of the Masters in Teaching Program 2006-2008,* Olympia, Washington.

Richardson, F. C., & Suinn, R. M. (1972). The Mathematics Anxiety Rating Scale: Psychometric data. Journal of Counseling Psychology, 19, 551-554.

Rittle – Johnson, B. & Schneider, M. (n. d.). *Developing conceptual and procedural knowledge of mathematics*. Oxford university press.

Rittle-Hohnson, B., & Siegler, R.S. (1998). The relations between conceptual and procedural knowledge in learning mathematics: A review. In C. Donlan (Ed.). *The development of mathematical skell* (pp. 75 – 110). Hove, England: Phychology Press.

Rittle-Johnson, B. & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: does one lead to the other? *Journal of Educational Psychology*, *91*, 175-189.

Rittle-Johnson, B., & Schneider, M. (2015). Developing conceptual and procedural knowledge in mathematics. In R. Cohen Kadosh & A. Dowker (Eds.), *Oxford handbook of numerical cognition* (pp. 1102-1118). Oxford, UK: Oxford University Press. doi:10.1093/oxfordhb/9780199642342.013.014

Rittle-Johnson, B., Fyfe, E., & Loehr, A. (2016). Improving conceptual and procedural knowledge: *The impact of instructional content within a mathematics* lesson. *British Journal of Educational Psychology, 86.* doi: 10.1111/bjep.12124

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93,* 346–362. doi:10.1037// 0022-0663.93.2.346.

Ross, A. A. (2010). *The effects of constructivist teaching approaches on middle school students' algebraic understanding* (Doctoral dissertation, Texas A & M University)

Saunders, M., Lewis, P., & Thornhill, A. (2009). Understanding research philosophies and approaches. *Research Methods for Business Students, 4*. 106-135.

Schneider, M., & Stern, E. (n.d). Conceptual and Procedural knowledge of a mathematics Problem: *Their Measurement and Their Causal Interrelations. Max Planck Institute for Human Development Lentzeallee* 94, 14195 Berlin, Germany

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for information*, *22*(2), 63-75.

Star, J. R. (2005). Re-conceptualizing procedural knowledge. *Journal for research in mathematics education*, 404-411.

Stivers, J. (2010). *Project-based learning*. ESPY 505.

Taherdoost, H. (2016). Sampling Methods in Research Methodology: How to Choose a Sampling Technique for Research. *International Journal of Academic Research in Management, 5*(2), 18-27, ISSN: 2296-1747© Helvetic Editions LTD, Switzerland. Retrieved from https://www.researchgate.net/publication/31999824

Von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning.* Washington, DC: Falmer.

Welder, R. M. (2006, January). Prerequisite knowledge for the learning of algebra. In *Conference on Statistics, Mathematics and Related Fields, Honolulu, Hawaii*. Wits School of Education, University of the Witwatersrand.

Presented at the ICMI Regional Congress, University of the Witwatersrand, June, 2005.

Yilmaz, K. (2011). The cognitive perspective on learning: Its theoretical underpinnings and implications for classroom practices. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *84*(5), 204-212.

Zakaria, E., Yaakob, M. J., Maat, S. M., & Adnan, M. (2010). Conceptual knowledge and mathematics achievement of matriculation students. *Procedia-Social and Behavioral Sciences*, *9*, 1020-1024.

Zuya, H. E. (2017). Prospective Teachers' Conceptual and Procedural Knowledge in Mathematics: The Case of Algebra. *American Journal of Educational Research*, *5*(3), 310-315.

APPENDIXES

## Appendix – A

### Name of the Schools

Kathmandu metropolitan city

| SN | Name of Schools | No. of students (Grade – 08) |
|---|---|---|
| 1 | Mangala Devi Ma V | 31 |
| 2 | Guheshwori Ma V | 30 |
| 3 | Padmakanya Vidyashram Secondary School | 103 |
| 4 | Siddheshwor Ma V | 63 |
| 5 | Bhimsengola Ma V | 48 |
| 3 | Nepal Adarsha Ma V | 32 |
| 4 | Sarada Ma V | 77 |
| 5 | Tangal Secondary School | 83 |
| 6 | Viswa Niketan Ma V | 244 |
| 7 | Shram Rastriya Ma V | 113 |
| 8 | Bal Byabasahi Kendra Secondary School | 55 |
| 9 | Pashupati Mitra Ma V | 96 |
| 10 | Bansbari Secondary School | 51 |
| 11 | Guheshwori Bal Sikshya Ma V | 33 |
| 12 | Ratna Rajya Ma V | 390 |
| 13 | Jagan Nath Ma V | 39 |
| 14 | Bandi Bikash Adharbhut Vidyalaya | 11 |

| | | |
|---|---|---|
| 15 | Mahendra Rastriya Ma V | 83 |
| 16 | Koteshwor Saraswati Ma V | 120 |
| 17 | Bal Sewa Ma V | 17 |
| 18 | Nawa Yug Ma V | 13 |
| 19 | Shanti Nikunja Ma V | 81 |
| 20 | Jana Kalyan Ma V | 130 |
| 21 | Bijay Smarak Ma V | 58 |
| 22 | Nawa Jagriti Secondary School | 30 |
| 23 | Bhanu Ma Vi | 21 |
| 24 | Tyouda Secondary School | 24 |
| 25 | Himalaya Ma V | 9 |
| 26 | Paropakar Adarsha Ma V | 42 |
| 27 | Nawa Adarsha Ma V | 29 |
| 28 | Nandi Secondary School | 64 |
| 29 | Shivapuri Secondary School | 163 |
| 30 | Mahendra Boudha Secondary School | 156 |
| 31 | Buddha Jyoti Bal Udhyan Adharbhut Vidyalaya | 7 |
| 32 | Sahid Sukra Ma V | 33 |
| 33 | Shanti Vidya Griha Secondary School | 90 |
| 34 | Bhakta Vidyashram Ma V | 14 |
| 35 | Saraswati Niketan Ma V | 35 |
| 36 | Jana Prabhat Ma V | 77 |
| 37 | Pancha Kanya Ma V | 14 |
| 38 | Kanya Mandir Secondary School | 41 |
| 39 | Kanti Ishwori Rajya Laxmi Ma V | 20 |

| | | |
|---|---|---|
| 40 | Prabhat Ma V | 23 |
| 41 | Gyanodaya Ma V | 227 |
| 42 | Gitamata Ma V | 302 |
| 43 | Siddhi Ganesh Ma V | 68 |
| 44 | Ranibari ( Ranidevi )Basic School | 19 |
| 45 | Juddhodaya Madhyamic Vidhyalaya | 116 |
| 46 | Jana Jagriti Gyan Rashmi Secondary School | 40 |
| 47 | Mahankal Ma V | 17 |
| 48 | Nil Barahi Ma V | 69 |
| 49 | Tarun Ma V | 267 |
| 50 | Kanya Madhyamik Vidhyalaya | 19 |
| 51 | Nepal Yuwak Ma V | 40 |
| 52 | Padmodaya Ma V | 63 |
| 53 | Janapath Ma V | 63 |
| 54 | Jana Bikash Ma V | 32 |
| 55 | Sanskrit Ma V | 58 |
| 56 | Sitala Ma.Vi. | 52 |
| 57 | Nandi Ratri ma.Vi. | 29 |
| 58 | Kuleswor Ma.Vi. | 16 |
| 59 | Shanti Shiksha Mandir Ma.Vi. | 21 |
| 60 | Dhumrabaraha Ma.vi | 57 |
| 61 | Shanti Vidya Griha Secondary School | 90 |
| | Total | **4468** |

# Appendix – B

## Name of Sample Schools

| SN | Name of Schools | No. of Students |
| --- | --- | --- |
| 1 | Geeta Mata Ma Vi, Dallu | 40 |
| 2 | Nil Barahi HSS, Tankeshwor | 40 |
| 3 | Tangal HSS, Shivpuri | 40 |
| 4 | Vishwa Niketan HSS, Tripureshwor | 40 |
| 5 | Koteshwor HSS, Koteshwor | 40 |
| 6 | Ratna Rajya HSS, New Baneshwor | 40 |
| 7 | Mahendra Bauddha HSS, Bauddha | 40 |
| 8 | Tarun Ma Vi, Balaju | 40 |
| 9 | Paropakar Ma Vi, Tankeshwor | 40 |
| Total | | 360 |

## Appendix – C

### Name of Interviewees (Phase I & II)

| SN | Name of Students | Group (Marks) | Name of School |
|---|---|---|---|
| 1 | Depti Gurung, (F)* | 18 – 20 | Vishwa Niketan HSS |
| 2 | Dipesh Thakur (M)* | 18 – 20 | Koteshwor HSS |
| 3 | Mahima Lama (F)* | 9 – 11 | Koteshwor HSS |
| 4 | Diwash Shrestha (M)* | 9 – 11 | Vishwa Niketan HSS |
| 5 | Ashmita Adhikari (F)* | $\leq 5$ | Koteshwor HSS |
| 6 | Mohit Khadka (M)* | $\leq 5$ | Vishwa Niketan HSS |

*Gender of the students

## Appendix – D

### Name of Interviewees (Teachers, Phase II)

| SN | Name of Teachers | Name of Schools |
|---|---|---|
| 1 | Dilliram Paudel | Vishwa Niketan HSS |
| 2 | Rati Shrestha | Gita Mata HSS |
| 3 | Deepak Pandit | Bhakta Bishram Ma Vi |
| 4 | Mahesh Khadka | Koteshwor HSS |

# Appendix – E

## Pilot Testing Questionnaires

PILOT TESTING

Student's Name:-………………………………..

Name of School:-………………………………

Procedural Knowledge Questions

Factorize the following expressions:

| | |
|---|---|
| b) $x^2 + 7x + 12$ | a) $4x^2 – 64$ |

Solve for x:

| | |
|---|---|
| a) $x^2 – 4x + 3 = 0$ | b) $3 (x + 5) = 2x + 35$ |

Conceptual Knowledge Questions

Tick (√) the best answer.

1. What does factorization mean?

a) A process of expressing the expression into the division of two or more of its factors.

b) A process of expressing the expression into the difference of two or more its factors.

c) A process of expressing the expression into the product of two or more of its factors.

d) A process of expressing the expression into the addition of two or more of its factors.

2. What are $x^2$, $7x$ and $12$ in $x^2 + 7x + 12$?

a) Terms b) Factors c) Values d) Coefficients

3. In an algebraic expression defined on x, why 'x' is called a variable;

a) It has exactly one value

b) It has one or more particular value

c) x is a constant

d) x has an infinite value

4. Which type of expression is it $(x^2 + 7x + 12)$?

a) Monomial b) Binomial c) Trinomial d) cannot be determined

5. The degree of $2x^2 + 14x - 2y + 12$ is 2 because

a) It contains exactly two terms with variable x

b) It contains two variables x and y

c) The highest power of the variable x is 2

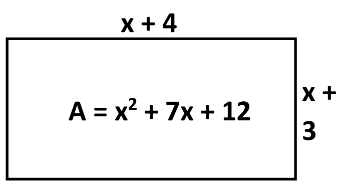
d) 2 is common in all terms

6. What are $(x + 3)$ and $(x + 4)$ in $x^2 + 7x + 12 = (x + 3).(x + 4)$?

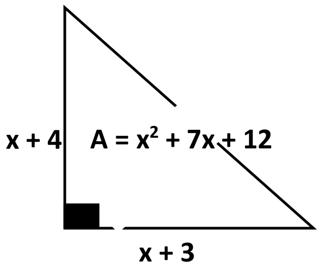
a) Factors b) Divisors c) Remainders d) Both (a) and (b)

7. Which of the following picture describes the diagram of $x^2 + 7x + 12 = (x + 3). (x + 4)$?

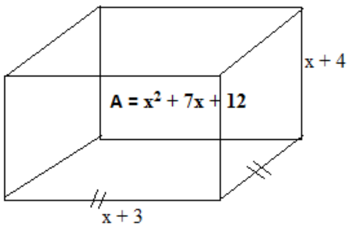
a)



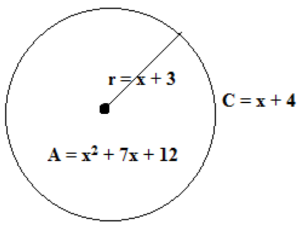

b)




c)




d)



8. Which of the following is equivalent to $x^2 + 7x + 12$?

a) $- x^2 + 7x – 12$ b) $x^2 – 7x + 12$

c) $x^2 - 7x – 12$ d) $12 – (- 7x) – (-x^2)$

9. If the area and the breadth of a rectangular table are $24x^2y – 18xy^2$ and $4x – 3y$, then the length of this table is:

a) $6xy$ b) $4x + 3y$ c) $4x – 3y$ d) $– 6xy$

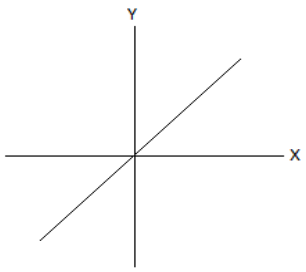
10. For the positive integer value of x, the correct ascending order of: $\frac{1}{x}, \left(\frac{1}{x^2}\right), \left(\frac{1}{x^3}\right)$ is:

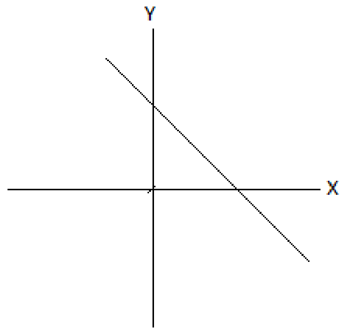
a) $\frac{1}{x} < \left(\frac{1}{x^2}\right) < \left(\frac{1}{x^3}\right)$ b) $\left(\frac{1}{x^3}\right) < \left(\frac{1}{x^2}\right) < \frac{1}{x}$

b) $\frac{1}{x} < \left(\frac{1}{x^3}\right) < \left(\frac{1}{x^2}\right)$ d) $\left(\frac{1}{x^2}\right) < \left(\frac{1}{x^3}\right) < \frac{1}{x}$

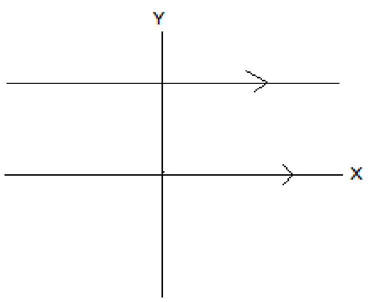
11. Which of the following describes the graph of the equation $x – y = 0$?

a)

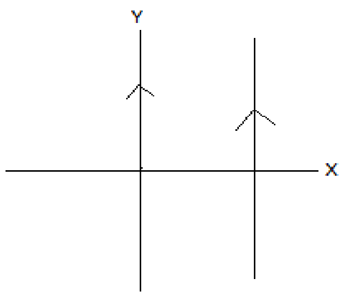



c)




d)




12. Mary has some fruits (apples). Julie has 3 times as many apples as Mary. They have 36 apples in all. Which of these equations represents their apples?

a) $3x = 36$ b) $x + 3 = 36$

c) $x + 3x = 36$ d) $3x + 36 = x$

13. If $x = y$ , then $x + a = y + a$ because;

a) Equals added to equal are equal

b) Order doesn't matter when adding two numbers.

c) Same number can be added to any quantity to get equal quantity

d) Quantity are equal when we add any quantity to equal quantity

14. Let xy be a two digit number, then which of the following is the exact value of (xy)

a) $10x - y$ b) $10x + y$ c) $x + y$ d) $x - y$

15. Which of the following statements is NOT TRUE about the equation $y = 2t$, if t is a positive number?

a) It shows how y changes for different values of t.

b) It shows a linear relationship between y and t.

c) It shows that the value of y is independent of the value of t.

d) It shows that as t increases, y also increases.

16. Jiya is exactly five years older than Rina. Let J stand for Jiya's age and R stands For Rina's age. Which of the following is an equation to compare Jiya's and Rina's age?

a) $J + 5 = R$ b) $J = R + 5$ 3) $J = R - 5$ d) None

17. Tanka is 7 years younger than twice the age of his cousin Rabi. Tanka is 15 years old. Which equation could be used to determine Rabi's age?

a) $15 = 2a + 7$ b) $15 = 7 - 2a$ c) $15 = 2a - 7$ d) $15 = 2 - 5a$

18. Which of the following is true if seven is subtracted from twice a number gives 17 (Suppose number as x).

a) $2x - 7 = 17$ b) $2(x - 7) = 17$ c) $7 - 2x = 17$ d) $7(2 - x) = 17$

19. Which equation represents the following statement?

"Twenty five more than a number is equal to three times the number."

(Let the number = y)

a) $25(y + 2) = 3y$ b) $2y + 25 = 3y$ c) $y + 25 = 3y$ d) $2y = 3y + 25$

20. Observe the following solution.

Solve: $2(x + 3) = x + 21$

Step 1: $2x + 6 = x + 21$

Step 2: $3x + 6 = 21$

Step 3: $3x = 15$

Step 4: $x = 5$

Which is the first incorrect step in the solution shown above?

a) Step 1 b) Step 2 c) Step 3 d) Step 4

21. Observe the following solution.

Solve: $5(x – 3) = x + 5$

Step 1: $5x – 15 = x + 5$

Step 2: $4x – 15 = 5$

Step 3: $4x = 20$

Step 4: $x = 20 – 4$

Step 5: $x = 16$

Which is the first incorrect step in the solution shown above?

a) Step 1 b) Step 2 c) Step 3 d) Step 4

22. Which of the following pair are the examples of linear equation?

a) $x – y = 2$ & $y = 3x$ b) $x^2 = y$ & $x^2 + y^2 = 25$

c) $x^3 – y^3 = 0$ and $y = x^3$ d) $xy = y$ and $(x^2 + y^2)^2 = 15$

23. Linear equation in one variable has

a) Only one variable with any power b) Only one term with a variable

c) Only one variable with power 1 d) Only one constant term

24. A linear equation in one variable has

a) Only one solution    b) two solution

c) More than two solution    d) no solution

25. The breadth of a rectangular handkerchief is 10 cm less than its length. If the perimeter if 110 cm, which of the following equation gives the perimeter,

a) $110 = 2 \{x + (x - 10)\}$    b) $110 = 4\{x + (x - 10)\}$

c) $110 = x.(x - 10)$    d) $110 = 2\{x .(x - 10)\}$

26. In grade 9, there are 28 students in total. If the number of boys is less than 7 that of the girls, then which of the following equation represents the total number of students

a) $28 = x.(x - 7)$    b) $28 = x + (x - 7)$

b) $28 = x.(x + 7)$    d) $28 = x - (x + 7)$

## Appendix - F

## Survey Questions

Student's Details

Student's Name:-……………………………………….

School's Name:-………………………………………….

Tick (√) the following.

| Sex | (a) Male | (b) Female | (c) Other |
|---|---|---|---|

Parent's Occupation

| **Father's Occupation** | (a) Teaching | (b) Government Job | (c) Business | (d) Agriculture | (f) Other |
|---|---|---|---|---|---|
| **Mention if others** | | | | | |

| **Mother's Occupation** | (a) Housewife | (b) Business | (c) Teaching | (d) Agriculture | (f) Other |
|---|---|---|---|---|---|
| **Mention if others** | | | | | |

Parent's Education

| | | | | | | |
|---|---|---|---|---|---|---|
| **Father's Education** | (a) Literate | (b) SLC | (c) 10+2 | (d) Bachelor | (e) Masters | (e) Other |
| **Mother's Education** | (a) Literate | (b) SLC | (c) 10+2 | (d) Bachelor | (e) Masters | (e) Other |

(Note: The provided response will not be used for other purpose except for this study.

I appreciate for your valuable response)

## Procedural Knowledge Questions

Factorize the following expressions:

d) $x^2 + 7x + 12$

c) $4x^2 – 64$

Solve for x:

a) $x^2 – 4x + 3 = 0$

b) $3 (x + 5) = 2x + 35$

## Conceptual Knowledge Questions

Tick (√) the best answer.

1. Factorization of any expression is a

a) Process of expressing the expression into the division of two or more of its factors.

b) Process of expressing the expression into the product of two or more of its factors.

c) Process of expressing the expression into the difference of two or more of its factors.

d) Process of expressing the expression into the addition of two or more of its factors.

2. What are $x^2$, 7x and 12 in $x^2 + 7x + 12$?

a) Factors b) Values c) Terms d) Coefficients

3. The degree of $2x^2 + 14x - 2y + 12$ is 2 because

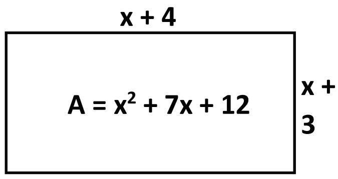
a) It contains exactly two terms with variable x

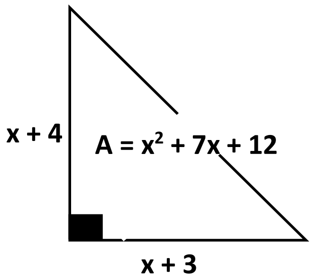
b) It contains two variables x and y

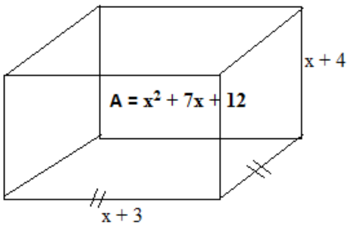
c) The highest power of the variable x is 2

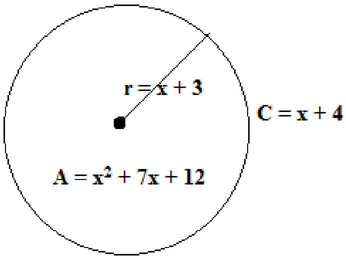
d) 2 is common in all terms

4. Which of the following describes the diagram of in $x^2 + 7x + 12 = (x + 3). (x + 4)$?

a) 


b) 


c) 


d) 


5. Which of the following is equivalent to $x^2 + 7x + 12$?

a) $- x^2 – 7x – 12$

b) $– 7x + 12 + x^2$

c) $x^2 - 7x – 12$

d) $12 – (- 7x) – (-x^2)$

6. What is the type of this expression $(x^2 + 7x + 12)$?

a) Monomial

b) Binomial

c) Trinomial

d) cannot be determined

7. For the positive integer value of x, the correct ascending order of: $\frac{1}{x}$, $\left(\frac{1}{x^2}\right)$, $\left(\frac{1}{x^3}\right)$ is:

a) $\frac{1}{x} < \left(\frac{1}{x^2}\right) < \left(\frac{1}{x^3}\right)$

b) $\left(\frac{1}{x^3}\right) < \left(\frac{1}{x^2}\right) < \frac{1}{x}$

c) $\frac{1}{x} < \left(\frac{1}{x^3}\right) < \left(\frac{1}{x^2}\right)$

d) $\left(\frac{1}{x^2}\right) < \left(\frac{1}{x^3}\right) < \frac{1}{x}$

8. If the area and the breadth of a rectangular table are $24x^2y – 18xy^2$ and $4x – 3y$, then the length of this table is:

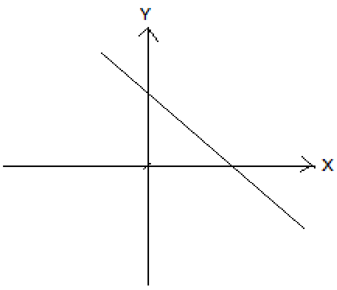
a) $6xy$

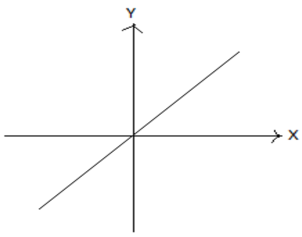
b) $4x + 3y$

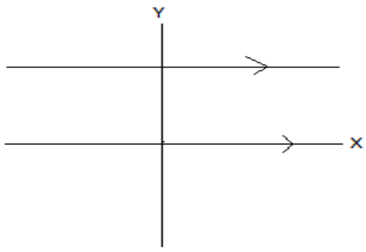
c) $4x – 3y$

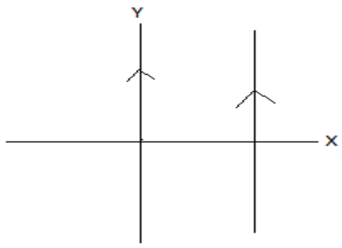
d) $– 6xy$

9. Which of the following describes the graph of the equation $x – y = 0$?

a) 


) 

c) 


d) 


10. Mary has some fruits (apples). Julie has 3 times as many apples as Mary. They have 36 apples in all. Which of these equations represents their apples?

a) $3x = 36$ b) $x + 3 = 36$

c) $x + 3x = 36$ d) $3x + 36 = x$

11. If $x = y$ , then $x + a = y + a$ because;

a) Quantity are equal when we add any quantity to equal quantity

b) Equals added to equal are equal

c) Order doesn't matter when adding two numbers.

d) Same number can be added to any quantity to get equal quantity

12. Jiya is exactly five years older than Rina. Let J stands for Jiya's age and R stands For Rina's age. Which of the following is an equation to compare Jiya's and Rina's age?

a) $J + 5 = R$ b) $J = R + 5$ 3) $J = R – 5$ d) $J = 5 \times R$

13. Which equation represents the following statement?

"Twenty five more than a number is equal to three times the number."

(Let the number = y)

a) $25 (y + 2) = 3y$ b) $2y + 25 = 3y$ c) $y + 25 = 3y$ d) $2y = 3y + 25$

14. Which of the following is true if seven is subtracted from twice a number gives 17 (Suppose number as x).

a) $2x - 7 = 17$ b) $2(x - 7) = 17$ c) $7 - 2x = 17$ d) $7(2 - x) = 17$

15. Tanka is 7 years younger than twice the age of his cousin Rabi. Tanka is 15 years old. Which equation can be used to determine Rabi's age?

a) $15 = 2a + 7$ b) $15 = 7 - 2a$ c) $15 = 2a - 7$ d) $15 = 2 - 5a$

16. Observe the following solution.

Solve: $2(x + 3) = x + 21$

Solution: - Step 1: $2x + 6 = x + 21$

Step 2: $3x + 6 = 21$

Step 3: $3x = 15$

Step 4: $x = 5$

Which is the first incorrect step in the solution shown above?

a) Step 1 b) Step 2 c) Step 3 d) Step 4

17. Which of the following pair are the examples of linear equation?

a) $x^3 - y^3 = 0$ and $y = x^3$ b) $x^2 = y$ & $x^2 + y^2 = 25$

c) $x - y = 2$ & $y = 3x$ d) $xy = y$ and $(x^2 + y^2)^2 = 15$

18. A linear equation in one variable has

a) Only one solution b) two solution

c) More than two solution d) no solution

19. The breadth of a rectangular handkerchief is 10 cm less than its length. If the perimeter is 110 cm, which of the following equation gives the perimeter,

a) $110 = 2\{x + (x - 10)\}$ b) $110 = 4\{x + (x - 10)\}$

c) $110 = x.(x - 10)$ d) $110 = 2\{x .(x - 10)\}$

20. Which of the following statements is 'NOT TRUE' about the equation $y = 2t$, if t is a positive number?

a) It shows that the value of y is independent of the value of t.

b) It shows how y changes for different values of t.

c) It shows a linear relationship between y and t.

d) It shows that as t increases, y also increases.

## Appendix – G

### Survey Questions (Nepali)

विद्यार्थीको नाम, थर :- ...............................................................................

विद्यालयको नाम :- ....................................................................................

(√) चिह्न लगाउनुहोस्।

| Sex | (a) Male | (b) Female | (c) Other |
|---|---|---|---|

Parent's Occupation

| Father's Occupation | (a) Teaching | (b) Government Job | (c) Business | (d) Agriculture | (f) Other |
|---|---|---|---|---|---|
| | | | | | |

| Mother's Occupation | (a) Housewife | (b) Business | (c) Teaching | (d) Agriculture | (f) Other |
|---|---|---|---|---|---|
| | | | | | |

Parent's Education

| | | | | | | |
|---|---|---|---|---|---|---|
| Father's Education | (a) Literate | (b) SLC | (c) 10+2 | (d) Bachelor | (e) Masters | (e) Other |
| Mother's Education | (a) Literate | (b) SLC | (c) 10+2 | (d) Bachelor | (e) Masters | (e) Other |

(Note: The provided response will not be used for other purpose except for this study. I appreciate for your valuable response)

Procedural Knowledge Questions

खण्डीकरण गर्नुहोस्:

a) $x^2 + 7x + 12$

b) $4x^2 - 64$

<u>हल गर्नुहोस</u>:

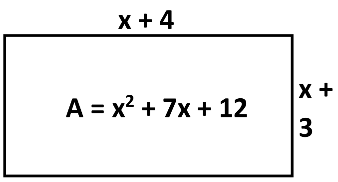
| | |
|---|---|
| a) $x^2 - 4x + 3 = 0$ | b) $3 (x + 5) = 2x + 35$ |

## <u>Conceptual Knowledge Questions</u>

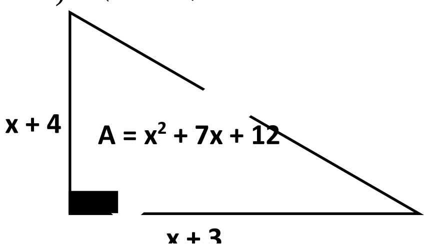
सबैभन्दा सहि उत्तरमा (√) चिह्न लगाउनुहोस्

1. खण्डीकरण भनेको के हो?
   a) अभिव्यञ्जकको २ बा सो भन्दा बढि यसका खण्डहरुलाई भागको रुपमा व्यक्त गर्ने प्रक्रिया
   b) अभिव्यञ्जकको २ बा सो भन्दा बढि यसका खण्डहरुलाई गुणनको रुपमा व्यक्त गर्ने प्रक्रिया
   c) अभिव्यञ्जकको २ बा सो भन्दा बढि यसका खण्डहरुलाई अन्तरको रुपमा व्यक्त गर्ने प्रक्रिया
   d) अभिव्यञ्जकको २ बा सो भन्दा बढि यसका खण्डहरुलाई जोडको रुपमा व्यक्त गर्ने प्रक्रिया
2. $x^2 + 7x + 12$ मा $x^2$, $7x$ र 12 हरु के के हुन् ?
   a) खण्डहरु b) मानहरु c) पदहरु d) गुणाङ्कहरु
3. अभिव्यञ्जक $2x^2 + 14x - 2y + 12$ को डिग्री 2 हुन्छ किनभने
   a) यसमा चल x भएको 2 बटा पदहरु भएकोले
   b) यसमा चलहरु x र y भएकोले
   c) यसमा चल x को घाताङ्क सबैभन्दा बढि 2 भएकोले
   d) सबै पदहरुमा 2 साझा भएकोले
4. तलकाभध्ये कुनचाही चित्र ले $x^2 + 7x + 12 = (x + 3). (x + 4)$ लाई जनाउँछ?

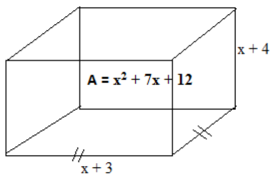
a)



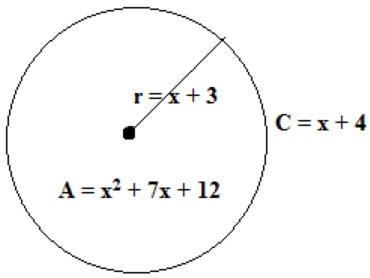

b)




c)




d)



5. तलका अभिव्यञ्जकहरुमध्ये कुन चाहीँ $x^2 + 7x + 12$ सँग बराबर हुन्छ?

a) $- x^2 – 7x – 12$ b) $– 7x + 12 + x^2$

c) $x^2 - 7x – 12$ d) $12 – (- 7x) – (-x^2)$

6. $(x^2 + 7x + 12)$ कुन प्रकारको भभिव्यञ्जक हो?

a) एकपदिय b) द्बिपदिय c) त्रिपदिय d) परिभाषित गर्न नसकिने

7. यदि x एउटा धनात्मक संख्या भए, $\frac{1}{x}$, $\left(\frac{1}{x^2}\right)$, $\left(\frac{1}{x^3}\right)$ लाई बढ्दो क्रममा राख्नुहोस:

a) $\frac{1}{x} < \left(\frac{1}{x^2}\right) < \left(\frac{1}{x^3}\right)$ b) $\left(\frac{1}{x^3}\right) < \left(\frac{1}{x^2}\right) < \frac{1}{x}$

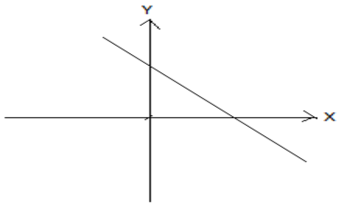
c) $\frac{1}{x} < \left(\frac{1}{x^3}\right) < \left(\frac{1}{x^2}\right)$ d) $\left(\frac{1}{x^2}\right) < \left(\frac{1}{x^3}\right) < \frac{1}{x}$

8. एउटा आयतकार चउरको क्षेत्रफल र चौडाई क्रमश: $24x^2y – 18xy^2$ र $4x – 3y$ छन् भने यसको लम्बाइ कति हुन्छ?

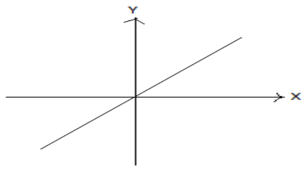
a) $6xy$ b) $4x + 3y$ c) $4x – 3y$ d) $– 6xy$

9. तलकामध्ये कुनचाहीँ ग्राफले $x – y = 0$ को समिकरणलाई प्रतिनिधित्व गर्छ?

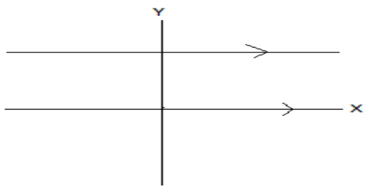
a)



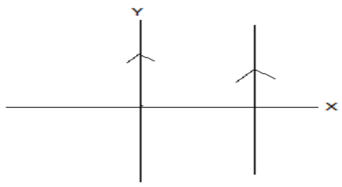

b)




c)




d)




10. प्रीति सगँ केही स्याउहरु छन्। जुलीसगँ प्रीतिको भन्दा 3 गुणा बढि स्याउहरु छन्। यदि दुवैजना गरि जम्मा 36 स्याउहरु भए, तलकामध्ये हुनचाहीँ समिकरणले उनीहरुसँग भएको जम्मा स्याउहरुको समिकरणलाई प्रतिनिधित्व गर्छ?

a) $3x = 36$ b) $x + 3 = 36$

c) $x + 3x = 36$ d) $3x + 36 = x$

11. यदि $x = y$ भए $x + a = y + a$ हुन्छ किनकी:

a) बराबरमा जुनसुकै संख्या जोड्दा पनि परिणाम बराबर आउँछ

b) बराबरमा बराबर जोड्दा परिणाम पनि बराबर हुन्छ

c) 2 बटा सख्ँया जोड्दा यसको क्रमले (order) कुनै अर्थ राख्दैन

d) कुनै बराबर सँख्या जुनसुकै 2 वटा संख्या सगँ जोड्दा बराबर हुन्छ

12. जियाको उमेर रीनाको भन्दा 5 बर्ष बढि छ। मानौं J र R क्रमश: जिया र रीनाको उमेरहरु हुन् भने तलकामध्ये कुनचाहीँ समिकरणलाई जिया र रीनाको उमेर तुलना गर्न प्रयोग गर्न सकिन्छ?

a) $J + 5 = R$ b) $J = R + 5$ 3) $J = R – 5$ d) $5 \times R$

13. तलकामध्ये कुनचाहीँ समिकरणले दिइएको भनाइलाई प्रतिनिधित्व गर्छ?

"कुनै संख्या र 25 को योगफल सोहि संख्याको 3 गुणासँग बराबर छ" (मानौ आबश्यक संख्या = y)

a) $25 (y + 2) = 3y$ b) $2y + 25 = 3y$ c) $y + 25 = 3y$ d) $2y = 3y + 25$

14. दिइएको भनाइको लागि तलकामध्ये कुनचाही सहि छ?

'कुनै संख्याको 2 गुणाबाट 7 घटाउँदा 17 हुन्छ' (मानौ आबश्यक संख्या = x)

a) $2x - 7 = 17$ b) $2(x - 7) = 17$ c) $7 - 2x = 17$ d) $7(2 - x) = 17$

15. टंकको उमेर उसको भतिज रविभन्दा 2 गुणा भन्दा पनि 7 वर्षले कम छ । यदि टंकको उमेर अहिले 15 बर्ष भए, तलकामध्ये कुनचाहीँ समिकरणले रविको उमेर पत्ता लगाउन सकिन्छ? (जहाँ टंकको उमेर = a)

a) $15 = 2a + 7$ b) $15 = 7 – 2a$ c) $15 = 2a – 7$ d) $15 = 2 – 5a$

16. तलको समाधानलाई अबलोकन गर्नुहोस्

हल गर्नुहोस: $2 (x + 3) = x + 21$

Step 1: $2x + 6 = x + 21$

Step 2: $3x + 6 = 21$

Step 3: $3x = 15$

Step 4: $x = 5$

माथि दिइएको समाधानमा कुनचाही step गलत छ?

a) Step 1 b) Step 2 c) Step 3 d) Step 4

17. तलकामध्ये कुनचाहीँ एक जोडी समिकरण 'रेखिय समिकरण' को उदाहरण हो?

a) $x^3 – y^3 = 0$ & $y = x^3$ b) $x^2 = y$ & $x^2 + y^2 = 25$

c) $x – y = 2$ & $y = 3x$ d) $x^4 = y$ & $(x^2 + y^2)^2 = 15$

18. एक चलयुक्त रेखिय समिकरणमा, चलको मान

a) एउटामात्र हुन्छ b) 2 वटा हुन्छ

c) 2 भन्दा बढि हुन्छ d) मान हुदैन

19. एउटा आयतकार रुमालको चौडाइ, लम्बाइभन्दा 10 cm कम छ। यदि यसको परिमिति 110 cm भए, तलकामध्ये कुनचाहीँ समिकरणले यसको परिमितिलाई प्रतिनिधित्व गर्छ?

a) $110 = 2 \{x + (x – 10)\}$ b) $110 = 4\{x + (x – 10)\}$

c) $110 = x.(x – 10)$ d) $110 = 2\{x .(x – 10)\}$

20. यदि $y = 2t$ जहाँ t एउटा धनात्मक सख्याँ हुदाँ तलबकामध्ये कुनचाहीँ भनाई गलत छ?

a) यसले y को मान t मा भर नपर्ने सम्बन्धलाई जनाउँछ

b) यसले y को मान t को मानअनुसार परिवर्तन हुने सम्बन्धलाई जनाउँछ

c) यसले y र t को रेखिय सम्बन्धलाई जनाउँछ

## Appendix –H

## Interview Check List

Name of student: -

Implicit measure

| SN | Indicators | Response |
|---|---|---|
| 1 | Ability to provide Examples | |
| 2 | Representational Knowledge (Linking with pictures) Q. No. : 4 & 9 | |
| 3 | Compare Quantities (Q. No. 5 & 7) | |
| 4 | Other Methods to solve Problems | |
| 5 | How much H/S knows about such knowledge | |

Explicit Measure

| SN | Indicators | Response |
|---|---|---|
| 1 | Ability to define concept (Factorization, linear equations etc.) | |
| 2 | Explain why procedure work (Q. No. 16) | |
| 3 | Critical thinking questions (Q. No. 11) | |

# Appendix – I

## Semi – Structured Questionnaires (Phase – II)

Name of Student: - …………………………

Interview Questionnaires

| Questions | Response |
|---|---|
| **What do you focus while solving problems?** | A step by step algorithmic procedures<br>understanding of underlying concept or<br>both |
| **What should be the process of learning mathematics?** | It should be learn through algorithmic step by step manner no matter what the concept is.<br>The core meaning should be learned. |
| **What is your opinion about learning mathematics?** | I search the meaning of each concept or ask with teacher.<br>I am good in mathematics.<br>I do not enjoy learning mathematics.<br>I am weak in mathematics.<br>Mathematics is very difficult to learn comparing other subject |
| **How much time do you spend in learning mathematics (algebra) at your home?** | I don't study at home<br>0 – 1 hour<br>Max. 2 hours<br>More than 2 hours |

Perception of Students about their Teachers' Instruction

a) H/S only emphasize on doing routine problems.

i. Sometimes ii. Always ii. never

b) H/S forces us to learn and read steps and formulae

i. Sometimes b. Always c. never

c) H/S emphasizes on pictures, diagrams, and different materials while teaching mathematics

i. Sometimes ii. Always iii. never

d) H/S teaches us mathematics using contextual examples

i. Sometimes ii. Always iii. never

e) H/S emphasizes on problems related to our day to day life

i. Sometimes ii. Always iii. never

f) H/S encourages every students to learn mathematics

i. Sometimes ii. Always iii. never

g) H/S always prioritize the problems (this is/isn't important for the exam)

i. Sometimes ii. Always iii. never

h) H/S emphasizes on practical side of problem solving

i. Sometimes ii. Always iii. never

i) There is mathematics workshop and competition in the school

i. Sometimes ii. Always iii. never

j) My teacher' approach of teaching is;

i. Lecture method without listening to students

ii. Inquiry method (there is chance to ask questions)

iii. Collaborative method (emphasis on group study)

iv. Problem solving method (Focusing on only solving problems)

## Appendix – J

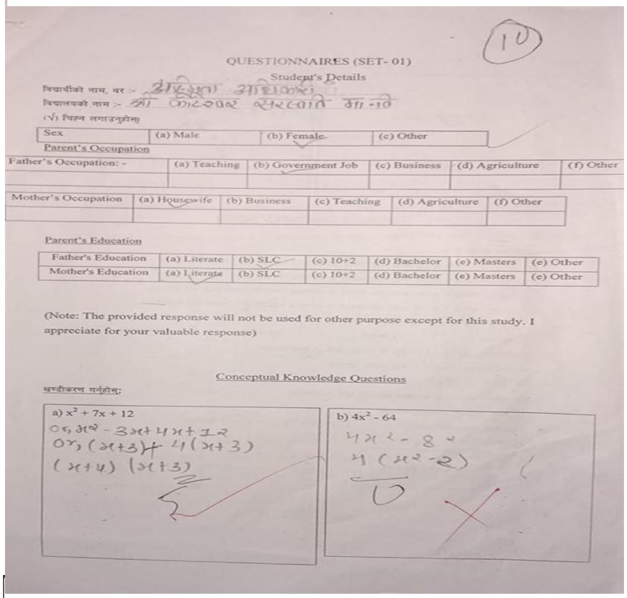
## Semi – Structured Questionnaires (Phase – II: For Teachers)

Open Group Discussion

| | |
|---|---|
| **Your emphasis on teaching and learning mathematics** | **Steps and Procedures should be memorized**<br>**Should focus more on learning concepts**<br>**Both are equally important** |
| **Instruction focus** | Lecture b) inquiry c) Group discussion d) Collaborative and critical discussion. Mention if there is any.<br>Reasons:<br>2. |
| **Contextual Examples Focus (Related to daily life)** | Never b) Sometimes c) Always<br>Notes: |
| **Use of Manipulatives and teaching materials (Audio, visuals aids)** | Never b) Sometimes c) Always<br>Notes: |
| **Practical, Project and Field Work Activities to Students** | Never b) Sometimes c) Always<br>Notes: |

# Appendix – K

## Sample Responses Sheet



QUESTIONNAIRES (SET- 01)

Student's Details

विद्यार्थीको नाम, थर :-

विद्यालयको नाम :-

(√) चिह्न लगाउनुहोस्

| Sex | (a) Male | (b) Female | (c) Other |
|---|---|---|---|

Parent's Occupation

| Father's Occupation: - | (a) Teaching | (b) Government Job | (c) Business | (d) Agriculture | (f) Other |
|---|---|---|---|---|---|
| | | | | | |

| Mother's Occupation | (a) Housewife | (b) Business | (c) Teaching | (d) Agriculture | (f) Other |
|---|---|---|---|---|---|
| | | | | | |

Parent's Education

| Father's Education | (a) Literate | (b) SLC | (c) 10+2 | (d) Bachelor | (e) Masters | (e) Other |
|---|---|---|---|---|---|---|
| Mother's Education | (a) Literate | (b) SLC | (c) 10+2 | (d) Bachelor | (e) Masters | (e) Other |

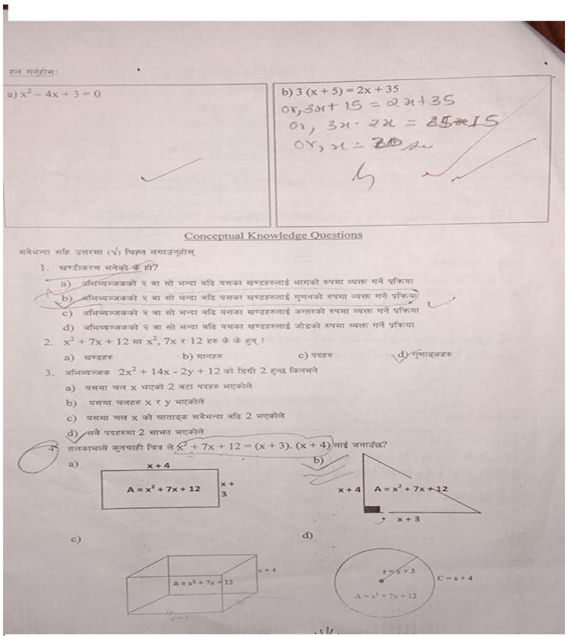
(Note: The provided response will not be used for other purpose except for this study. I appreciate for your valuable response)

Conceptual Knowledge Questions

खण्डीकरण गर्नुहोस्:

a) $x^2 + 7x + 12$

b) $4x^2 - 64$



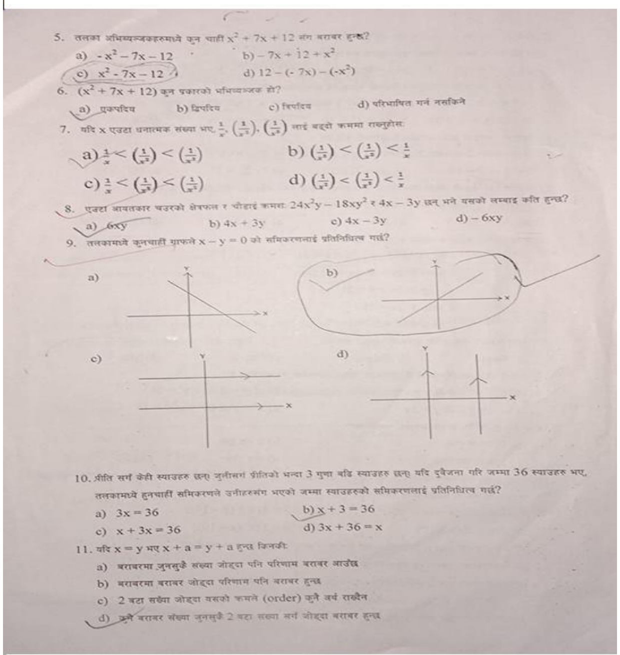
हल गर्नुहोस्:

a) $x^2 - 4x + 3 = 0$

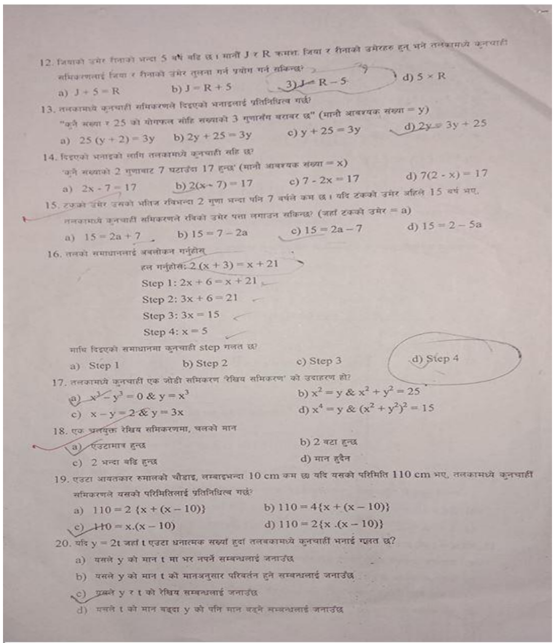
b) $3(x + 5) = 2x + 35$

Conceptual Knowledge Questions

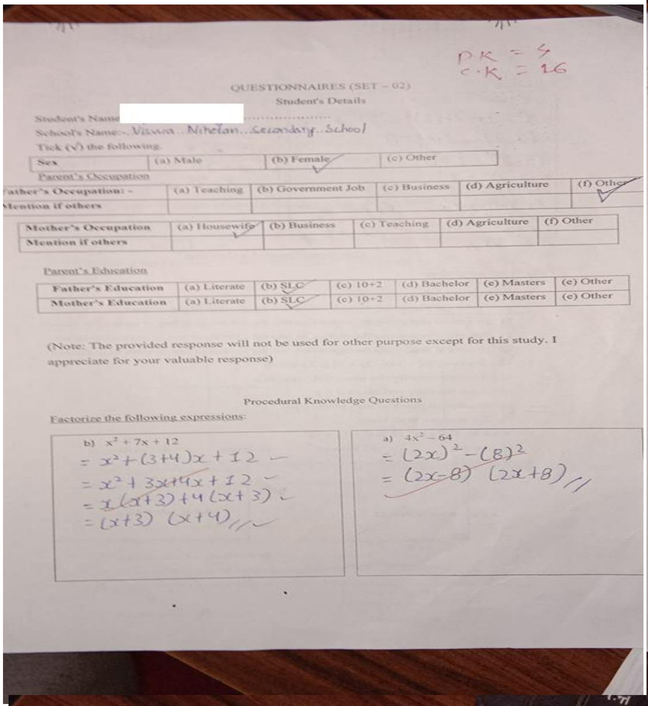
1. खण्डीकरण भनेको के हो?
2. $x^2 + 7x + 12$ मा $x^2$, $7x$ र $12$ हरू के के हुन्?
3. अभिव्यञ्जक $2x^2 + 14x - 2y + 12$ को डिग्री 2 हुन्छ किनभने
4. तलकामध्ये कुनचाहिँ चित्र ले $x^2 + 7x + 12 = (x + 3).(x + 4)$ लाई जनाउँछ?



5. तलका अभिव्यञ्जकहरूमध्ये कुन चाहिँ $x^2 + 7x + 12$ सँग बराबर हुन्छ?
   a) $-x^2 - 7x - 12$ b) $-7x + 12 + x^2$ c) $x^2 - 7x - 12$ d) $12 - (-7x) - (-x^2)$
6. $(x^2 + 7x + 12)$ कुन प्रकारको अभिव्यञ्जक हो?
7. यदि x एउटा धनात्मक संख्या भए, $\frac{1}{x}$, $\left(\frac{1}{x^2}\right)$, $\left(\frac{1}{x^3}\right)$ लाई बढ्दो क्रममा राख्नुहोस्
8. एउटा आयतकार चउरको क्षेत्रफल र चौडाइ क्रमश: $24x^2y - 18xy^2$ र $4x - 3y$ छन् भने यसको लम्बाइ कति हुन्छ?
   a) $6xy$ b) $4x + 3y$ c) $4x - 3y$ d) $-6xy$
9. तलकामध्ये कुनचाहिँ ग्राफले $x - y = 0$ को समीकरणलाई प्रतिनिधित्व गर्छ?
10. a) $3x = 36$ b) $x + 3 = 36$ c) $x + 3x = 36$ d) $3x + 36 = x$
11. यदि $x = y$ भए $x + a = y + a$ हुन्छ किनकि



12. a) $J + 5 = R$ b) $J = R + 5$ c) $J = R - 5$ d) $5 \times R$
13. a) $25(y + 2) = 3y$ b) $2y + 25 = 3y$ c) $y + 25 = 3y$ d) $2y = 3y + 25$
14. a) $2x - 7 = 17$ b) $2(x - 7) = 17$ c) $7 - 2x = 17$ d) $7(2 - x) = 17$
15. a) $15 = 2a + 7$ b) $15 = 7 - 2a$ c) $15 = 2a - 7$ d) $15 = 2 - 5a$
16. हल गर्नुहोस्: $2(x + 3) = x + 21$
    Step 1: $2x + 6 = x + 21$
    Step 2: $3x + 6 = 21$
    Step 3: $3x = 15$
    Step 4: $x = 5$
    a) Step 1 b) Step 2 c) Step 3 d) Step 4
17. a) $x^2 - y^2 = 0$ & $y = x^3$ b) $x^2 = y$ & $x^2 + y^2 = 25$ c) $x - y = 2$ & $y = 3x$ d) $x^4 = y$ & $(x^2 + y^2)^2 = 15$
18. एउटा रेखीय समीकरणमा, चलको मान
19. एउटा आयतकार रुमालको चौडाइ, लम्बाइभन्दा 10 cm कम छ यदि यसको परिमिति 110 cm भए,
    a) $110 = 2\{x + (x - 10)\}$ b) $110 = 4\{x + (x - 10)\}$ c) $110 = x.(x - 10)$ d) $110 = 2\{x.(x - 10)\}$
20. यदि $y = 2t$ जहाँ t एउटा धनात्मक संख्या हुँदा तलकामध्ये कुनचाहिँ भनाइ गलत छ?


PK = 4
C.K = 16

QUESTIONNAIRES (SET – 02)

Student's Details

Student's Name [redacted]

School's Name:- Vidya Niketan Secondary School

Tick (√) the following.

| Sex | (a) Male | (b) Female | (c) Other |
|---|---|---|---|

Parent's Occupation

| Father's Occupation: - | (a) Teaching | (b) Government Job | (c) Business | (d) Agriculture | (f) Other |
|---|---|---|---|---|---|
| Mention if others | | | | | |

| Mother's Occupation | (a) Housewife | (b) Business | (c) Teaching | (d) Agriculture | (f) Other |
|---|---|---|---|---|---|
| Mention if others | | | | | |

Parent's Education

| Father's Education | (a) Literate | (b) SLC | (c) 10+2 | (d) Bachelor | (e) Masters | (e) Other |
|---|---|---|---|---|---|---|
| Mother's Education | (a) Literate | (b) SLC | (c) 10+2 | (d) Bachelor | (e) Masters | (e) Other |

(Note: The provided response will not be used for other purpose except for this study. I appreciate for your valuable response)

Procedural Knowledge Questions

Factorize the following expressions:

b) $x^2 + 7x + 12$

$= x^2 + (3+4)x + 12$

$= x^2 + 3x + 4x + 12$

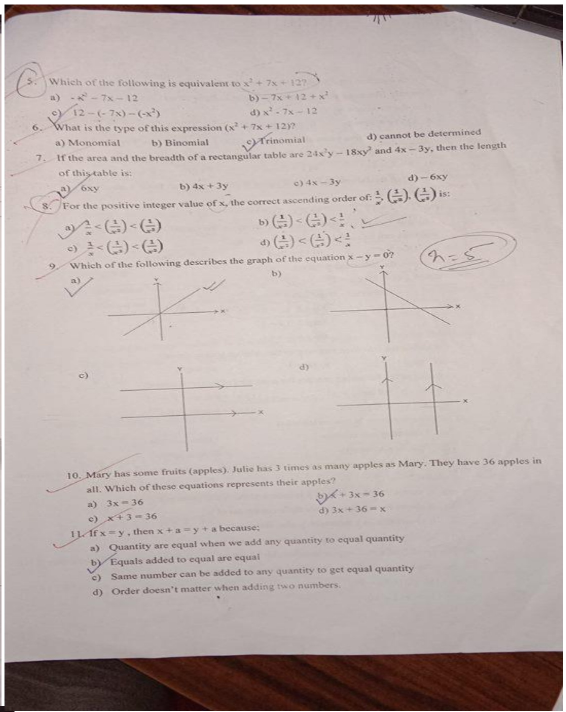
$= x(x+3) + 4(x+3)$

$= (x+3)(x+4)$

a) $4x^2 - 64$

$= (2x)^2 - (8)^2$

$= (2x-8)(2x+8)$


5. Which of the following is equivalent to $x^2 + 7x + 12$?

a) $-x^2 - 7x - 12$ b) $-7x + 12 + x^2$ c) $12 - (-7x) - (-x^2)$ d) $x^2 - 7x - 12$

6. What is the type of this expression $(x^2 + 7x + 12)$?

a) Monomial b) Binomial c) Trinomial d) cannot be determined

7. If the area and the breadth of a rectangular table are $24x^2y - 18xy^2$ and $4x - 3y$, then the length of this table is:

a) $6xy$ b) $4x + 3y$ c) $4x - 3y$ d) $-6xy$

8. For the positive integer value of x, the correct ascending order of: $\frac{1}{x}, \left(\frac{1}{x^2}\right), \left(\frac{1}{x^3}\right)$ is:

a) $\frac{1}{x} < \left(\frac{1}{x^2}\right) < \left(\frac{1}{x^3}\right)$ b) $\left(\frac{1}{x^3}\right) < \left(\frac{1}{x^2}\right) < \frac{1}{x}$

c) $\frac{1}{x} < \left(\frac{1}{x^3}\right) < \left(\frac{1}{x^2}\right)$ d) $\left(\frac{1}{x^2}\right) < \left(\frac{1}{x^3}\right) < \frac{1}{x}$

9. Which of the following describes the graph of the equation $x - y = 0$?

a) b) c) d)

n = 5

10. Mary has some fruits (apples). Julie has 3 times as many apples as Mary. They have 36 apples in all. Which of these equations represents their apples?

a) $3x = 36$ b) $x + 3x = 36$ c) $x + 3 = 36$ d) $3x + 36 = x$

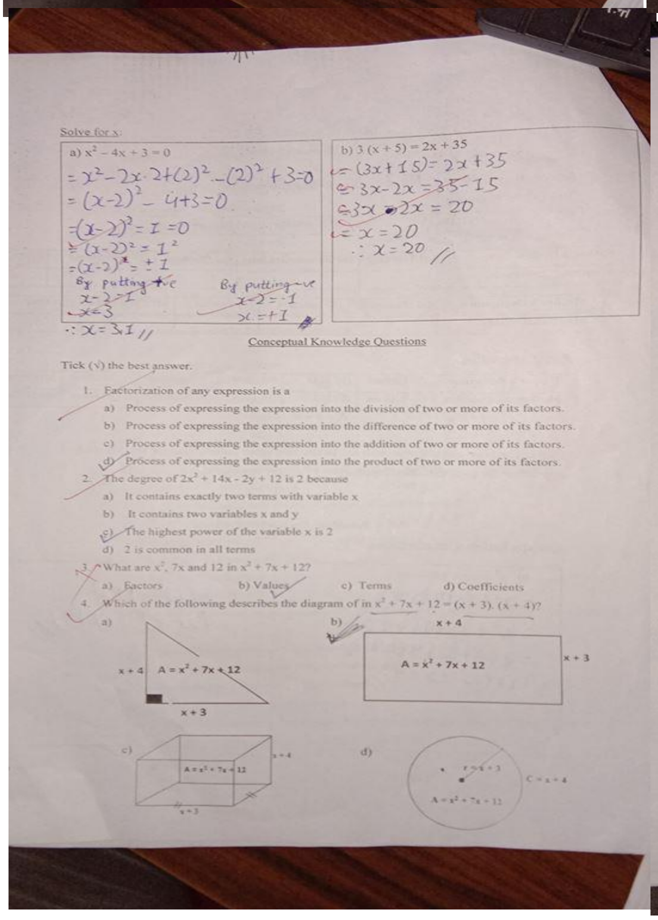
11. If $x = y$, then $x + a = y + a$ because;

a) Quantity are equal when we add any quantity to equal quantity

b) Equals added to equal are equal

c) Same number can be added to any quantity to get equal quantity

d) Order doesn't matter when adding two numbers.


Solve for x:

a) $x^2 - 4x + 3 = 0$

$= x^2 - 2 \cdot x \cdot 2 + (2)^2 - (2)^2 + 3 = 0$

$= (x-2)^2 - 4 + 3 = 0$

$= (x-2)^2 - 1 = 0$

$= (x-2)^2 = 1^2$

$= (x-2) = \pm 1$

By putting +ve: $x - 2 = 1$, $x = 3$

By putting −ve: $x - 2 = -1$, $x = 1$

$\therefore x = 3, 1$

b) $3(x+5) = 2x + 35$

$= 3x + 15 = 2x + 35$

$= 3x - 2x = 35 - 15$

$= 3x - 2x = 20$

$= x = 20$

$\therefore x = 20$

Conceptual Knowledge Questions

Tick (√) the best answer.

1. Factorization of any expression is a

a) Process of expressing the expression into the division of two or more of its factors.

b) Process of expressing the expression into the difference of two or more of its factors.

c) Process of expressing the expression into the addition of two or more of its factors.

d) Process of expressing the expression into the product of two or more of its factors.

2. The degree of $2x^2 + 14x - 2y + 12$ is 2 because

a) It contains exactly two terms with variable x

b) It contains two variables x and y

c) The highest power of the variable x is 2

d) 2 is common in all terms

3. What are $x^2$, 7x and 12 in $x^2 + 7x + 12$?

a) Factors b) Values c) Terms d) Coefficients

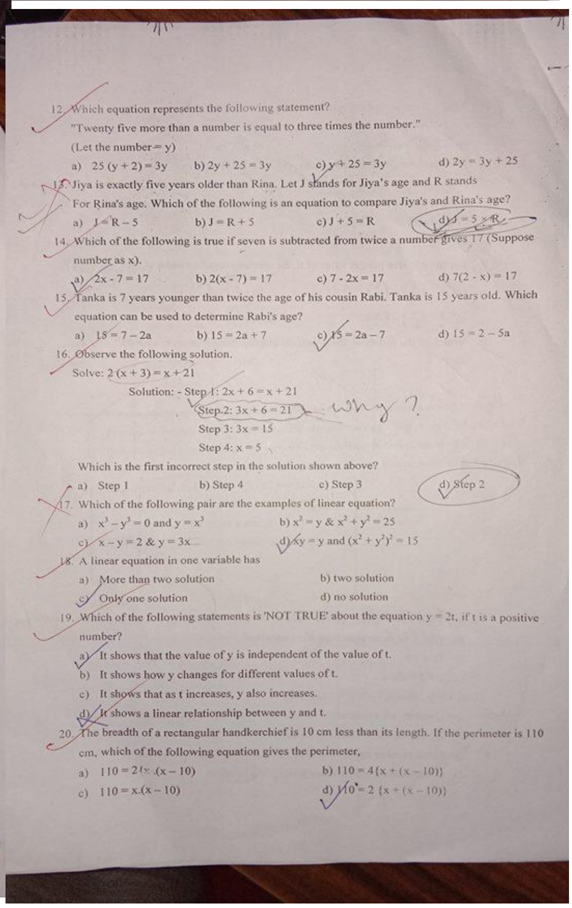
4. Which of the following describes the diagram of in $x^2 + 7x + 12 = (x + 3).(x + 4)$?

a) x + 4, A = $x^2$ + 7x + 12, x + 3

b) x + 4, A = $x^2$ + 7x + 12, x + 3

c) A = $x^2$ + 7x + 12, x + 4, x + 3

d) r = x + 3, C = x + 4, A = $x^2$ + 7x + 12


12. Which equation represents the following statement?

"Twenty five more than a number is equal to three times the number."

(Let the number = y)

a) $25(y + 2) = 3y$ b) $2y + 25 = 3y$ c) $y + 25 = 3y$ d) $2y = 3y + 25$

13. Jiya is exactly five years older than Rina. Let J stands for Jiya's age and R stands For Rina's age. Which of the following is an equation to compare Jiya's and Rina's age?

a) $J = R - 5$ b) $J = R + 5$ c) $J + 5 = R$ d) $J = 5 \times R$

14. Which of the following is true if seven is subtracted from twice a number gives 17 (Suppose number as x).

a) $2x - 7 = 17$ b) $2(x - 7) = 17$ c) $7 - 2x = 17$ d) $7(2 - x) = 17$

15. Tanka is 7 years younger than twice the age of his cousin Rabi. Tanka is 15 years old. Which equation can be used to determine Rabi's age?

a) $15 = 7 - 2a$ b) $15 = 2a + 7$ c) $15 = 2a - 7$ d) $15 = 2 - 5a$

16. Observe the following solution.

Solve: $2(x + 3) = x + 21$

Solution: - Step 1: $2x + 6 = x + 21$

Step 2: $3x + 6 = 21$ why ?

Step 3: $3x = 15$

Step 4: $x = 5$

Which is the first incorrect step in the solution shown above?

a) Step 1 b) Step 4 c) Step 3 d) Step 2

17. Which of the following pair are the examples of linear equation?

a) $x^3 - y^3 = 0$ and $y = x^3$ b) $x^2 = y$ & $x^2 + y^2 = 25$

c) $x - y = 2$ & $y = 3x$ d) $xy = y$ and $(x^2 + y^2)^2 = 15$

18. A linear equation in one variable has

a) More than two solution b) two solution

c) Only one solution d) no solution

19. Which of the following statements is 'NOT TRUE' about the equation $y = 2t$, if t is a positive number?

a) It shows that the value of y is independent of the value of t.

b) It shows how y changes for different values of t.

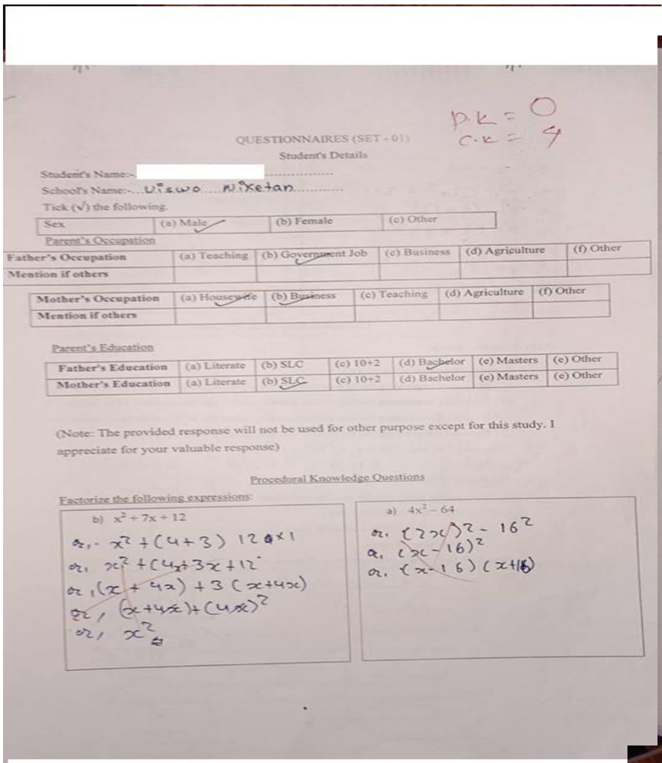
c) It shows that as t increases, y also increases.

d) It shows a linear relationship between y and t.

20. The breadth of a rectangular handkerchief is 10 cm less than its length. If the perimeter is 110 cm, which of the following equation gives the perimeter,

a) $110 = 2(x.(x - 10)$ b) $110 = 4\{x + (x - 10)\}$

c) $110 = x.(x - 10)$ d) $110 = 2\{x + (x - 10)\}$



D.K = 0
C.K = 4

QUESTIONNAIRES (SET - 01)

Student's Details

Student's Name:- [redacted]

School's Name:- Viswo Niketan

Tick (√) the following.

| Sex | (a) Male | (b) Female | (c) Other |
|---|---|---|---|

Parent's Occupation

| Father's Occupation | (a) Teaching | (b) Government Job | (c) Business | (d) Agriculture | (f) Other |
|---|---|---|---|---|---|
| Mention if others | | | | | |

| Mother's Occupation | (a) Housewife | (b) Business | (c) Teaching | (d) Agriculture | (f) Other |
|---|---|---|---|---|---|
| Mention if others | | | | | |

Parent's Education

| Father's Education | (a) Literate | (b) SLC | (c) 10+2 | (d) Bachelor | (e) Masters | (e) Other |
|---|---|---|---|---|---|---|
| Mother's Education | (a) Literate | (b) SLC | (c) 10+2 | (d) Bachelor | (e) Masters | (e) Other |

(Note: The provided response will not be used for other purpose except for this study. I appreciate for your valuable response)

Procedural Knowledge Questions

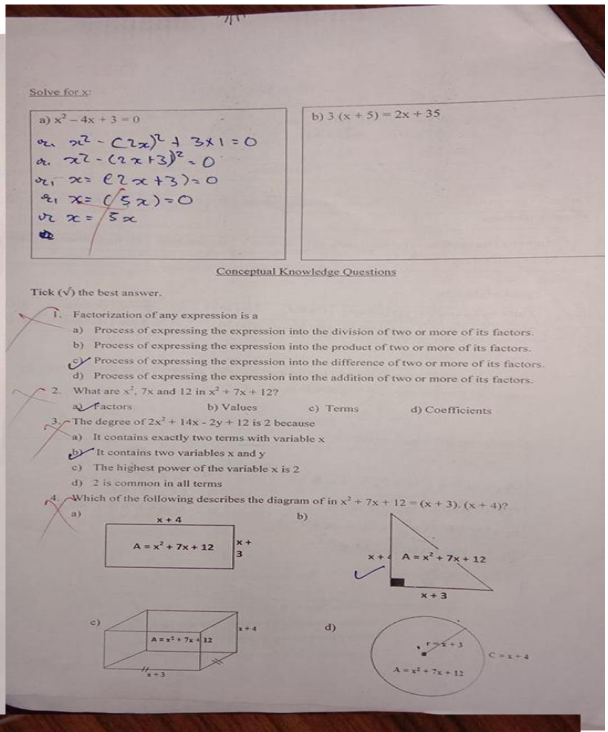
Factorize the following expressions:

b) $x^2 + 7x + 12$

or, $x^2 + (4+3)\ 12 \times 1$
or, $x^2 + (4x + 3x + 12$
or, $(x + 4x) + 3(x + 4x)$
or, $(x + 4x) + (4x)^2$
or, $x^2$

a) $4x^2 - 64$

or, $(2x)^2 - 16^2$
or, $(x - 16)^2$
or, $(x - 16)(x + 16)$



Solve for x:

a) $x^2 - 4x + 3 = 0$

or, $x^2 - (2x)^2 + 3 \times 1 = 0$
or, $x^2 - (2x + 3)^2 = 0$
or, $x = (2x + 3) = 0$
or, $x = (5x) = 0$
or, $x = 5x$

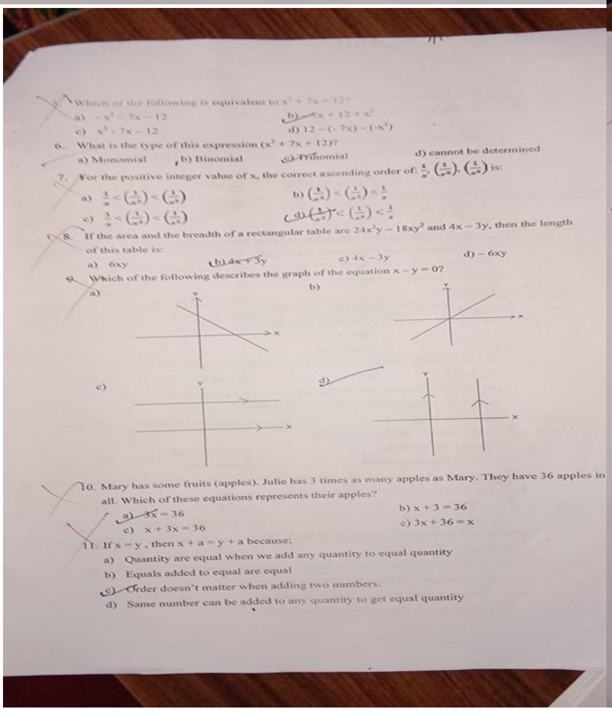
b) $3(x + 5) = 2x + 35$

Conceptual Knowledge Questions

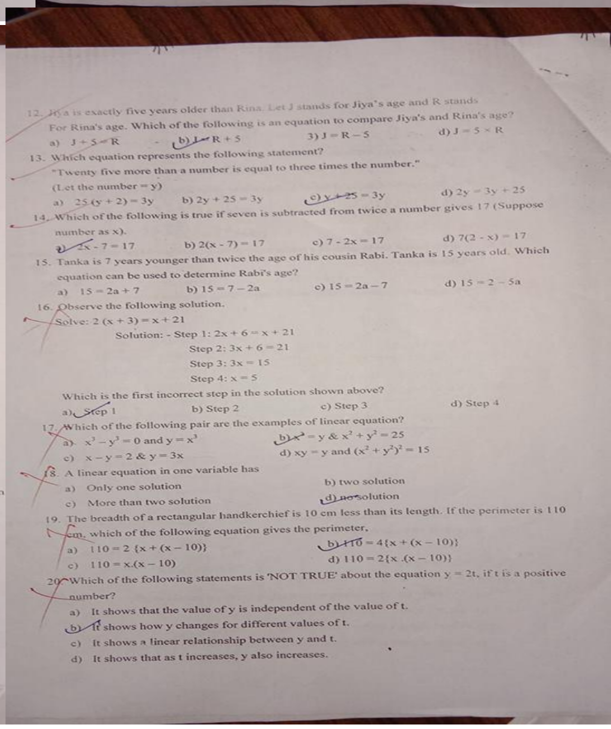
Tick (√) the best answer.

1. Factorization of any expression is a
   a) Process of expressing the expression into the division of two or more of its factors.
   b) Process of expressing the expression into the product of two or more of its factors.
   c) Process of expressing the expression into the difference of two or more of its factors.
   d) Process of expressing the expression into the addition of two or more of its factors.
2. What are $x^2$, 7x and 12 in $x^2 + 7x + 12$?
   a) Factors b) Values c) Terms d) Coefficients
3. The degree of $2x^2 + 14x - 2y + 12$ is 2 because
   a) It contains exactly two terms with variable x
   b) It contains two variables x and y
   c) The highest power of the variable x is 2
   d) 2 is common in all terms
4. Which of the following describes the diagram of in $x^2 + 7x + 12 = (x + 3).(x + 4)$?
   a) b) c) d)





5. Which of the following is equivalent to $x^2 + 7x + 12$?
   a) $-x^2 - 7x - 12$ b) $7x + 12 + x^2$
   c) $x^2 - 7x - 12$ d) $12 - (-7x) - (-x^2)$
6. What is the type of this expression $(x^2 + 7x + 12)$?
   a) Monomial b) Binomial c) Trinomial d) cannot be determined
7. For the positive integer value of x, the correct ascending order of: $\frac{1}{x}, \left(\frac{1}{x}\right), \left(\frac{1}{x}\right)$ is:
   a) b) c) d)
8. If the area and the breadth of a rectangular table are $24x^2y - 18xy^2$ and $4x - 3y$, then the length of this table is:
   a) 6xy b) $4x + 3y$ c) $4x - 3y$ d) $-6xy$
9. Which of the following describes the graph of the equation $x - y = 0$?
   a) b) c) d)
10. Mary has some fruits (apples). Julie has 3 times as many apples as Mary. They have 36 apples in all. Which of these equations represents their apples?
    a) $3x = 36$ b) $x + 3 = 36$
    c) $x + 3x = 36$ d) $3x + 36 = x$
11. If $x = y$, then $x + a = y + a$ because:
    a) Quantity are equal when we add any quantity to equal quantity
    b) Equals added to equal are equal
    c) Order doesn't matter when adding two numbers.
    d) Same number can be added to any quantity to get equal quantity



12. Jiya is exactly five years older than Rina. Let J stands for Jiya's age and R stands for Rina's age. Which of the following is an equation to compare Jiya's and Rina's age?
    a) $J + 5 = R$ b) $J = R + 5$ c) $J = R - 5$ d) $J = 5 \times R$
13. Which equation represents the following statement?
    "Twenty five more than a number is equal to three times the number."
    (Let the number = y)
    a) $25(y + 2) = 3y$ b) $2y + 25 = 3y$ c) $y + 25 = 3y$ d) $2y = 3y + 25$
14. Which of the following is true if seven is subtracted from twice a number gives 17 (Suppose number as x).
    a) $2x - 7 = 17$ b) $2(x - 7) = 17$ c) $7 - 2x = 17$ d) $7(2 - x) = 17$
15. Tanka is 7 years younger than twice the age of his cousin Rabi. Tanka is 15 years old. Which equation can be used to determine Rabi's age?
    a) $15 = 2a + 7$ b) $15 = 7 - 2a$ c) $15 = 2a - 7$ d) $15 = 2 - 5a$
16. Observe the following solution.
    Solve: $2(x + 3) = x + 21$
    Solution: - Step 1: $2x + 6 = x + 21$
    Step 2: $3x + 6 = 21$
    Step 3: $3x = 15$
    Step 4: $x = 5$
    Which is the first incorrect step in the solution shown above?
    a) Step 1 b) Step 2 c) Step 3 d) Step 4
17. Which of the following pair are the examples of linear equation?
    a) $x^2 - y^2 = 0$ and $y = x^3$ b) $x = y$ & $x^2 + y^2 = 25$
    c) $x - y = 2$ & $y = 3x$ d) $xy = y$ and $(x^2 + y^2)^2 = 15$
18. A linear equation in one variable has
    a) Only one solution b) two solution
    c) More than two solution d) no solution
19. The breadth of a rectangular handkerchief is 10 cm less than its length. If the perimeter is 110 cm, which of the following equation gives the perimeter.
    a) $110 = 2\{x + (x - 10)\}$ b) $110 = 4\{x + (x - 10)\}$
    c) $110 = x.(x - 10)$ d) $110 = 2\{x.(x - 10)\}$
20. Which of the following statements is 'NOT TRUE' about the equation $y = 2t$, if t is a positive number?
    a) It shows that the value of y is independent of the value of t.
    b) It shows how y changes for different values of t.
    c) It shows a linear relationship between y and t.
    d) It shows that as t increases, y also increases.

*The End!!!*